Proposition I.47

The square on the hypotenuse of a right triangle is equal to the squares formed by the legs of a right triangle.
Let the given straight line be AB
Steps:
1) Draw $\square ABED$ on AB, $\square BCGF$ on BC, and $\square ACIH$ [Prop.I.46]
2) Draw a perpendicular line to AC from point B. Label this point as point J. [Prop.I.12]
3) Extend line BJ on the side of J so that it intersects HI at point K [Postulate 2]
4) Draw straight lines from B to G, B to D, A to G, C to D, B to I, and B to H. [Postulate 1]
Claim: Half of the square BCGF, which is $\triangle BCG$ is equal to half of the rectangle JCIK, which is $\triangle JCI$.
By proving this, we will prove the square BCGF is equal to JCIK and WOLOG, square ABED= rectangle AJKH.
This is equivalent to the proof where the two squares on the legs are equal to the square on the hypotenuse.
Proof:
$\angle FBC= \text{right angle}$ and $\angle BCG= \text{right angle}$ [Def.20]
Thus, $\angle FBC+\angle BCG= 2 \text{ right angles}$ [CN.2]
$AF\parallel CG$ [Postulate 5]
$:\triangle CGB=\triangle CGA$ [Prop.I.38]
CG=BC [Def.20]
AC=CI [Def. 20]
\[ \angle JCI = \text{right angle} \quad \text{[Def. 20]} \]
\[ \angle JCI = \angle BCG \quad \text{[CN. 1]} \]
\[ \angle BCJ = \angle BCJ \]
\[ \angle BCJ + \angle JCI = \angle BCG + \angle BCJ \quad \text{[CN. 2]} \]
\[ \angle BCJ + \angle JCI = \angle BCJ \quad \text{and} \quad \angle BCJ + \angle BCG = \angle ACG \quad \text{[CN. 4]} \]
\[ \angle ACG = \angle BCI \quad \text{[CN. 1]} \]
\[ \triangle CGA = \triangle BCI \quad \text{[Prop. I. 4]} \]

As \( \angle BJC \) is a right angle, \( \angle KCI = \text{right angle} \quad \text{[Def. 10]} \)

BK \parallel CI \quad \text{[Postulate 5]}

\[ \because \triangle JCI = \triangle BCI \quad \text{[Prop. I. 38]} \]
\[ \therefore \triangle JCI = \triangle BCG \quad \text{[CN. 1]} \]

Q.E.D.

Comments:
1. "We could have stated that \( \triangle CGA = \triangle BCI \) through a proof of rotation but we haven’t proved that yet so we needed to use SAS (Prop. I. 4)"