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This version: February 16, 2016

Abstract

We study optimal macroprudential policies in an heterogeneous agent economy with collateral constraints. An enforcement problem results in a collateral constraint that depends on the market value of the borrower’s assets. First, we analyze the optimal allocation chosen by a Pareto Planner that cannot solve the enforcement problem faced by the markets and can only affect borrower’s policies while taking the supply of credit as given. Second, we study the problem of a Ramsey Planner that can choose taxes that affect both the demand and supply of credit. While the standard macroprudential motive would make it optimal for the Planner to reduce total borrowing, the competitive equilibrium of our model can also feature under-borrowing when the borrower’s initial wealth is low enough. Since the model explicitly considers both borrowers and savers, the choice of the loan-to-value ratio and the taxes on assets and borrowing induce distributional effects. We find that the presence of over-borrowing critically depends on only taking into account the welfare of borrowers while disregarding that of savers.


Keywords: Macroprudential Policy, Heterogenous Agents, Credit and Financial Policy, Political Economy

*We are grateful to Nobuhiro Kiyotaki, Enrique Mendoza, and Matthias Paustian for very helpful comments. We also thank seminar participants at the Federal Reserve Board for very useful feedback. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System, the Federal Reserve Bank of Boston, or of any other person associated with the Federal Reserve System.

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1 Introduction

The literature on optimal policy in models with financial constraints has followed two different approaches. One, following the work of Lorenzoni (2008) and Bianchi (2012), emphasizes the need to restrict borrowing in normal times in order to reduce frequency and the severity of crises ex post. The argument behind this type of macroprudential intervention is that, in the absence of regulation, agents’ fail to internalize the effect of their borrowing decisions on asset prices and financial constraints. A planner can then improve welfare by reducing borrowing in good times. The second approach focuses instead on policy intervention during a crisis and explores the ways in which the relaxation of binding financial constraints can limit the contraction of borrowing during a crisis and hence result in a quicker recovery. Papers that fall in the second category are for example Gertler and Kiyotaki (2009) and Gertler and Karadi (2011).\(^1\)

In this paper we develop a framework that allows us to analyze the interaction between macroprudential policies when financial constraints are slack and ex-post intervention when constraints are binding. The model considers the presence of domestic borrowers and savers, where their interaction is modeled in general equilibrium. This is a distinct and novel feature of our analysis for several reasons.

First, the literature has often focused on small open economies where the welfare of international lenders is disregarded. By contrast, we consider the effect that policies exert both on the welfare of borrowers and lenders. Such an approach is suited for economies such as the U.S. where a large allocation of credit and saving occurs between domestic savers and lenders.\(^2\) This approach allows to better understand the effects that policy has on both savers and borrowers as well as their interaction. For instance, macroprudential policies in our framework affects the interest rate at which agents lend and borrow.

\(^1\)Geneakoplos and Polemarchakis (1986) showed the presence of externalities in settings alike to Kiyotaki and Moore (1998).

\(^2\)For instance Iacoviello (2005) estimates a model for the US economy and finds that the presence of both borrowers and lenders is required to match the data.
Second, and most importantly, by explicitly modelling both borrowers and savers, our approach can examine the distributional effects that macroprudential policies generate and the associated political economy issues. This is an issue of first order importance as borrowers and savers are on the opposite sides of trading credit. Policies that are beneficial to one group are not necessarily so to the other. For instance, the finding in the literature that there is over-borrowing is only matched in our framework by the claim that not only borrowers borrowed too much, but that savers saved too much. Limiting the amount of borrowing carries welfare consequences for all agents, not only borrowers, that must be examined.

This paper shows that different optimal policy concepts being used in the literature are tightly connected to the ability of the planner to tax borrowers and lenders and on the available tax instruments. The collateral constraint, despite depending on asset prices, has often been considered to be equivalent to a technological constraint—such as the feasibility constraint or the production function—that cannot be overcome in optimal policy. We show that such planner’s problem, often referred to as constrained efficient planner, is equivalent to a Ramsey planner problem that can tax borrowers but not lenders. Once lenders are considered to be domestic agents that can be taxed, a Ramsey planner will find optimal to affect both the credit demand and supply margins.

We find that once borrowers and savers are taken into account, there is both a region of over-borrowing and under-borrowing for which the correct policy levels are quite different. The level of macroeconomic uncertainty plays a key role in determining the presence of underborrowing and overborrowing. Absent uncertainty, either the economy is efficient, or there is underborrowing in case the loan-to-value ratios are binding. Once there is uncertainty in the economy, both overborrowing and underborrowing are possible. If the constraints are not binding today, but may be binding tomorrow, then agents try to borrow too much. The reason is that borrowing is still unrestricted, but if a crisis hits and the collateral constraint becomes binding then ex-post the planner would have preferred that ex-ante there would had been less borrowing.

The paper is organized as follows. Section 2 presents the main model in a finite horizon environment. Section 3 discusses the theoretical results regarding the planner’s problem. Section 4 presents the main results. Section 5 considers the infinite horizon version of the
model. Section 6 concludes.

2 The Three Period Model

The Environment: The economy lasts three periods: \( t = 1, 2, 3 \). There are two goods in the economy: consumption goods \( c \) and housing goods \( h \). Housing is in fixed supply, and agents undertake production of consumption goods by employing a technology that uses as only input housing goods (chosen in the previous period) and features decreasing returns:

\[
y_t = A_t F(h_{t-1}) = A_t h_{t-1}^\gamma,
\]

where \( \gamma < 1 \) controls the curvature of the production function and \( A_t \) is a measure of productivity that is stochastic at \( t = 2 \) only.

