$\boxed{4/9/14}$

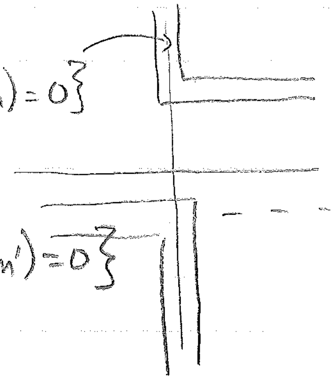
Recall: Given $(\underset{\uparrow\, b_1(Y)=0}{Y}, \underset{\text{nullhomol.}}{K})$, we would like to understand

$p \in \mathbb{Z}^+$, $p >> 0$

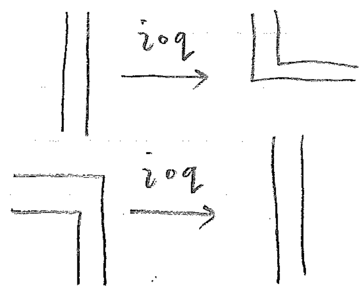
Theorem stated last time.

$\widehat{CF}(Y_{-p}(K), s_m) \cong \mathcal{C}[S]\{\min(i, j-m)=0\}$

$\widehat{CF}(Y_p(K), s_{m'}) \cong \mathcal{C}[S]\{\max(i, j-m')=0\}$

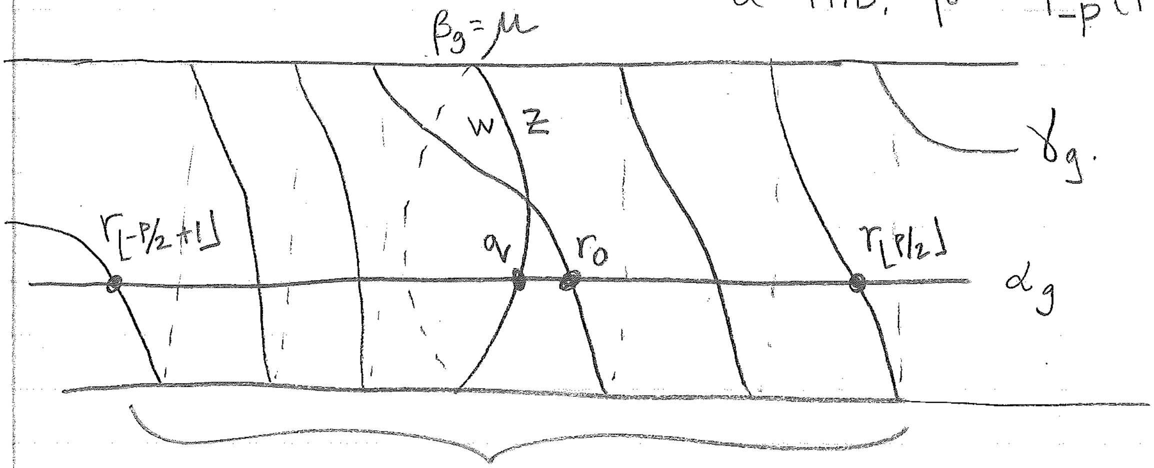


and cobordism maps and



Topological setup: $(\Sigma, \vec{\alpha} = \vec{\alpha}_0 \cup \alpha_g, \vec{\beta} = \vec{\beta}_0 \cup \beta_g, w, z)$ a H.D. for (Y, K).

$(\Sigma, \vec{\alpha} = \vec{\alpha}_0 \cup \alpha_g, \vec{\gamma} = \vec{\beta}_0 \cup \gamma_g, w)$ a H.D. for $Y_{-p}(K)$.



Recall that we have

$$\mathfrak{s}_w : \mathbb{T}_{\vec{\alpha}} \cap \mathbb{T}_{\vec{\gamma}} \longrightarrow Spin^c(Y_{-p}(K)).$$

$$
\begin{array}{c|c}
 & \begin{array}{ccc} \beta_1 \cdots & \beta_{g-1} & \beta_g \end{array} \\
\begin{array}{c} \alpha_1 \\ \vdots \\ \alpha_{g-1} \\ \alpha_g \end{array} &
\left(\begin{array}{c|c} \mathbb{T}_{\vec{\alpha}_0} \cap \mathbb{T}_{\vec{\beta}_0} & \\ \hline & q \end{array}\right)
\end{array}
\qquad
\begin{array}{c|c}
 & \begin{array}{ccc} \beta_1 \cdots & \beta_{g-1} & \gamma_g \end{array} \\
\begin{array}{c} \alpha_1 \\ \vdots \\ \alpha_{g-1} \\ \alpha_g \end{array} &
\left(\begin{array}{c|c} \mathbb{T}_{\vec{\alpha}_0} \cap \mathbb{T}_{\vec{\beta}_0} & \\ \hline & r_i\text{'s} \end{array}\right)
\end{array}
$$

