Some new supercuspidal representations

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This is a written version of my talk at the conference in honor of Roger Howe's 70th birthday at Yale in June 2015. The write-up retains the expository nature of the lecture, with references for more details given along the way. I thank Beth Romano for a careful reading of an earlier draft.

1 Supercuspidal representations from 1977 to 2007

Let K be a p-adic field with residue field k, and let $G = \mathbf{G}(K)$, where $\mathbf{G}$ is a connected reductive algebraic group defined over K. An irreducible complex representation of G is *supercuspidal* if it may be realized in the space of smooth functions on G whose support is compact modulo the center of G. Supercuspidal representations are fundamental to the study of harmonic analysis on G.

The early work on of supercuspidal representations seems to have been confined to GL_2 and related groups. See [2] for the history of this period. Then in 1977 Roger Howe [12] constructed all of the supercuspidal representations of $\mathrm{GL}_n(K)$, assuming p does not divide n. Over the next thirty years, Howe's work was extended in two main directions:

$G = GL_n(K)$, but no (or minimal) restriction on p. This is work of Moy [15], Carayol [4] then decisively Bushnell-Kutzko [3], whose approach was extended by Stevens [24] to determine all the supercuspidals for classical groups, assuming $p > 2$. All of these methods rely on the fact that $\mathrm{GL}_n(K)$ is the unit group of a central-simple K-algebra.

General G, but large p. Work of Adler [1] and then Yu [26] extended Howe's construction to general reductive groups, and Kim [14] showed that Yu's construction gives all supercuspidal representations of G for $p \gg 0$. This assumption on p arises first in certain tameness conditions on the inducing data in Yu's construction, analogous to Howe's assumption that $p \nmid n$, and then again from use of the exponential map in Kim's proof of exhaustion.

2 Simple Supercuspidal Representations

In 2010 a new construction of certain supercuspidal representations was given in [7]. The construction yields very few representations, but has some advantages: a) it works uniformly for all p and all G, and b) it is extremely simple.

I will describe the construction assuming $\mathbf{G}$ is split over K, almost simple and simply-connected, as in [7]. For other cases see [21, 2.6]. Let I_+ be the unique Sylow pro-p subgroup of an Iwahori subgroup of G. There is a natural quotient

$$I_+ \twoheadrightarrow \bigoplus_{\psi} k_\psi, \tag{1}$$

where the k_ψ are copies of the additive group of the residue field k, indexed by simple affine roots. Let $\chi : I_+ \to \mathbb{C}^\times$ be a character of I_+ which factors through the quotient (1) and is nontrivial on each line k_ψ. Extend χ to $Z \cdot I_+$ arbitrarily, where Z is the center of G. Then the (compactly) induced representation

$$\pi_\chi = \mathrm{ind}_{ZI_+}^G \chi \tag{2}$$

is irreducible and supercuspidal. These are the *simple supercuspidal representations*.

Remark 1. The representation (2) is an affine analogue of the Gelfand-Graev representation. However, the latter is highly reducible. The irreducibility of (2) stems from a property of affine root systems that does not hold for ordinary root systems. Namely, if C, C' are two alcoves, then C' lies on the positive side of some wall of C that is not a wall of C'. See [7, 9.1].

Remark 2. If $G = \mathrm{GL}_n(K)$ the representations (2) are a simple case of Carayol's construction [4]. If G is general and $p \gg 0$ tthe representations (2) can also be realized by Adler's construction [1], as shown in [13]. If p is small for G, for example if $p = 2$ and we are outside type A, the representations (2) seem to be new.

Remark 3. If $|k| = 2$ then G has just one simple supercuspidal representation. We shall return to this in section 4 below.

3 Local Langlands Correspondence

The Local Langlands Correspondence (LLC) for p-adic groups is a kaleidoscope of conjectures admitting diverse viewpoints. I like to think of the LLC as an interaction between three areas of mathematics:

representations of p-adic groups

↓

p-adic Galois theory ⟶ Complex Lie groups

These interactions have a tame aspect:

$$\begin{array}{ccc} & \boxed{\text{depth-zero representations}} & \\ & \downarrow & \\ \boxed{\text{tame extensions}} & \longrightarrow & \boxed{\text{complex geometry}} \end{array}$$

And a wild aspect, which is our concern here:

$$\begin{array}{ccc} & \boxed{\text{positive-depth representations}} & \\ & \downarrow & \\ \boxed{\text{wild extensions}} & \longrightarrow & \boxed{p\text{-local structure}} \end{array}$$

Here, "p-local structure" is a term borrowed from finite group theory. In our context it refers to normalizers of finite p-subgroups of complex Lie groups. The vertical arrow is the conjectured map that takes a representation π of G to its Langlands parameter

$$\varphi_\pi : \mathscr{W}_K \to {}^L G,$$

where $\mathscr{W}_K$ is the Weil (or Weil-Deligne) group of K and ${}^L G$ is the L-group of G. I draw the arrows this way because representations of p-adic groups appear to behave more smoothly than their parameters. In effect, the representations π parameterize their parameters, not the other way around. For example, we have just seen that simple supercuspidal representations π_χ have a uniform construction independent of p, but soon we will see that their parameters φ_{π_χ} change drastically when p is small.

