Proposition I.39

Proposition Equal Triangles which are on the same base are also in the same parallels.
For our proof, we have $\triangle ABC = \triangle DBC$ in area, and they are on the same base BC. We’d like to prove that $AD \parallel BC$.

Proof. Proof by contradiction. Let $AE$ be drawn s.t. $AE \parallel BC$ [I.31]. Suppose $E$ is not $D$. Connect $EC$. As $AE \parallel BC$, $\triangle ABC = \triangle EBC$ in area [I.37]. So $\triangle DBC = \triangle EBC$ in area [C.N.1]. But this contradicts C.N.5. Hence $E$ is $D$, and so $AD \parallel BC$. \qed