Proposition I.44

To a given straight line, to apply, in a given angle, a parallelogram equal to a given triangle.

For the purpose of this proof all equalities will relate to equality of area.

Steps:
1. Construct $BEFG = \triangle C$ in $\angle EBG = \angle D$ [I.42]
2. Extend \( FG \) to \( H \) and let \( AH \parallel BG \) [I.32]
3. Join \( HB \) [Post. 1]
4. Since \( FE \parallel HA \) with transversal \( FH \) then \( \angle AHF + \angle HFE = \angle \angle \) [I.29]
   so, \( \angle BHG + \angle GFE < \angle \angle \), so we know that \( FE \) and \( HB \) meet, at point \( K \) [post. 5]
5. Now, extend \( FE \) and \( HB \) [post. 2]
6. Through \( K \), draw \( KL \parallel EA \) and extend \( HA \) and \( GB \) to point \( L \) and \( M \) [I.31]

Proof:
Parallellograms \( LMBA = BEFG \) [I.43]
Since parallelogram \( BEFG = \triangle C \), we know that parallelogram \( LMBA = \triangle C \) [cn. 1]
Then, \( \angle D = \angle GBE \), but \( \angle GBE = \angle ABM \) [I.15]
Then, \( \angle D = \angle ABM \)
Therefore, parallelogram \( LMBA = \triangle C \), on line \( AB \) with \( \angle ABM = \angle D \)

Q.E.F.