Proposition XIII.9

If the side of the hexagon and that of the decagon inscribed in the same circle are added together, then the whole straight line has been cut in extreme and mean ratio, and its greater segment is the side of the hexagon.

Given:
A circle $ABC$ with diameter $AB$, $BC$ equal to the side length of a regular decagon inscribed in $ABC$, and $CD$ equal to the side of a regular hexagon inscribed in circle $ABC$. 
Claims:
1) $BD$ is cut at $C$ in extreme and mean ratio, and
2) $CD$ is the greater segment.

Proof:
Let $E$ be the center of $ABC$. [III.1]
Join $EC$ and $ED$.
The circumference of $ABC$ is 10 times circumference $BC$.
So $(\text{circumference } ACB) = 5(\text{circumference } BC)$.
$\text{(circumference } AC) = 4(\text{circumference } BC)$.
$(\text{circumference } AC) : (\text{circumference } BC) = \angle AEC : \angle CEB$.
$\therefore \angle AEC = 4\angle CEB$.
$\angle EBC = \angle ECB$ since $BE = CE$. [I.5]
$\angle AEC = \angle EBC + \angle ECB = 2\angle ECB$. [I.32]

Since $CD$ = side of regular hexagon inscribed in $ABC$, and since $EC$ = radius of circle $ABC$:
$CD = EC$. [IV.15 cor.]

So $\angle CED = \angle CDE$. [I.5]
$\angle ECB = \angle CED + \angle CDE = 2\angle CDE$. [I.32]
$\therefore \angle AEC = 4\angle CDE$.
So $\angle CEB = \angle EDC$.

In $\triangle BEC$ and $\triangle BDE$:
$\angle EBD$ is common, and $\angle CEB = \angle EDC$, so $\angle BDE = \angle ECB$. [I.32]
Therefore the triangle $\triangle BEC$ and $\triangle BDE$ are equiangular.
$DB : BE = EB : BC$ [VI.4]
$BE = CD$, so $DB : CD = DC : BC$.
Thus $BD$ is cut in extreme mean ratio, and $CD$ is the greater segment.
Q.E.D.