MT 430 Intro to Number Theory
PROBLEM SET 1

Due Thursday 1/24

Problem 1. Find the greatest common divisor of 2940 and 4004.

Problem 2. Write the greatest common divisor of 198 and 423 in the form

\[ 198x + 423y, \]

where \( x, y \in \mathbb{Z} \).

Problem 3. Find numbers \( x \) and \( y \) satisfying \( 43x + 64y = 1 \).

Problem 4. Prove that the product of 3 consecutive integers is divisible by 6.

Problem 5. Prove that \( 4 \nmid (n^2 + 2) \) for any integer \( n \).

Problem 6. Prove that if \( n \) is odd, then \( n^2 - 1 \) is divisible by 8.

Problem 7. Prove that any square is of the form \( 3k \) or \( 3k + 1 \) but not of the form \( 3k + 2 \).

Problem 8. Show that \( (n! + 1, (n + 1)! + 1) = 1 \).

Problem 9. Prove that for every integer \( n \geq 1 \) one has the equality

\[
1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{1}{2n-1} - \frac{1}{2n} = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}
\]