Proposition VI.6: If two triangles have one angle equal to one angle and the sides about the equal angles proportional, then the triangles are equiangular and have those angles equal opposite the corresponding sides.

Given: \( \angle BAC = \angle EDF \) and \( BA : AC = ED : DF \)

Claim: \( \triangle ABC \) is equiangular to \( \triangle DEF \) and thus \( \angle ABC = \angle DEF \) and \( \angle ACB = \angle DFE \)

Proof: Construct \( \angle FDG = \angle BAC \) and \( \angle DFG = \angle ACB \) [I.23].

[We know that point G exists, by Post 5 as by construction \( \angle DFG + \angle FDG < \angle \angle \) ]

Since \( \angle FDG = \angle BAC \) and \( \angle DFG = \angle ACB \), then \( \angle ABC = \angle DCF \) [I.32].

Therefore, \( \triangle ABC \) is equiangular to \( \triangle DGF \) and \( BA \cdot AC = GD \cdot DF \) [VI.4].

Recall that \( BA : AC = ED : DF \) by assumption, thus \( ED : DF = GD \cdot DF \) [V.11].

Then \( ED = GD \) [V.9].

Since \( DF \) is shared, \( ED = GD \), and \( \angle FDG = \angle BAC = \angle EDF \) [CN 1], \( \angle FDG = \angle EDF \) [I.4] and \( \angle DFG = \angle DFE, \angle DGF = \angle DEF \), and \( EF = GF \).

Since \( \angle DFG = \angle ACB = \angle DFE \) [CN 1] and \( \angle BAC = \angle EDF \) by assumption, \( \angle ABC = \angle DEF \) [I.32].

Therefore, we can conclude that \( \triangle ABC \) is equiangular to \( \triangle DEF \).