Proposition 3.37

If a point be taken outside a circle and from the point there fall on the circle two straight lines, if one of them cut the circle, and the other fall on it, and if further the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference be equal to the square on the straight line which falls on the circle, the straight line which falls on it will touch the circle.

Consider circle ABC with D on the outside. Suppose AB cuts the circle, and DB falls on the circle. We want to show that if

\[ AD \cdot DC = DB^2 \]

then DB touches the circle.

Proof

1. Draw the line DE that touches the circle (Prop. 3.17).
2. Draw the center F of the circle (Prop. 3.1).
3. Draw B and BF and DF.

From Proposition 3.18, we have \( \angle DEF \) is a right angle. From Proposition 3.36 we have the following

\[ BE^2 = AD \cdot DC \]
\[ DB^2 = AD \cdot DC. \]

Therefore (by C.N. 1) we have that DB = DE. We also have that BF = FE by the definition of the radius of a circle. Thus by Proposition 1.8 we have \( \triangle DBF \cong \triangle DEF \) and therefore

\[ \angle DBF = \angle DEF = 90^\circ. \]

From Porism 3.16 we have that DB must touch the circle. Q.E.D.