Next week: Class W, F

p. 1

3/31/14

Discussing $CFK^\infty(Y,K)$ for $K \subseteq Y$
$(\mathbb{Z}\oplus\mathbb{Z})$-filtered   nullhomol.? $b_1(Y)=0$.

<u>Claim</u>: $CFK^\infty(Y,K)$ has data not only of

- $\mathbb{Z}$-filtered complex $\widehat{CF}(Y,K)$
  (also $\underbrace{CF^\infty(Y,K),\ CF^-(Y,K),\ CF^+(Y,K)}_{\substack{\text{induced } \mathbb{Z}\text{-filtrations} \\ \text{on } CF^\infty(Y),\, CF^-(Y),\, CF^+(Y),\ \text{resp.}}}$)

but also of

- $CF(Y_N(K))$ ($\wedge, \infty, +, -$ versions) for $|N| >> 0$,
- Maps induced by 2-handle cobordisms
  (assume $N \in \mathbb{Z}^+$)

$$CF(Y) \longrightarrow CF(Y_{-N}(K))$$

$$CF(Y_N(K)) \longrightarrow CF(Y).$$

<u>First, some notation</u>: Fix admissible 2-pointed H.D. $(\Sigma, \vec{\alpha}, \vec{\beta}, w, z)$ compatible with (Y,K).

Denote by: $\mathcal{C} := CFK^\infty(Y,K)$

For $\mathfrak{s} \in Spin^c(Y)$, $\mathcal{C}[\mathfrak{s}] := CFK^\infty(Y,K,\mathfrak{s})$

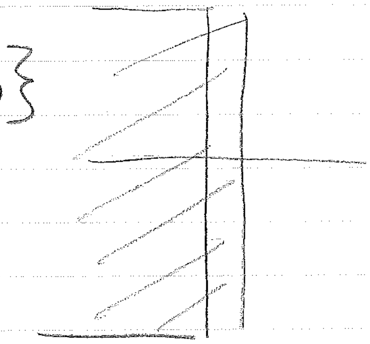
For any subset $S \subseteq \mathbb{Z}\oplus\mathbb{Z}$ that defines a <u>subquotient complex</u> of $\mathcal{C}$
quotient of a subcomplex by another subcpx.

(i.e. for which $\exists$ $S_1 \subseteq S_2 \subseteq \mathbb{Z}\oplus\mathbb{Z}$ such that

- S_1, S_2 are closed under partial order: $(i,j) \in S_* \Rightarrow (i',j') \in S_*$ $\forall$ $(i',j') \leq (i,j)$
  &
- $S = S_2 - S_1$, so

$\mathcal{F}_{S_1} \subseteq \mathcal{F}_{S_2} \subseteq \mathcal{C}$, and $\mathcal{F}_{S_2}/\mathcal{F}_{S_1}$ is a subquotient)

$$\mathcal{C}\{S\} := \mathcal{F}_{S_1} / \mathcal{F}_{S_2}.$$



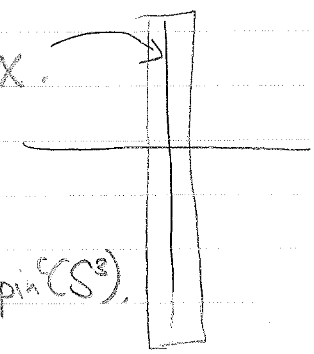
<u>Example</u>: Consider $S = \{(i,j) \in \mathbb{Z}\oplus\mathbb{Z} \mid i = 0\}$

Then $S = S_1 - S_2$ for

$S_1 = \{(i,j) \in \mathbb{Z}^2 \mid i \leq 0\}$

$S_2 = \{(i,j) \in \mathbb{Z}^2 \mid i < 0\}$.

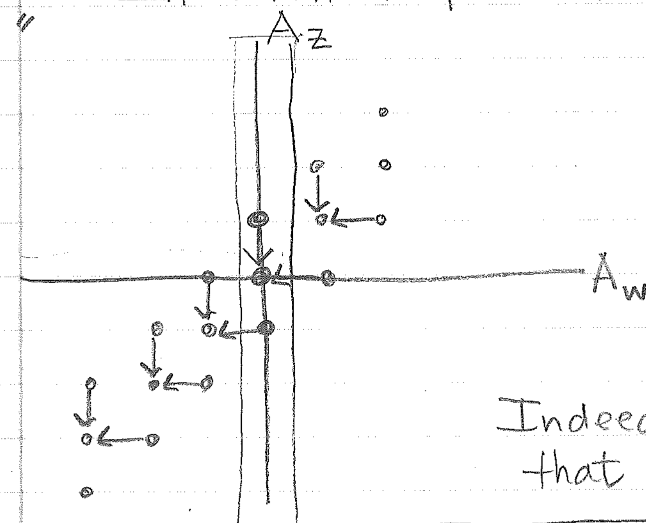
So S defines a subquotient complex.



