Mathematics MT808
Geometry/Topology I: Algebraic Topology
Fall 2011

Instructor: Eli Grigsby
357 Carney
grigsbyj@bc.edu

Office hours: Mon 12:30-3:30
357 Carney

Course Meetings: MWF 10 AM - 11 AM
Gasson 208

Course Description: This is a first graduate course on geometry and topology, focusing on the basics of algebraic topology. Topologists are interested in those properties of abstract spaces that remain unchanged as the spaces are deformed without tearing or gluing. An effective way to study these properties is to associate algebraic objects (e.g., numbers, vector spaces, groups, rings) to the topological spaces and prove that these algebraic objects are invariant under deformations. If we are lucky, the *algebraic* properties of the invariants then give us information about the *topological* properties of the spaces.

The formal prerequisites for this course are undergraduate algebra and point-set topology. Topics covered will include (but are not limited to) homotopy, fundamental group, covering spaces, homology, cohomology, and Poincaré duality. To get a sense of whether you are adequately prepared for the course, I suggest reading the preface and skimming Chapter 0 of Hatcher's book (see below). A good reference for some of the point-set topology background (in addition to those references mentioned by Hatcher) is Chapters 2 and 3 of *Topology* by James R. Munkres. For the algebra, I like *Algebra* by Michael Artin.

Because of my own research interests, many of the examples I present will be motivated by questions in *low-dimensional topology*, the study of smooth manifolds of dimension ≤ 4. Since many of these topics lie outside of the scope of Hatcher's text (see below), I will offer additional references throughout the semester as needed.

Textbook: Chapters 0-3 of *Algebraic Topology* by Allen Hatcher, published by Cambridge University Press. Also available for free on-line at:
http://www.math.cornell.edu/ hatcher/AT/ATpage.html

Grading: There will be homework assignments due every 1-2 weeks, one midterm exam, and a take-home final. These will count toward the final grade as follows:

- Homework: 40 %
- Midterm: 20 %
- Final: 40 %

Course Material: All handouts for the course as well as announcements, homework and reading assignments, etc. will be available through the Blackboard Vista site for the course. I will also maintain a calendar on the site, warning you of upcoming due dates, quizzes, exams, etc. I will also regularly comment on the material in the "Discussion" area of the BB vista website. You are encouraged to make use of the "Discussion" feature of BB vista as well: to post and answer each others' questions, provide feedback on the pace/content of the course, organize study groups, etc. Please check the site frequently.

Reading: I will not be able to cover all details of the material in lecture, so I will rely on you to read the relevant sections of the text. Reading assignments will be posted in the calendar section of Blackboard Vista in advance of every lecture.

Homework: Unless your name is Will Hunting (and probably even if it is), you cannot learn the material in this course without doing problems. I will assign a lot (one long-ish assignment every 1-2 weeks posted in the calendar section of Blackboard Vista). I know that it is easy to get behind on homework, but please keep in mind that expending effort on the homework is the surest path to succeeding in the course (both in terms of your knowledge and your grade). You are *strongly* encouraged to work together on problems. However, each person must write up and submit his/her own solutions. *Late homework will be accepted, but can only receive a maximum score of 50%.* In addition, it may not be possible for me to grade late homework in a timely fashion.

Academic integrity policy: Thanks to Lord Google, it's pretty easy to find solutions on-line to almost any problem I can think to assign. My advice: don't. You won't learn any algebraic topology that way, and though you may ace the homework, you'll struggle with the exams. I encourage you to read BC's policy on academic integrity at www.bc.edu/integrity.

Rough schedule of topics:

Week 1:	9/7	Introduction to Algebraic Topology with Motivating Examples
	9/9	Homeomorphism, Homotopy-Equivalence, CW complexes
Week 2:	9/12	Fundamental Group 1: Definition & First Examples
	9/14	FG 2: Applications (Brouwer FP theorem, Fund. Thm. of Algebra)
	9/16	FG 3: Invariance under homotopy equivalence, dependence on basepoint
Week 3:	9/19	FG 4: Van Kampen Theorem
	9/21	FG 5: First Applications of VKT (S^n, CW complexes)
	9/23	FG 6: More Apps of VKT (knot group, braid group)
Week 4:	9/26	Covering Spaces 1: Definition & First Examples
	9/28	CS 2: Path lifting, homotopy lifting, FG action
	9/30	CS 3: More Examples
Week 5:	10/3	CS 4: Classification
	10/5	Homology 1: Simplicial homology & Examples
	10/7	Hom 2: Singular homology
Week 6:	10/10	Columbus Day: No class
	10/12	Hom 3: Examples
	10/14	Hom 4: Homotopy invariance of homology
Week 7:	10/17	Midterm?
	10/19	Hom 5: H_1 is the abelianization of π_1 for path connected spaces
	10/21	Hom 6: Relative homology, Excision, Long Exact Sequence of a Pair
Week 8:	10/24	Hom 7: Snake Lemma, Mayer-Vietoris, Five Lemma
	10/26	Hom 8: Cellular homology of a CW complex, Degrees of sphere maps
	10/28	Hom 9: Euler characteristic, Lefschetz Fixed Point Theorem
Week 9:	10/31	Hom 10: Homology with Coefficients, Universal Coefficient Theorem
	11/2	Hom 11: Künneth theorems
	11/4	Hom 12: More Examples
Week 10:	11/7	Cohomology 1: Definition, Properties, & First Examples
	11/9	Cohom 2: Long Exact Sequences and Universal Coefficient Theorem
	11/11	Cohom 3: Cup Product, Künneth theorems
Week 11:	11/14	Cohom 4: Orientations and & Examples
	11/16	Cohom 5: More Orientations, Fundamental Class
	11/18	Cohom 6: Brief Introduction to Smooth Manifolds
Week 12:	11/21	Cohom 7: Introduction to Poincarè Duality
	11/23	Thanksgiving holiday: No class
	11/25	Thanksgiving holiday: No class
Week 13:	11/28	Cohom 8: Cap product, Compactly supported cohomology
	11/30	Cohom 9: Duality between cup product and intersection of submanifolds
	12/2	Cohom 10: Poincareè-Lefschetz Duality for manifold with boundary
Week 14:	12/5	Cohom 11: Alexander duality
	12/7	Cohom 12: Additional topics (e.g., Poincaré-Hopf index theorem)
	12/9	Cohom 13: Additional topics (apps. to low-dimensional topology)