If $G = GL_n$ the map $\pi \to \varphi_\pi$ is a bijection [8], [10], uniquely determined by independent numerical invariants $\varepsilon(\pi)$ and $\varepsilon(\varphi_\pi)$ and their extension to pairs (π, π') [9]. The invariants $\varepsilon(\varphi_\pi)$ exist for general groups, once we choose a representation of ${}^L G$, but analogues of $\varepsilon(\pi)$ have not been defined for general G.

However, for *square-integrable* π we have a partial substitute for $\varepsilon(\pi)$, namely the power $N(\pi)$ of $q = |k|$ in the formal degree of π (with respect to a certain Haar measure on G). According to [11], as reformulated in [7, 7.2] the mapping $\pi \to \varphi_\pi$ should have the property that $N(\pi)$ is determined by the Swan conductor $\mathsf{sw}(\varphi_\pi, \hat{\mathfrak{g}})$ of the adjoint representation

$$\mathscr{W}_K \xrightarrow{\varphi_\pi} {}^L G \xrightarrow{\mathrm{Ad}} \mathrm{Aut}(\hat{\mathfrak{g}}),$$

where $\hat{\mathfrak{g}}$ is the Lie algebra of ${}^L G$. See [7, (43)] for the precise conjectural formula for $N(\pi)$ in terms of φ_π. Here we simply note that this formula places strong restrictions on φ_π. For example, if π is compactly induced from ZJ, where $J < I_+$, the degree conjecture implies that

$$\mathsf{sw}(\varphi_\pi, \hat{\mathfrak{g}}) \geq \mathrm{rank}(\mathfrak{g}), \tag{3}$$

with equality if and only if π is a simple supercuspidal representation.

4 An example in exceptional groups

Let $\mathscr{W} \supset \mathscr{I} \supset \mathscr{P}$ be the Weil group, its inertia and wild inertia subgroups, respectively, of K.

Let π be a simple supercuspidal represenation of G. What does (3) tell us about the corresponding parameter $\varphi_\pi : \mathscr{W} \to {}^LG$? For example, what are the images $\mathscr{W}_\pi \supset \mathscr{I}_\pi \supset \mathscr{P}_\pi$ of $\mathscr{W}, \mathscr{I}, \mathscr{P}$ under φ_π?

First of all, one can check [7, 9.5] that φ_π must be *totally ramified*, in the sense that $\mathscr{I}_\pi$ has trivial invariants in $\hat{\mathfrak{g}}$.

Next, if p does not divide the order of the Weyl group W, the images $\mathscr{W}_\pi \supset \mathscr{I}_\pi \supset \mathscr{P}_\pi$ have a simple description. For example, $\mathscr{W}_\pi$ normalizes a maximal torus $T \subset {}^LG$ such that $\mathscr{P}_\pi < T$, and and $\mathscr{I}_\pi$ projects to a cyclic subgroup of the Weyl group W of T generated by a Coxeter element (or a twisted analogue of the Coxeter element if $\mathbf{G}$ is not split).

Suppose now that $p = 2$. In this case the structure of φ_π will depend on the 2-local structure of ${}^LG$. If $K = \mathbb{Q}_2$, so that G has a unique simple supercuspidal representation π, then $\varphi_\pi : \mathscr{W} \longrightarrow {}^LG$ should be uniquely characterized by equation (3). In many cases φ_π is known [7, 6.3], but we consider here a new case (which turns out to be a family of three cases).