p of them

and our basepoints have been chosen so that for every $(\vec{x}_0, r_m) \in \mathbb{T}_\alpha \cap \mathbb{T}_\gamma$ supported in winding region, we have

i.e., of the form $(\vec{x}_0, r_m)$ for $\vec{x}_0 \in \mathbb{T}_{\vec{\alpha}_0} \cap \mathbb{T}_{\vec{\beta}_0}$

$$(*)\ s_w(\vec{x}_0, r_m) = s_m - A_{w,z}(\vec{x}_0, q)$$

(Think: For $(\vec{x}_0, q)$ of Alexander grading 0, $s_w(\vec{x}_0, r_m) = s_m$)

As usual, generators in $\mathbb{T}_{\vec{\alpha}} \cap \mathbb{T}_{\vec{\gamma}}$ split into "ε"-classes ("Spin^c"-classes)

ε-equivalent if $[\gamma_a - \gamma_b] = 0 \in H_1$

Identified with Spin^c structures once we fix base point w.

Following Rasmussen thesis, Sec. 4, we call an ε-class a "good" ε-class if all generators in this class are supported in the winding region.

Suppose $(\vec{x}_0, q) \in \mathbb{T}_\alpha \cap \mathbb{T}_\beta$ has $A_{w,z}(\vec{x}_0, q) = 0$, and $(\vec{x}_0, r_0)$ is in a good ε-class.

By $(*)$, all other generators in this class are of the form

$$(\vec{y}_0, r_{A_{w,z}(\vec{y}_0, q)})$$

index on r is the Alexander grading of $(\vec{y}_0, q)$, by correspondence

$$\underset{s_m}{\mathrm{Spin}^c(Y_0(K))} \xrightarrow{\text{quotient}} \underset{s_m}{\mathrm{Spin}^c(Y_{-p}(K))}$$

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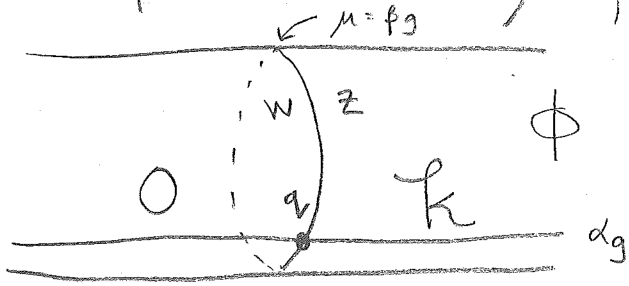
How to understand this: If $A_{w,z}(\vec{y}_0, q) = k > 0$, then we have a domain

$$\phi \in \pi_2((\vec{y}_0, q), (\vec{x}_0, q)) \subseteq (\Sigma, \vec{\alpha}, \vec{\beta})$$

w/ local picture near $\mu = \beta_g$:



$\subseteq (\Sigma, \vec{\alpha}, \vec{\beta})$

For Y

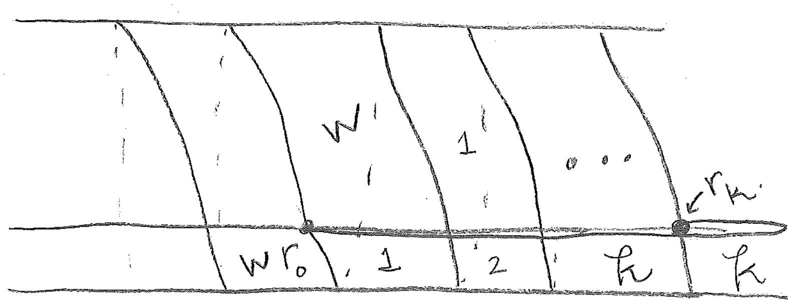
We have an analogous "perturbed"

$$\phi' \in \pi_2((\vec{y}_0, r_k), (\vec{x}_0, r_0)) \subseteq (\Sigma, \vec{\alpha}, \vec{\gamma})$$

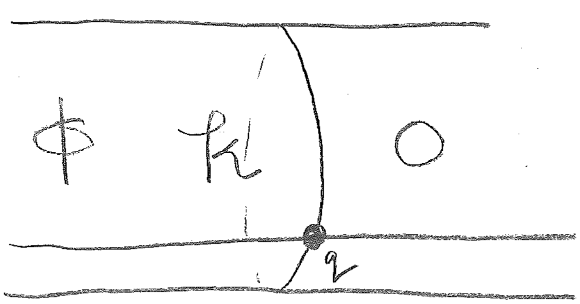
Index = Alexander grading of corresponding gen. in $\mathbb{T}_\alpha \cap \mathbb{T}_\beta$

