Proposition VI.3 11/27/17
(a) If an angle of a triangle is bisected by a sl cutting the base, then the segments of the base will have the same ratio as the remaining sides of the triangle.
(b) And, if the segments of the base have the same ratio as the remaining sides of the triangle, then the sl joining the vertex to the point of section will be the angle bisector.

Part (a)
Let ABC be a △, ∠BAC be bisected by AD.
Claim: BD:CD = BA:AC

Let CE be drawn through C, parallel to DA and let BA be produced to E.
Since sl AC falls upon parallels AD, EC, then ∠ACE = ∠CAD (Prop. I.29) (alt. interior)
∠BAD = ∠ACE (AD bisects ∠BAC)
Since sl BE falls upon parallels AD, EC, then ∠BAD = ∠AEC (Prop. I.29) (Corresponding angles)
∠ACE = ∠BAD ⇒ ∠ACE = ∠AEC
⇒ AE = AC (Prop. I.6)
Since AD||EC, EC being a side of △BCE, then BD:DC = BA:AE (Prop. VI.2)
But AE = AC, so BD:DC = BA:AC

Part (b)
Let BD:DC = BA:AC
Join AD
Claim: sl AD bisects ∠BAC

Since BD:DC = BA:AC and as BD:DC = BA:AE for AD||EC, which is side of △BCE (Prop. VI.2)
⇒ BA:AC = BA:AE (Prop. V.11)
Therefore, AC = AE (Prop. V.9)
So, ∠AEC = ∠ACE (Prop. I.5)
∠AEC = ∠BAD and ∠ACE = ∠CAD (Prop. I.29)
Therefore, ∠BAD = ∠CAD
Therefore, ∠BAC is bisected by sl AD. //