$\mathcal{C}\{i=0\}$

Shorthand: Drop the "$(i,j) \in \mathbb{Z}^2 \mid$"

In L.H. trefoil example: Unique $s \in Spin^c(S^3)$.



(Check!)

Indeed: It's clear by definition that $\forall$ pairs (Y,K), $s \in Spin^c$

$$\boxed{\mathcal{C}[s]\{i=0\} \cong \widehat{CF}(Y,K,s)}$$

as $\mathbb{Z}$-filtered complexes (we are only considering those holomorph. disks with domains ϕ satisfying $n_w(\phi) = 0$).

BEWARE: Maslov-(hom.) grading not really evident on these CFK^∞ diagrams, beyond info. we get from arrows.

Also: $\boxed{\mathcal{C}[s]\{i=0, j \leq m\} \cong \mathcal{F}_m[s]}$

3/31/14

In fact, we can also compute

"large N" surgeries.

$$\widehat{HF}(Y_N(K)) \text{ for } |N| >> 0$$

as the homology of certain natural subquotient complexes of $CFK^\infty(Y, K)$.

Recall: Let $n \in \mathbb{Z}$. $H_1(Y_n(K)) \cong H_1(Y) \times \mathbb{Z}/n\mathbb{Z}$

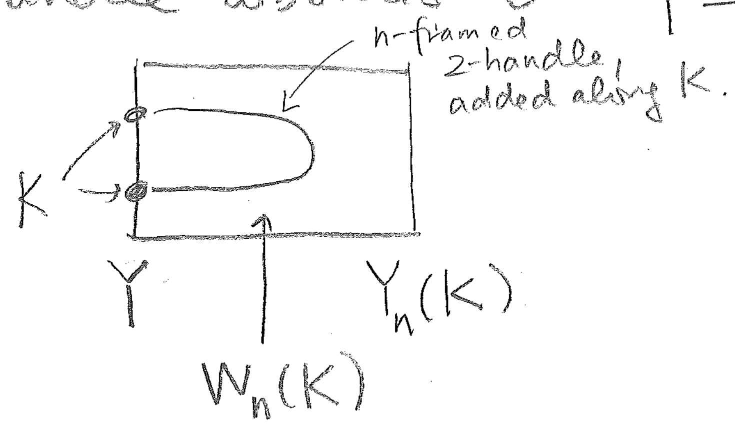
generated by μ (but $n\mu = \lambda = 0$)

Indeed, we can make an identification

$$\mathrm{Spin}^c(Y) \times \mathbb{Z}/n\mathbb{Z} \longrightarrow \mathrm{Spin}^c(Y_n(K))$$

$$(s, [m]) \longmapsto s_m$$

as follows. Let $W_n(K)$ be the 4D 2-handle cobordism $Y \to Y_n(K)$.



There is a unique $t \in \mathrm{Spin}^c(W_n(K))$
- extending $s \in \mathrm{Spin}^c(Y)$
- satisfying $\langle c_1(t), [F] \rangle + n = 2m$.

Will try to post some notes on
Spin/Spin^c structures following Gompf–Stipsicz

(Recall $\mathrm{Spin}^c(W) \xrightarrow{c_1}$ Characteristic elts.
of Inters. form Q_W
since they're integ. lifts of w_2

Now restrict t to $Y_n(K)$.

Theorem: (4.1 & 4.4 in O-S Knots; see also Rasmussen's thesis, Chp. 4)

Let $K \subseteq Y^3$ a nullhomolog. knot in a c.c.o. 3-mfd. Y^3 w/ $b_1(Y)=0$. Then $\exists\ N \in \mathbb{Z}^+$ such that $\forall\ p \geq N$, we have isomorphisms

$$\widehat{HF}(Y_{-p}(K), \mathfrak{s}_m) \cong H_*(\mathcal{C}[\mathfrak{s}]\{\min(i, j-m)=0\})$$

$$\widehat{HF}(Y_p(K), \mathfrak{s}_m) \cong H_*(\mathcal{C}[\mathfrak{s}]\{\max(i, j-m)=0\})$$

where in the above $m \in \mathbb{Z}$ satisfies $-\frac{p}{2} < m \leq \frac{p}{2}$ and $\mathfrak{s}_m \in \mathrm{Spin}^c(Y_{-p}(K))$ is as described above.

