Proposition I.34

Statement:
In a parallelogram, the opposite sides and angles are equal to one another, and the diameter bisects the area.

Claim:
In parallelogram $ABCD$,
(a) $AB = CD$, $AC = BD$, $\angle ABD = \angle ACD$ and $\angle BAC = \angle CDB$.
(b) $BC$ bisects the area of parallelogram $ABCD$.

Proof:
First we will prove Claim (a):
Because $ABCD$ is a parallelogram, $AB \parallel CD$ and $AC \parallel BD$.
Then $\angle ABC = \angle BCD$ and $\angle ACB = \angle CBD$. [I.29]
We see $BC = BC$.
Because $\angle ABC = \angle BCD$, $\angle ACB = \angle CBD$, and $BC = BC$, we have that $\triangle ABC = \triangle BDC$. [I.26]
Then $AB = CD$, $AC = BD$, and $\angle BAC = \angle CDB$.
Because $\angle ABD = \angle ABC + \angle CBD$ and $\angle ACD = \angle ACB + \angle BCD$, and $\angle ABC = \angle BCD$ and $\angle ACB = \angle CBD$, we see $\angle ABD = \angle ACD$. [C.N. 2]
Thus we have shown that $AB = CD$, $AC = BD$, $\angle ABD = \angle ACD$ and $\angle BAC = \angle CDB$, and Claim (a) is proved.
Next we will prove Claim (b):
We know that $AB = CD$, $BC = BC$, and $\angle ABC = \angle BCD$.
Thus $\triangle ABC = \triangle BCD$. [I.4]
We see that parallelogram $ABCD = \triangle ABC + \triangle BCD$.
Thus parallelogram $ABCD = \triangle ABC + \triangle ABC$.
Because $\triangle ABC = \triangle ABC$, $BC$ bisects the area of parallelogram $ABCD$. 
Thus we have proved Claim (b).
Claim (a) and Claim (b) are proved, therefore Proposition 34 is proved. QED.