Let $G = F_4(\mathbb{Q}_2)$. Even though φ_π is not yet known, equation (3) is strong enough to determine the only possible image $\mathscr{W}_\pi$ of φ_π, along with the complete ramification filtration of $\mathscr{W}_\pi$. The result involves the following nonabelian 2-group in $F_4(\mathbb{C})$. Start with the dual pair $G_2 \times \mathrm{SO}_3 \subset F_4(\mathbb{C})$. Let $J \simeq 2^3, Z \simeq 2^2$ be the unique maximal 2-tori (elementary abelian 2-subgroups) in each factor. Then the subgroup $Y = J \times Z$ is the unique maximal 2-torus in $F_4(\mathbb{C})$, and is its own centralizer. The normalizer $N(Y)$ acts linearly on Y, giving a mapping $N(Y) \to \mathrm{GL}(Y)$. This map is not surjective, because there are two conjugacy-classes of involutions in $F_4(\mathbb{C})$; one class C has centralizers Spin_8, and we have $Y \cap C = Z - \{1\}$. Hence the image of $N(Y)$ in $\mathrm{GL}(Y)$ is contained in the parabolic subgroup $\mathrm{GL}(Y,Z) = \{g \in \mathrm{GL}(Y) :\ gZ = Z\}$; in fact this image is the whole of $\mathrm{GL}(Y,Z)$. The subgroup $X < N(Y)$ acting trivially on Z and Y/Z is a 2-group of nilpotence class 2, of order $|X| = 2^{5+6}$ fitting into the extension

$$1 \longrightarrow Y \longrightarrow X \longrightarrow \mathrm{Hom}(J,Z) \longrightarrow 1.$$

In $\mathrm{GL}(J) \times \mathrm{GL}(Z)$ we have a unique class of elements $\sigma = (\sigma_J, \sigma_Z)$ of order 21, whose components are regular elements of order 1+Coxeter number (namely 7 and 3) in G_2 and SO_3. Since the centralizer of Z is Spin_8, we have

$$X \cdot \langle \sigma \rangle < \mathrm{Spin}_8 \cdot \langle \sigma_Z \rangle < F_4.$$

Proposition 1. *Let $K = \mathbb{Q}_2$ and let ${}^L G = F_4(\mathbb{C})$, $\hat{\mathfrak{g}} = \mathrm{Lie}({}^L G)$. Suppose $\varphi : \mathscr{W} \to {}^L G$ is a totally ramified Langlands parameter and let $\mathsf{sw}(\varphi, \hat{\mathfrak{g}})$ be the Swan conductor of the representation $\mathrm{Ad}\,\varphi : \mathscr{W} \to \mathrm{GL}(\hat{\mathfrak{g}})$ obtained by composing φ with the adjoint representation of ${}^L G$. Then we have*

$$\mathsf{sw}(\varphi, \hat{\mathfrak{g}}) \geq 4 (= \mathrm{rank}\, G)$$

with equality if and only if the following hold:

a) $\varphi(\mathscr{I}) = X \cdot \langle \sigma \rangle$ and $\varphi(\mathscr{P}) = X$.
b) The ramification filtration of $\varphi(\mathscr{P})$ is given by $X > Y > Z$, each occurring once, 32 and 128 times, respectively.

This result is proved in [19] by examining the possibilities for the smallest nontrivial ramification subgroup of $\varphi(\mathscr{P})$; it ultimately comes down to the uniqueness of the Hamming code in dimension 7.

Let π be the unique simple supercuspidal representation of $F_4(\mathbb{Q}_2)$, and let φ_π be its conjectural Langlands parameter. From (3), we must have $\mathsf{sw}(\varphi_\pi, \hat{\mathfrak{g}}) = 4$, so that $\mathscr{W}_\pi$ can only be as described in Prop. 1. Thus, based on the existence of the simple supercuspidal representation π of $F_4(\mathbb{Q}_2)$, the LLC predicts a Galois extension $L/\mathbb{Q}_2$, namely the kernel field of φ_π, of degree $6 \cdot 21 \cdot 2^{11}$. The information in b) determines the discriminant valuation $d_{L/\mathbb{Q}_2}$.

Prop. 1 remains true, with the same groups X, Y, Z, when G replaced by $E_6(\mathbb{Q}_2)$ with the rank 4 replaced by 6. It also remains true if G is replaced by the twisted form ${}^3D_4(\mathbb{Q}_2)$ of $\mathbb{Q}_2$-rank 2, and X, Y, Z are replaced by their images $X/Z, Y/Z, 1$ in $\mathrm{Aut}(\mathfrak{so}_8)$.

5 Epipelagic Representations

It turns out that simple supercuspidal representations are a special case of a more general construction that relies on the Geometric Invariant Theory (GIT) of Moy-Prasad filtrations [16], and that again makes no assumptions on p.

