Proposition I.38

Triangles which are on equal bases and in the same parallels are equal to one another.

Given: Triangle \( \triangle ABC \) and Triangle \( \triangle DEF \) with \( BC = EF \) and both line segments on line \( L_1 \) and points \( A \) and \( D \) fall on line \( L_2 \) and \( L_1 \) is parallel to \( L_2 \).
Claim: Triangle \( \triangle ABC = \triangle DEF \)

Construction Steps:
1. Through point \( B \), let \( BG \) be drawn parallel to \( CA \) \[I.31\]
2. Through point \( F \), let \( FG \) be drawn parallel to \( DE \) \[I.31\].

Proof:
\( GB \) parallel to \( AC \), \( DC \) parallel to \( HF \), \( L_1 \) parallel to \( L_2 \), we have two parallelograms \( GABC \) and \( DHEF \) [definition of parallelogram]. Since \( BC = EF \), we know that parallelograms \( GABC = DHEF \) \[I.36\].
Since \( AB, DF \) are diameters of \( GABC \) and \( DHEF \) respectively, triangle \( \triangle ABC = \frac{1}{2} GABC \), triangle \( \triangle DEF = \frac{1}{2} DHEF \) \[I.34\].
Since parallelogram $GABC$ is equal to parallelogram $DHEF$, therefore triangle $ABC = DEF$ [c.n. 1].

Special Case: If the two triangles are on the same base and within the same parallels [I.37].

Note from discussion:
What if the bases of triangles are on opposite parallels?