Exercise 1. Let $G$ be a Lie group and let $H \subset G$ be a closed subgroup. Give $G/H$ the topology whose open sets are those with open pre-image in $G$. Prove that $G/H$ is Hausdorff. (Given distinct points $x, y \in G/H$ there are disjoint open sets $U, V$ in $G/H$ such that $x \in U$ and $y \in V$.)

Exercise 2. Complex projective space $\mathbb{CP}^n$ is the set of lines in $\mathbb{C}^{n+1}$. Let $[z_0, z_1, \ldots, z_n]$ be the complex line through the vector $(z_0, z_1, \ldots, z_n) \in \mathbb{C}^{n+1}$. A set $V \subset \mathbb{CP}^n$ is open iff the set of all vectors in $\mathbb{C}^{n+1}$ belonging to lines in $V$ is open in $\mathbb{C}^{n+1} = \mathbb{R}^{2n+2}$. Let $V_i = \{[z_0, z_1, \ldots, z_n] \in \mathbb{CP}^n : z_i \neq 0\}$. Show that $\mathbb{CP}^n$ is a $2n$-dimensional manifold by defining maps $\varphi_i : \mathbb{R}^{2n} \to V_i$ making a chart $\{(\varphi_i, \mathbb{R}^{2n}, V_i) : i = 0, 1, \ldots, n\}$ on $\mathbb{CP}^n$, and compute the transition functions.

Exercise 3. Let $G = U_n = \{g \in \text{GL}_n(\mathbb{C}) : \bar{g}g = g^{-1}\}$ be the compact unitary group, with maximal torus $T$ consisting of the diagonal matrices in $G$.

a) Show that $T$ meets every conjugacy class in $G$.

b) Show that every maximal torus in $G$ is conjugate to $T$.

Exercise 4. Let $U_n$ and $U_1$ be embedded in $U_{n+1}$ as

$$
\begin{bmatrix}
U_1 & 0 \\
0 & U_n
\end{bmatrix}.
$$

a) Show that $U_{n+1}/U_n$ is diffeomorphic to $S^{2n+1}$.

b) Show that $U_{n+1}/(U_1 \times U_n)$ is diffeomorphic to $\mathbb{CP}^n$. 