Proposition III.22

The opposite angles of quadrilaterals in circles are equal to two right angles.

Construction Steps:
1. Draw circle ABCD and let the vertices of quadrilateral ABCD lie on the circumference of the circle such that quadrilateral ABCD is inside of the circle.
2. Join A to C and B to D [Post 1]

Claim: \( \angle ABC + \angle ADC = \perp \perp \) and \( \angle BAD + \angle CDB = \perp \perp \)

Proof:
\( \angle CAB = \angle CDB \) because both angles are formed from the same segment \( BC \) [III.21]. Similarly, from segment \( AB \), we have \( \angle BCA = \angle BDA \) [III.21].

We know that \( \angle ADC = \angle ADB + \angle BDC \). Using substitution, we have \( \angle ADC = \angle BCA + \angle BAC \) [c.n. 1]. Adding \( \angle ABC \) to both sides [c.n. 2], we have:
\( \angle ADC + \angle ABC = \angle BCA + \angle BAC + \angle ABC \).

But, we know that \( \angle BCA + \angle BAC + \angle ABC = \perp \perp \) [I.32].

Thus, \( \angle ADC + \angle ABC = \perp \perp \), which are opposite angles in the quadrilateral which lie on the inside of the circle.

Following a similar proof, we know that \( \angle DAB + \angle DCB = \perp \perp \).

Therefore, the sum of opposite angles in a quadrilateral in a circle are equal to two right angles.