Proposition VI.19

Similar triangles are to one another in the duplicate ratio of the corresponding sides.

Given:
\( \triangle ABC \sim \triangle DEF \)

Claim:
\[
\frac{A(\triangle ABC)}{A(\triangle DEF)} = \frac{(BC)^2}{(EF)^2} = \frac{(AB)^2}{(DE)^2} = \frac{(AC)^2}{(DF)^2}
\]
Proof:
Without loss of generality, we want to show $A(\triangle ABC) : A(\triangle DEF) = (BC)^2 : (EF)^2$
WLOG, assume $BC > EF$.
Pick $G$ on $BC$ such that $EF : BG = BC : EF$ [VI.11]
We know that $BC : EF = AB : DE$ [V.16]
∴ $EF : BG = AB : DE$ [V.11]
∴ $A(\triangle ABG) = A(\triangle DEF)$ [VI.15]
Because $BC : EF = EF : BG$, $BC : BG = (BC)^2 : (EF)^2$ [V. def. 9]
Therefore $A(\triangle ABC) : A(\triangle AGC) = BC : BG = (BC)^2 : (EF)^2$ [VI.1]
Since $A(\triangle ABG) = A(\triangle DEF)$, we have:
$A(\triangle ABC) : A(\triangle DEF) = A(\triangle ABC) : A(\triangle ABG) = (BC)^2 : (EF)^2$ [c.n. 1]
Q.E.D.