Proposition I.42

To construct, in a given rectilineal angle, a parallelogram equal to a given triangle.

1. Bisect line BC at point E. \[\text{Prop. 1.10}\]
2. Construct line through A parallel to line BC. \[\text{Prop. 1.31}\]
3. Let F be a point on this line so that $\angle CEF = \angle D$. \[\text{Prop. 1.23}\]
4. Construct line CG parallel to line EF. \[\text{Prop. 1.31}\]
5. Then parallelogram ECGF is equal to triangle ACB.

Proof

Triangle ABE is equal to triangle AEC. \[\text{Prop 1.38}\]
Triangle ABC is equal to triangle ABE plus triangle AEC. \[\text{C.N. 1.1}\]
Therefore, triangle ABC is equal to 2 triangle AEC.
Next, triangle FEC is equal to triangle AEC. \[\text{Prop 1.37}\]
Parallelogram ECGF is equal to 2 triangle FEC. \[\text{Prop. 1.41}\]
Therefore, parallelogram ECGF is equal to 2 triangle AEC. \[\text{C.N. 1}\]
Thus parallelogram ECGF is equal to triangle ABC. \[\text{Q.E.D.}\]