Midterm 1 will cover the material of all lectures up to and including February 14. More precisely, the following topics are covered:

1. Divisibility. Division algorithm.
2. Euclidean algorithm. Greatest common divisors.
3. Prime numbers.
4. Uniqueness of factorization.
7. Euler’s $\phi$-function.
10. Euler’s criterion.

To prepare for the midterm, review your lecture notes and redo the first three problem sets (also read and work through the posted solutions).

Here are a few extra practice problems:

**Problem 1.** Find the greatest common divisor of 72 and 231. Write it in the form $d = 72x + 231y$.

**Problem 2.** Write 1 as a linear combination of 12 and 5. What is the multiplicative inverse of 5 modulo 12?

**Problem 3.** Find a number $x$ which satisfies
\[
\begin{align*}
  x &\equiv 5 \pmod{7} \\
  x &\equiv 2 \pmod{4} \\
  x &\equiv 2 \pmod{3}
\end{align*}
\]

**Problem 4.** Let $p$ be a prime. For which values of $k$ does $p$ divide $\binom{p}{k}$?

**Problem 5.** Which of the following numbers are representable as a sum of two integer squares: 41, 122, 150?

**Problem 6.** Give a definition of Euler’s $\phi$-function. Compute $\phi(360)$.

**Problem 7.** Does equation $x^4 - x^3 + 1 = 0$ have any integer solutions?

**Problem 8.** Solve $x^3 - 2x + 4 \equiv 0 \pmod{3^3}$.

**Problem 9.** Let $p > 2$ be a prime. Prove that the map $x \mapsto x^{p-2}$ is a bijection of the complete residue system modulo $p$ onto itself.