Problem Set n^o 8 MAT 883: Knot Theory Due December 6, 2015

(1) Derive a succinct formula for the determinant of the pretzel link $P(p_1, \dots, p_n)$.
(2) Prove that if B and W are chessboard surfaces for a connected link projection D, then $b_1(B) + b_1(W) = c(D)$.
(3) Prove that the signature of a non-trivial positive link is negative.
(4) Derive Howie's theorem: a knot $K \subset S^3$ is alternating if and only if there exist spanning surfaces $B, W \subset X_K$ such that $\chi(B) + \chi(W) + \frac{1}{2} i(\partial B, \partial W) = 2$. Here $i(\cdot, \cdot)$ denotes the geometric intersection number between a pair of slopes on a torus.
(5) Derive Banks and Hirasawa-Sakuma's theorem: every minimum genus Seifert surface for a special alternating knot is obtained by applying Seifert's algorithm to a special alternating diagram of it.