1. Consider the function \( f(x) = x^2e^{-2x} \), defined for \(-\infty < x < \infty\).

a) Compute \( f'(x) \) and find the point(s) \( x \) where \( f'(x) \) is positive, negative and zero.

b) Compute \( f''(x) \) and find the point(s) \( x \) where \( f''(x) \) is positive, negative and zero.

c) Compute the limits of \( f(x) \) as \( x \to \infty \) and \( x \to -\infty \).

d) Use your results from a)-c) to sketch the graph of \( f(x) \), labelling the points you found in a) and b).

2. Repeat Problem 1 for the function \( f(x) = \frac{x}{\log x} \), defined for \( x > 0 \). (In c) take only the limit as \( x \to \infty \).)

3. Suppose \( f(x) \) is a differentiable function.

a) Suppose \( f'(x) = (\cos x)f(x) \). Prove that \( f(x) = f(0)e^{\sin x} \).

b) Suppose \( f'(x) = (\sin x)f(x) \). State and prove a formula for \( f(x) \) analogous to that in part a).

4. In class we showed that \( \lim_{h \to 0} (1 + h)^{1/h} = e \).

a) Let \( x \) be any real number. Compute the limit
\[
\lim_{h \to 0} (1 + xh)^{1/h}.
\]
(Hint: Change to a limit as \( k \to 0 \), where \( k = xh \).)

b) Use a) to compute the limit
\[
\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n.
\]

5. Compute the following two limits.

a) \( \lim_{x \to 0^+} x \log x \). [Hint: Let \( x = e^{-y} \) and let \( y \to \infty \), using a result from class.]

b) \( \lim_{x \to 0^+} x^y \).

6. As a running coach, you are aware that aerobic exercise at high altitude is more difficult than at sea level because the drop in atmospheric pressure retards oxygen exchange in the lungs. Assume that the rate of change with respect to height of atmospheric pressure at any height is proportional to the pressure there. If the pressure is 100 mmHg at sea level and 90 mmHg at 5000 feet above sea level, what is the pressure at 10000 feet above sea level? (Give an exact answer without using a calculator.)