MT 320/01 Introduction to Abstract Algebra
Spring 2010 Syllabus

Schedule: MWF 1 pm, Fulton 425
Instructor: Prof. Mark Reeder, office: Carney 322
email: reederma@bc.edu
Office hours: MW 2-3, Th 3-4 (subject to change) and by appointment.
Course website:

`http://www2.bc.edu/~reederma/310S10.html`

Text: *Abstract Algebra: Theory and Applications* by Thomas Judson.
This is available for free download as a PDF file, at

`http://abstract.ups.edu/download.html`

A reformatted version with smaller margins and 100 fewer pages is available at

`http://fmwww.bc.edu/gross/MT216/aata.pdf`

We will cover all or most of the material in chapters 3,4,5,8,9,10,11,12,14,15,19,21.

Homework: It will be assigned each week, and part of it will be collected for grading about one week later. HOMEWORK WILL NOT BE ACCEPTED AFTER THE DAY IT IS DUE. Your lowest homework score will be dropped, so you can miss one assignment without harming your grade. The homework you hand in for grading must be typed in LaTeX (see course website).

In-class presentations: At the beginning of class, students can volunteer or be volunteered to present homework solutions on the board, thereby earning additional homework points.

Exams: We will have two in-class exams, and a final exam. Exams must be taken as scheduled, except for documented illness or family emergency.

If you have special issues regarding exams, I need to hear about them this month, with documentation from the appropriate offices of Boston College.

In-Class Exam Dates: **Friday Feb 19**, **Friday April 9**

Final Exam: Monday May 17, 9:00 am.

Grading: Homework: 30%, Exams: 20% each, Final: 30%.

After the final exam, these weights are used to combine your homework and exam scores into a number between 0 and 100. The highest score will probably be an A, and similar scores will receive similar grades. This procedure determines the rest of the grades. Individual exam scores are not curved. They just contribute to the final score, which is "curved" as described above.

Course Description

In high school, we learn *al-jabr* (an Arabic word: الجابر, meaning "to put back together") as the art of manipulating equations to solve for x. For example, to solve a quadratic equation, we use the quadratic formula:

$$ax^2 + bx + c = 0 \qquad \Rightarrow \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This raises some questions.

- What if $b^2 - 4ac < 0$? In that case we must use complex numbers, like $2 + 5i$. But what is i? Is it magic, or just made up? In fact, the meaning of i is geometric: i can be viewed as "rotation by 90 degrees". Every complex number can be understood similarly, without recourse to magic. So algebra leads to complex numbers, which are themselves some kind of geometric actions.

- How do we decide which of the signs $\pm$ to take? If the roots are $2 + \sqrt{2}$ and $2 - \sqrt{2}$, then neither is better than the other, unless the problem has additional constraints. So we should take both roots, understanding that there is a symmetry that exchanges them. Thus, Algebra leads to symmetry of the roots of polynomials.

- What about polynomials of degrees larger than 2? There would be more more symmetries because there are more roots. Sometimes there is a formula for the roots and sometimes not. And even if there is a formula, it can be too complicated to use. The symmetries of the roots can usually be found, however. And whether there is a formula for the roots depends on the structure of these symmetries. So Algebra leads to *structure*, in this case Group Theory, which can be computed, and away from numbers which cannot be known in an elementary sense.

Thus, to understand the quadratic formula and its generalization to higher degrees, we have to understand Algebra from a structural point of view. In this course, we'll begin with the structure of Groups: First by example, then by abstract theory. This is like passing from "three chickens" or "three eggs" to understanding the Platonic Three, which appears in many forms. Likewise, we can think of Groups in the Platonic realm, from which they manifest as symmetry in many ways. This is the structural notion of *isomorphism*. Next, we'll have the structure of Rings, which are where equations and their roots live. Finally, we'll apply Group Theory to study the symmetries of roots of polynomials, as well as other ancient problems, such as why Euclid (and we, in high school geometry) could construct polygons with $3, 4, 5, 6, 8, 10, 15$ sides, but not with 7 or 9 sides. In fact, this entire course could be thought of as a modern approach to the problems left unsolved in Euclid's *Elements*.

Tentative Course Outline

I. Groups (first examples) [Judson 3.1, 3.2, 4.1, 4.2]

Cyclic, unit group of $\mathbb{Z}_n$, $\mathbb{C}^\times$, GL_2, Quaternion Q_8, Symmetric S_n, Dihedral D_n, tetrahedral, octahedral, icosahedral

II. Groups (structure theory) [Judson 8.1, 8.2, 9.1, 9.2, 9.3]

Lagrange Thm, Isomorphisms, direct product, normal subgroups, Factor groups, Isomorphism Thms, abelian, solvable and simple groups, Fundmental Thm of Finite Abelian groups.

III. Groups (Applications to symmetry) [Judson 10.1, 10.2]

Symmetries of the plane, matrix groups

IV. Exam I.

V. Groups (Group actions and counting) [Judson 12.1, 12.2, 12.3]

Group actions, Class equation, Burnside Counting Theorem, Counting stuff

VI. Rings (first examples) [Judson 14.1, 14.2]

$\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, $\mathbb{Z}_n$, polynomial rings.

VII. Rings (structure theory) [Judson 14.3, 14.4]

Fields and integral domains, Direct product, homomorphisms, ideals, quotient rings, isomorphism theorems, maximal and prime ideals

VIII. Rings (Principal ideals) [Judson 15.1, 15.2, 15.3]

Principal ideal domains, divison algorithm, irreducible polynomials, Chinese Remainder Theorem.

IX. Exam 2

X. Fields [Judson 19.1, 19.2, 19.3]

Extension fields, Number Fields, Splitting fields, Geometric constructions

XI. Galois Theory (Fields and their symmetry groups) [Judson 21.1, 21.2, 21.3]

Field automorphisms, Fundamental Theorem of Galois Theory, Cubics, Solvability