MT 8821/01 Number Theory I
Introduction to Bruhat-Tits Theory
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Fall 2016

Schedule: MWF 1 pm, Maloney 560 (Gasson 208 is the official room)

Office hours: MF 2-3, Tue 3-4 but appointment is best. Email is good for quick questions and hints.

Prerequisites: First year graduate algebra is an essential prerequisite. It would also help to have some familiarity with some or all of the following topics: local fields (e.g. [S]), root systems (e.g. [B]), algebraic (or Chevalley) groups (e.g. [St]). I will give short introductions to these topics as needed.

Homework: It will be assigned from time to time and collected two weeks later, approximately. Please turn in some of it. Your homework must be typed in \LaTeX .

The Course: Structure of reductive algebraic groups over non-archimedean local fields, as developed first by Iwahori-Matsumoto [IM] and expanded by Bruhat-Tits in [BT] and [BT2]. See also [T] for a dense users guide. We will follow the general path of [BT] but not step-by-step.

Tenative course outline

I. Introduction: SL₂
   a) SL₂ as group functor and as Chevalley group
   b) SL₂(R), the upper half-plane, Bruhat, Cartan and Iwasawa decompositions.
   c) Local fields and norms
   d) SL₂(Q_p), its tree, Bruhat, Cartan and Iwasawa decompositions.

II. Weyl groups and root systems, and their affine versions [BT, 1.1]

III. The building of an affine BN pair [BT, 1.2-4]
   a) BN pairs and Bruhat decomposition
b) From BN pairs to buildings

c) Apartments and structural maps (linking up with part II).

d) Metric properties and contractibility

e) Stabilizers and maximal bounded subgroups.

f) Iwasawa and Cartan decompositions

IV. Introduction to Chevalley groups [St]

V. Valuated root group data and BN pairs [BT, 1.6,7,10]

a) Valuated root group data

b) Examples from split groups

c) From a discretely valued root group data to a BN pair

d) Examples from nonsplit classical groups

VI. Moy-Prasad filtrations (time permitting) [MP]