MT 430 Intro to Number Theory

PROBLEM SET 5

Due Thursday 3/21

Problem 1. Expand the following rational numbers into finite simple continued fractions:
\[
\frac{17}{3}, \frac{3}{17}, \frac{35}{8}.
\]

Problem 2. Convert the following continued fractions into rational numbers:
   (1) \langle 2, 1, 4 \rangle
   (2) \langle -3, 2, 12 \rangle
   (3) \langle 0, 1, 1, 100 \rangle

Problem 3. Given positive integers \(b, c, d\) with \(c > d\), prove that \(\langle a, c \rangle < \langle a, d \rangle\) but \(\langle a, b, c \rangle > \langle a, b, d \rangle\) for any integer \(a\).

Problem 4. Let \(a_0\) be a real number and let \(a_1, a_2, \ldots, a_n\) and \(c\) be positive real numbers. Prove that
\[
\langle a_0, a_1, \ldots, a_n \rangle > \langle a_0, a_1, \ldots, a_n + c \rangle
\]
holds if \(n\) is odd, but is false if \(n\) is even.

Problem 5. Evaluate the infinite continued fraction \(\langle 1, 1, 1, 1, \ldots \rangle\).

Problem 6. Expand each of the following irrational numbers into infinite simple continued fractions:
\[
\sqrt{2}, \sqrt{3}, \sqrt{2} - 1.
\]