For $Y_{-p}(K)$

"R" domains



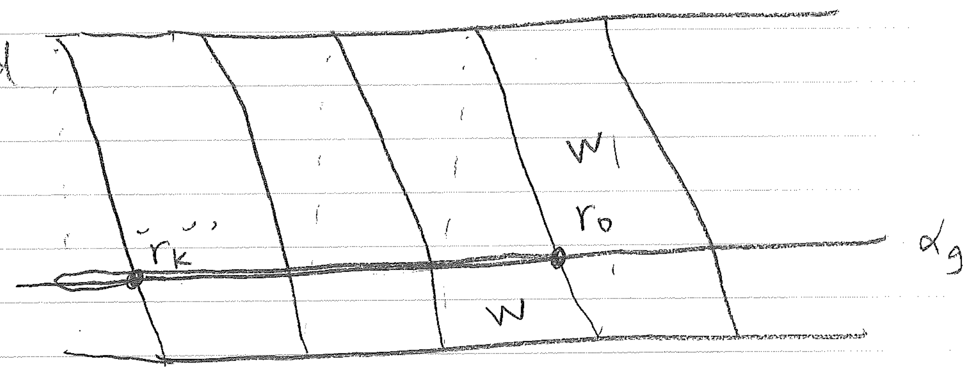
Similarly, if $A_{w,z}(\vec{y}_0, q) = k < 0$, we have:



$\subseteq (\Sigma, \vec{\alpha}, \vec{\beta})$

"L" domains

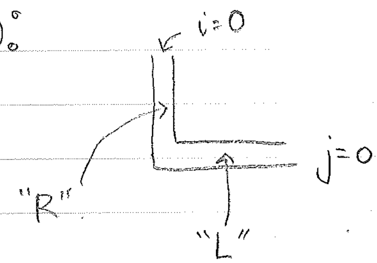
and



Since γ is a small perturbation (smoothing) of $\lambda - p\mu$ (and λ looks like α_g near μ),

ϕ admits a hol. representative iff ϕ' does.

(Notice that both the "R" and "L" domains ϕ' miss the w basepoint):



To get the other min-hooks, move basepoint w either R (get fewer "R" domains, so hook moves up) or L (get fewer "L" domains, so hook moves down).

Q1: Why does $\exists$ a good ε-class for $|p| >> 0$?
The set $\{A(\vec{x}) - A(\vec{y}) \mid \vec{x}, \vec{y} \in \mathbb{T}_\alpha \cap \mathbb{T}_\beta\}$ is bounded independent of p, and we've seen there is a nice relationship

A-gradings from $(\Sigma, \vec{\alpha}, \vec{\beta}, w, z)$ $\longleftrightarrow$ Spin^c structure on $Y_p(K)$ from $(\Sigma, \vec{\alpha}, \vec{\gamma}, w)$.

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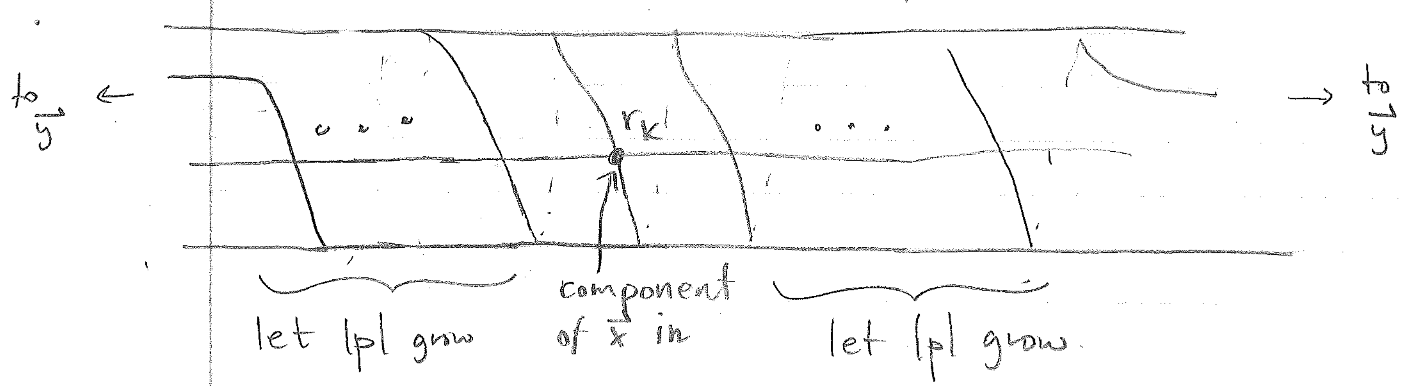
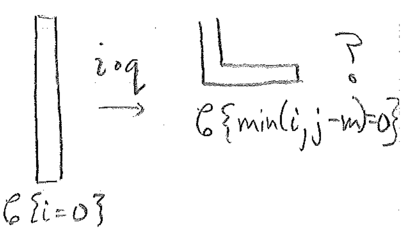
Fix an ε-class (+ basepoint w) $\Rightarrow$ $\mathfrak{s} \in \mathrm{Spin}^c(Y_{-p}(K))$.
We know $\exists$ $\vec{x}$ in winding region w/ $s_w(\vec{x}) = \mathfrak{s}$.
(since all spin^c structures on $Y_p(K)$ represented there)