Theorem is saying: For $p \in \mathbb{Z}^+$ sufficiently large, we can compute

$$\widehat{HF}(Y_{-p}(K), \mathfrak{s}_m)$$

$-\frac{p}{2} < m \leq \frac{p}{2}$

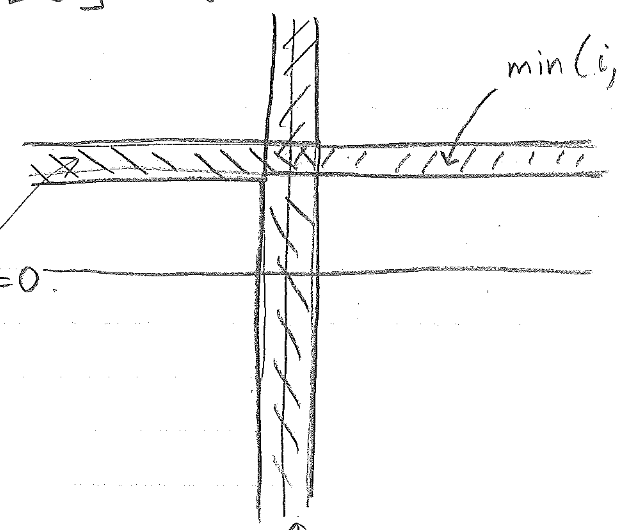
by looking @ the mth "min-hook" on the page of $CFK^\infty(Y, K)$ corresponding to $\mathfrak{s} \in \mathrm{Spin}^c(Y)$ and

$$\widehat{HF}(Y_p(K), \mathfrak{s}_m)$$

by looking @ the mth "max-hook" on $\mathfrak{s}$-page.

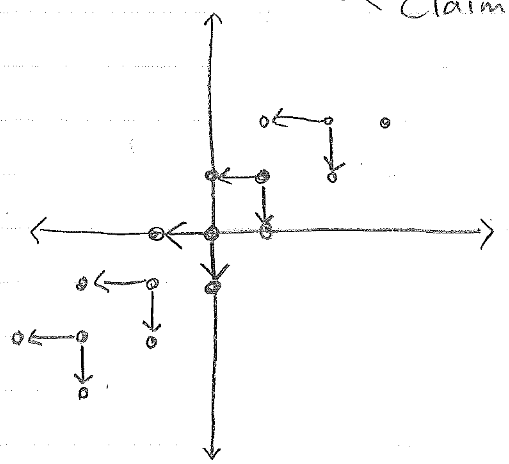
$\mathcal{C}[s]$: $CFK^\infty(Y, K)$ on page $s \in Spin^c(Y)$

3/31/14



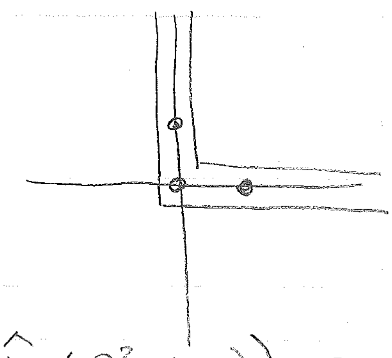
Example : S^3_{-5} (RH trefoil)

Claim: sufficiently large.



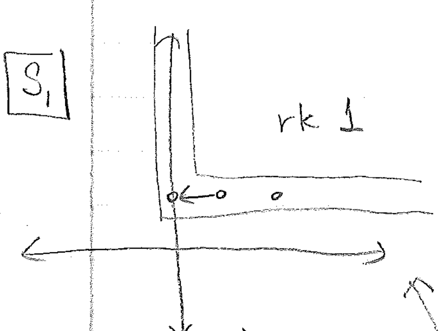
5 spin^c structures : $s_{-2}, s_{-1}, s_0, s_1, s_2$.

$\boxed{s_0}$

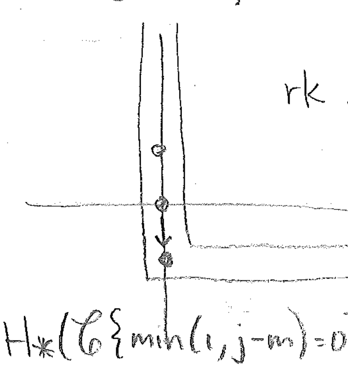


$\mathrm{rk}(\widehat{HF}(S^3_{-5}(K))) = 3.$

$\boxed{s_1}$



$\boxed{s_{-1}}$



Stabilizes :

$H_*(\mathcal{C}\{\min(i, j-m)=0\}) = H_*(\quad)$ $\forall\, m > 1.$

$H_*(\mathcal{C}\{\min(i, j-m)=0\}) = H_*(\quad)$ $\forall\, m < -1.$