The *apartment* of a maximal K-split torus $\mathbf{T}$ in $\mathbf{G}$ is an affine space $\mathscr{A}$ under the real span of the cocharacters of $\mathbf{T}$. Each point $x \in \mathscr{A}$ determines valuations on the root groups of T in G; these valuations define a compact open (parahoric) subgroup $G_x < G$, along with a filtration by (distinct) normal subgroups of G_x:

$$G_x > G_{x,r} > G_{x,s} > \cdots .$$

This is the *Moy-Prasad filtration* at x. The top two quotients $\mathsf{H}_x(k) := G_x / G_{x,r}$ and $\mathsf{V}_x(k) := G_{x,r}/G_{x,s}$ are the k-rational points of, respectively a connected reductive algebraic group H_x and a vector space V_x, both defined over k, and we have an algebraic representation $\mathsf{H}_x \to \mathrm{GL}(\mathsf{V}_x)$ induced by the conjugation action of G_x on $G_{x,r}$.

Fix a nontrivial character $\chi : k^+ \to \mathbb{C}^\times$. Then each linear functional $\lambda \in \check{\mathsf{V}}_x(k)$ gives a character $\chi_\lambda = \chi \circ \lambda$ of $G_{x,r}$. We say that λ is *stable* if the H_x-orbit of λ under the dual representation $\mathsf{H}_x \to \mathrm{GL}(\check{\mathsf{V}}_x)$ is Zariski closed, with finite stabilizers.

Theorem 1. *[21, Prop.2.4] If λ is stable then the compactly induced representation*

$$\pi_{x,\lambda} := \mathrm{ind}_{G_{x,r}}^{G} \chi_\lambda$$

is a finite direct sum of irreducible supercuspidal representations.

The representations constructed in Thm. 1 are called *epipelagic* supercuspidal representations, because they occur at the minimal positive depth of the Moy-Prasad filtration at x.

Though the construction in Thm. 1 makes no assumption on p, it still requires us to determine those points $x \in \mathscr{A}$ for which $\mathsf{V}_x(k)$ admits stable functionals.

For example, if x is the barycenter of an alcove in $\mathscr{A}$, then $r = 1/h$, where h is the Coxeter number of G; the group H_x is a torus, $G_{x,1/h} = I_+$, and $\mathsf{V}_x(k)$ is the quotient on the right side of (2). In this case, a character $\lambda \in \check{\mathsf{V}}_x(k)$ is stable precisely if it is nontrivial on each line k_ψ, as in section 2, and then $\pi_{x,\lambda}$ is a sum of simple supercuspidal representations, indexed by the extensions of χ_λ to Z.

For other points $x \in \mathscr{A}$, the GIT of $(\mathsf{H}_x, \check{\mathsf{V}}_x)$ is more subtle.

Theorem 2. *Assume that K has characteristic zero, but that p is arbitary. Then $\check{\mathsf{V}}_x$ has stable functionals under the action of H_x if and only if x is conjugate under the affine Weyl group of G to $\frac{1}{m}\check{\rho}$, where $\check{\rho}$ is the sum of a set of fundamental coweights, and m is the order of an elliptic regular element in the Weyl group W.*

Here $w \in W$ is *elliptic* if w fixes no vector in the root lattice and is *regular* if the cyclic group generated by w acts freely on the roots of G. In this situation we have $r = 1/m, s = 2/m$. For example if $G = E_8$ then the possible m's are the divisors of $20, 24$ and 30.

Thm. 2 was proved in [21] for $p \nmid |W|$, or $p = 0$, by relating $(\mathsf{H}_x, \check{\mathsf{V}}_x)$ to Vinberg's theory of graded Lie algebras [25], using the results of [20] to find the stable vectors in the latter setting. The extension of Thm. 2 to all p was proved for split groups by Fintzen-Romano [5] by lifting the GIT problem to the ring of integers $\mathscr{O}_K$, where stability is understood by [21] (since K has characteristic zero), and then showing that stability behaves well enough under reduction modulo the prime ideal of $\mathscr{O}_K$. Thm. 2 has also been extended to nonsplit groups by Fintzen [6].

For $p \nmid |W|$, it was shown in [13] that the epipelagic representations in Thm. 1 are already among those constructed (in a different way) in [1]. However, when p is small, epipelagic representations cannot be constructed by previously known methods; Thm. 1, combined with Fintzen-Romano's extension of the stability criterion to all p implies the existence of new supercuspidal representations of epipelagic type for small p.

Note, however, that Thm. 2 only guarantees the existence of stable vectors over the algebraic closure of k. This means one still requires k to be sufficiently large in order to obtain new supercuspidals (outside the simple supercuspidal case). This is no longer an issue for p large, as Thm. 2 has now been proved with $\check{\mathsf{V}}_x$ replaced by $\check{\mathsf{V}}_x(k)$ in [18]. For small p, it is still an open problem to determine the *k-rational* stable locus in $\check{\mathsf{V}}_x$, when x satisfies the conditions of Thm. 2.