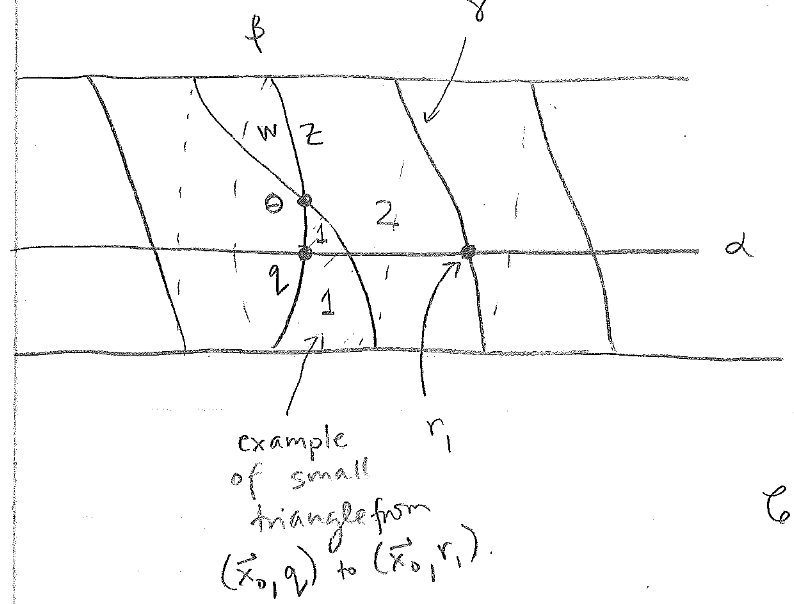
Suppose $\exists$ $\vec{y}$ not in winding region w/ $s_w(\vec{y}) = \mathfrak{s}$.
Then $\varepsilon(\vec{x}, \vec{y}) = 0 \in H_1(Y_{-p}(K))$. But for $|p| >> 0$,



$\langle PD\,\varepsilon(\vec{x}, \vec{y}), [\hat{F}] \rangle \neq 0$ since $\varepsilon(\vec{x}, \vec{y})$ will have to pick up a large # of copies of μ to "escape" winding region.

Q2: Why are the cobordism maps:





We see obvious "small" triangles relating generators/ differentials in good ε-class w/ $C\{\min(i, j-m) = 0\} \subseteq CFK^\infty(Y_1, K)$.

If spiral sufficiently "tight" (energy in each triangle v. small compared to energy in any other triangles), use "Energy Filtration Lemma" (Lemma 9.10, O-S "3-man.") to conclude that this map induces an isomorph. on homology.

<u>Lemma</u>: If $F: A \to B$ is a chain map of $\mathbb{R}$-filtered complexes that can be decomposed as $F = F_0 + \ell$ where F_0 is filtration-preserving, ℓ filtration-lowering, and filtration on B is bounded below, then if F_0 induces an isomorphism on homology, then so does F.

(Think: F_0 the map counting small triangles
F " " " all "
$\mathbb{R}$-Filtration is energy, prop. to area of triangles).

<u>Proof</u>: Choose a (finite, since B bounded below) collection of $\mathbb{R}$-levels such that

$$F = F_0 + \underbrace{F_1 + \dots + F_N}_{\ell}$$

We have a corresponding spectral sequence whose E^1 page is $MC(F_0)$, converging to $MC(F)$. Since F_0 induces an isomorphism on homology, $H_*(MC(F_0)) = 0 \implies H_*(MC(F)) = 0$ □

Note also that each "L" triangle hits w so does not contribute to the cobordism map on the "^" complex. Any other Δ in its homotopy class is obtained from one of these by adding/subtracting Σ (changes Maslov index) or adding/subtracting triply-periodic domains (still hit w).