Proposition IV.10

*To construct an isosceles triangle with each of the base angles equal to double the remaining angle.*
Construction:

1) Let there be a straight line $AB$
2) Cut $AB$ at a point $C$ s.t. $AB \cdot BC = AC^2$ (II.11)
3) Draw a circle with center $A$ and radius $AB$
4) Fit in the circle a straight line $BD$ s.t. $BD = AC$ where $AC < AB$
5) Join $AD$ and $DC$ (post. 1)
6) Circumscribe a circle about $\triangle ACD$

Proof:

$\therefore AC = BD$
$\therefore AC^2 = BD^2$
$\therefore AC^2 = AB \cdot BC$
$\therefore BD^2 = AB \cdot BC$

Claim that $BD$ touches circle $ACD$ (III.37) because $B$ is outside circle $ACD$ by construction and from $B$, $BA$ and $BD$ fall on circle $ACD$, one cutting it, one falling through it, and $BD^2 = AB \cdot BC$

Since $BD$ touches circle $ACD$ and $DC$ drawn across from point of contact at $D$, $\angle BDC = \angle DAC$ (III.32)

Then $\angle BDC + \angle CDA = \angle DAC + \angle CDA$ (c.n.2)

But $\angle BDC + \angle CDA = \angle BDA$

$\therefore \angle BDA = \angle DAC + \angle CDA$

$\angle BCD = \angle DAC + \angle CDA$ (I.32)

$\therefore BDA = \angle BCD$

Since $AB = AD$

$\therefore \angle BDA = \angle CBD$ (I.5)

$\therefore \angle BDA = \angle BCD = \angle CBD$

Since $\angle BCD = \angle CBD$ (I.6)

But $BD = CA$ by construction, $\therefore CA = DC$

$\therefore \angle DAC = \angle CDA$ (I.5)

$\therefore \angle BDA = \angle DAC + \angle CDA$

$\therefore BDA = 2 \cdot \angle DAC$

$\therefore \angle BCD = \angle DAC + \angle CDA$

$\therefore \angle BCD = 2 \cdot \angle DAC$

But $\angle BCD = \angle ABD$

$\therefore \angle ABD = 2 \cdot \angle DAC$