Mathematics W4052
Topics in Geometry and Topology: Knot Theory
Fall 2008

Instructor:	Eli Grigsby 525 Mathematics egrigsby@math.columbia.edu
Teaching Assistant:	Ina Petkova, ina@math.columbia.edu
Instructor Office hours: **(525 Mathematics)**	M 5:30 PM-6:30 PM Tu 1:30 PM-2:30 PM
TA Office hours:	M 3-4 PM (406 Mathematics) Tu 3-4 PM (408 Mathematics)

Course Materials: All announcements and handouts, including homework and reading assignments, etc., will be available through the course webpage at Courseworks: http://courseworks.columbia.edu

Course Outline: The study of algebraic and geometric properties of knots in the three-sphere, including but not limited to knot projections and Reidemeister's theorem, Seifert surfaces, braids, tangles, knot polynomials, and the fundamental group of knot complements. Depending on time and student interest, we will discuss more advanced topics like knot concordance, the relationship to 3-manifold topology, and other algebraic knot invariants.

Prerequisites: Linear Algebra (S2010z or similar) and Algebra (S4041xy) (may be taken concurrently). Topology (W4051x or similar) recommended but not required.

Text: Charles Livingston, *Knot Theory*, Volume 24, Carus Mathematical Monographs, on reserve at the math library. Any supplementary reading will be distributed either in class or at the course website. I also recommend Colin Adams, *The Knot Book* (easier), and Dale Rolfsen, *Knots and Links* (much harder, but a lot of fun).

Reading: During every lecture (and on the Courseworks page), I will assign reading to be completed by the following lecture. *All assigned reading is fair game for the exam, whether specifically addressed in lecture or not.* That said, I will do my best in lecture to emphasize the most relevant material.

Homework: Weekly problem sets will be assigned in class and posted on

Courseworks every Wednesday, to be turned in the following Wednesday at the beginning of class. Collaboration is encouraged for homework sets, but all work must be written up individually. *Late homeworks will not be accepted.*

Mini-lectures: At the end of every lecture, I will assign one person to give a 5-10 minute summary of the lecture at the beginning of the following class period. The purpose of this exercise is to:

1. ensure that I receive regular feedback on my lecturing pace,
2. ensure that you are sufficiently engaged in the lectures to synthesize the material,
3. give you practice discussing mathematics at the blackboard in front of others (perhaps the most important skill for a mathematician).

Midterm: There will be no midterm examinations for this course.

Final Paper and Presentation: Midway through the semester, you will each pick an advanced topic to investigate in greater depth. I will provide suggestions, but you should feel free to pick a topic on your own. You will then write a 4-6 page expository paper in clear, precise language and give a 20 minute presentation on the topic.

- Topic choice deadline: *Oct. 15*
- First draft deadline: *Nov. 10*
- Final draft deadline: *Nov. 24*
- Presentations: *Dec. 3-8*

Final Examination: There will be a take-home final, distributed in class on Monday, Dec. 8, due (either in my hands or slid under the door of my office) by 5 p.m. Friday, Dec. 12.

Grading: The course grades will be computed as follows: Homework 25%, Final Paper 20%, Final Presentation and Mini-lectures 20%, Take Home Final 35%.

Rough schedule of topics: (Flexible!)

W, 09/03	Motivation: Low-dimensional topology and knots
M, 09/08	What is a knot? Deformations, equivalence, orientations, diagrams
W, 09/10	Combinatorial techniques: Reidemeister moves and colorings
M, 09/15	More on colorings
W, 09/17	Oriented surfaces, triangulations, and classification
M, 09/22	Seifert surfaces, Seifert's algorithm, knot genus, and connected sum
W, 09/24	Morse theory and chain complexes
M, 09/29	More Morse theory and chain complexes, fundamental group
W, 10/01	Fundamental group of the knot complement
M, 10/06	Computing presentations of the knot group
W, 10/08	Symmetric group
M, 10/13	Knot labelings
W, 10/15	Seifert matrix, *Deadline: Topic choice*
M, 10/20	Linking number and the Alexander polynomial
W, 10/22	Braids, Bridges, Plats, Pretzels
M, 10/27	Numerical invariants, Interesting Knot Families
W, 10/29	Knot Symmetries
M, 11/03	*University Holiday*
W, 11/05	Knot concordance I
M, 11/10	Knot concordance II, *Deadline: First draft of paper*
W, 11/12	Skein theory and the Kauffman bracket
M, 11/17	Additional topic TBD
W, 11/19	Additional topic TBD
M, 11/24	Additional topic TBD, *Deadline: Final draft of paper*
W, 11/26	*Class to be rescheduled*
M, 12/01	Additional topic TBD
W, 12/03	*Final presentations, Day 1*
F, 12/05	*Final presentations, Day 2*
M, 12/08	*Final presentations, Day 3*, *Distributed: Take-home final*
F, 12/12	*Deadline: Take-home final, 5 p.m., Room 525 Mathematics*