6 An example in G_2

The results in this section are due to Beth Romano [22]. Let K be arbitrary, and let G have type G_2. The long element of $W(G_2)$ is elliptic and regular, of order $m = 2$. In this case x is a point in $\mathscr{A}$ such that

$$\mathsf{H}_x = \mathrm{SO}_4 = (\mathrm{SL}_2 \times \mathrm{SL}_2)/\mu_2, \quad \text{and} \quad \check{\mathsf{V}}_x = P_1 \boxtimes P_3,$$

where P_d is the representation of SL_2 on polynomials in two variables of degree d, and $\boxtimes$ denotes outer tensor product of representations. In fact this representation is defined over $\mathbb{Z}$: For any commutative ring A, we may regard elements $\lambda \in \check{\mathsf{V}}_x(A)$ as cubic polynomials in X,Y whose coefficients are linear polynomials in Z,W, all with A-coefficients. The discriminant $\mathrm{disc}_{X,Y}(\lambda)$ is then a quartic polynomial in Z,W, which itself has a discriminant $\mathrm{disc}_{Z,W}(\mathrm{disc}_{X,Y}(\lambda)) \in A$. When $A = \mathbb{Z}$, this bi-discriminant turns out to have all coefficients divisible by 2^8. Thus we have a polynomial function

$$\Delta(\lambda) = \frac{1}{2^8}\,\mathrm{disc}_{Z,W}(\mathrm{disc}_{X,Y}(\lambda))$$

on $\check{\mathsf{V}}_x$ whose values are integer polynomials in the coefficients of the monomials in X,Y,Z,W in λ.

It turns out that a vector $\lambda \in \check{\mathsf{V}}_x(k)$ is stable if and only if $\Delta(\lambda) \neq 0$ in k. For example, if $p \neq 3$, the functional $\lambda_0 := Z \otimes (X^3 + Y^3) + W \otimes X^2Y$ is stable. If $p \neq 2, 13$, the functional $\lambda_1 := W \otimes (X^3 + X^2Y + Y^3) + Z \otimes XY^2$ is stable. So $\check{\mathsf{V}}_x(k)$ contains stable vectors for all p-adic fields K of any residue characteristic p.

If $K = \mathbb{Q}_2$, so that $k = \mathbb{F}_2$, then there is just one $\mathsf{H}_x(\mathbb{F}_2)$-orbit of stable vectors in $\check{\mathsf{V}}_x(\mathbb{F}_2)$, namely $\mathsf{H}_x(\mathbb{F}_2) \cdot \lambda_0$, and there is just one epipelagic representation π_x. What is its Langlands parameter?

There is a tamely ramified tower of extensions with degrees as shown:

$$\mathbb{Q}_2 \overset{3}{\subset} \mathbb{Q}_8 \overset{7}{\subset} E,$$

where $\mathbb{Q}_8/\mathbb{Q}_2$ is unramified and $E = \mathbb{Q}_8(\sqrt[7]{2})$. Let $U_m = 1 + P^m$, where P is the prime ideal in $\mathscr{O}_E$. One checks that $U_1/U_1^2U_4 \simeq \mathbb{F}_8 \oplus \mathbb{F}_8$, where the summands are preserved by the action of $\mathrm{Gal}(E/\mathbb{Q}_8)$ and are non-isomorphic $\mathrm{Gal}(E/\mathbb{Q}_8)$-modules. By local class field theory, each summand corresponds to an extension L/E of type $(2,2,2)$ which is Galois over $\mathbb{Q}_2$ and which gives a homomorphism $\varphi : \mathscr{W}_{\mathbb{Q}_2} \to G_2(\mathbb{C})$ with kernel field L, mapping $\mathrm{Gal}(L/E)$ to the subgroup J of section 4. One of these parameters is trivial on U_2; let us call it φ_6, and the other one φ_2. Then Swan conductors and other invariants of the LLC imply that φ_6 is the parameter for the simple supercuspidal representation of $G_2(\mathbb{Q}_2)$ and φ_2 is the parameter for the epipelagic representation π_x found in this section.

Thus, we find some of the Galois theory of $\mathbb{Q}_2$ reflected in the structure of the complex Lie group $G_2(\mathbb{C})$, and this interaction is predicted by the existence of epipelagic representations, via the Local Langlands Conjecture.

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