The economy is populated by two types of agents which may differ either in their endowments, discount factors, or future production opportunities. All else equal, agents who are temporarily wealth-poor, impatient, or have relatively better production possibilities in the future will be borrowers in equilibrium. Accordingly, we will refer to the other group of agents as savers, and we will denote each group respectively as borrowers and savers.\(^3\) Both agents’ utility functions are given by:

\[
U^b(c_t) = E_1 \sum_{t=1}^3 \beta^t \log (c_t), \quad U^s(c'_t) = E_1 \sum_{t=1}^3 (\beta')^t \log (c'_t)\]  

Agents can trade a non state–contingent security which we will refer to as risk free loan. The budget constraint for an agent that enters time \( t \) with \( h_{t-1} \) housing goods and \( b_{t-1} \) loans is given by, in each of the three periods:

\[
c_t + q_t h_t + R_{t-1} b_{t-1} \leq \omega_t + A_t h_{t-1}^\gamma + q_t h_{t-1} + b_t,
\]

where \( b_t \) is the amount of loans and \( q_t \) is the price of housing goods. The amount \( \omega_t \) is an exogenous endowment of goods that agents receive in period 1. \( A_t \) is random in period 2.

We assume that an enforcement problem gives rise to a collateral constraint on the amount of loans that borrowers can take on. In particular, a borrower that owns an amount

\(^3\)We will follow the notational convention of denoting variables chosen by the lender with a ‘.
$h_t$ of housing goods at time $t$ will face the following collateral constraint

$$b_t \leq mq_t h_t,$$  

(4)

where $m$ is a parameter that measures the haircut applied to the housing collateral.

The Lender's Problem: Given an initial endowment $h'_{t-1}$ and indebtedness $b'_{t-1} R'_{t-1}$, the lender chooses $\{c_t\} \{b_t\}$ and $\{h_t\}$ to solve

$$\max E_1 \sum_{t=1}^3 \beta^t \log (c_t')$$

subject to:

$$c_t' + q_t h_t' + R_{t-1} b_{t-1}' \leq \omega_t' + A_t h_{t-1}' + q_t h_{t-1}' + b_t'$$  

(6)

The optimality conditions are given by

$$1 = \beta E_t \left\{ \frac{c_t'}{c_{t+1}'} \right\} R_t,$$

(7)

$$1 = \beta E_t \left\{ \frac{c_t A_{t+1} h_{t+1}'}{c_{t+1}'} + \frac{q_t}{q_t} \right\},$$

(8)

plus the budget constraint (10) with equality. The optimality conditions indicate that since the lender is unconstrained, he will equalize the discounted return on housing and loans.

The Borrower's Problem: Analogously, given an initial endowment $h_{t-1}$ and indebtedness $b_{t-1} R_{t-1}$, the borrower chooses $\{c_t\} \{b_t\}$ and $\{h_t\}$ to solve

$$\max E_1 \sum_{t=1}^3 \beta^t \log (c_t)$$

subject to:

$$c_t + q_t h_t - b_t \leq \omega_t + A_t h_{t-1}' + q_t h_{t-1} - b_{t-1} R_{t-1}$$

(10)

$$b_t \leq mq_t h_t$$

(11)

Letting $\lambda_t$ denote the Lagrange multiplier on the collateral constraint (11), the first order conditions for this problem are given by:

$$\frac{1}{c_t} = \beta E_t \left\{ \frac{R_t}{c_{t+1}} \right\} + \lambda_t,$$

(12)
\[ \frac{q_t}{c_t} = \beta E_t \left\{ \frac{A_{t+1} F'(h_t) + q_{t+1}}{c_{t+1}} \right\} + m \lambda_t q_t, \quad (13) \]

together with the complementary slackness condition on (11) and the budget constraint (10) with equality. Inspection of equations (12) and (13) reveals the effect of the collateral constraint on the competitive equilibrium. A binding collateral constraint, \( \lambda_t > 0 \), prevents the borrower from undertaking investment in housing even though the marginal benefit of such investment is strictly higher than the marginal cost of obtaining funds to finance it:

\[ \beta E_t \left\{ \frac{c_t A_{t+1} F'(h_t) + q_{t+1}}{q_t} \right\} > \beta E_t \left\{ \frac{c_t}{c_{t+1}} R_t \right\}. \]

The collateral constraint is therefore preventing mutually beneficial trade between borrowers and savers. In our welfare analysis below we will explore different ways in which a planner, although forced to respect the collateral constraint and to operate through the same markets as private agents, can reduce the extent of such unexploited trade opportunities.

The competitive equilibrium for this economy is defined as:

**Definition 1** A competitive equilibrium is an allocation \( \{c_t, c'_t, h_t, h'_t, b_t, b'_t\} \) and prices \( \{R_t, q_t\} \) such that given prices and initial conditions the allocation solves agents’ problem and all markets clear. The equilibrium allocation and prices are determined by the system of equations (6) – (13) plus the complementary slackness condition on the collateral constraint (11).

### 2.1 The Planner’s Problem

There are two sources of inefficiency in this model: market incompleteness and the collateral constraint. Our welfare analysis will take as a benchmark the allocation chosen by a Pareto Planner that cannot undo either of these inefficiencies. This allows us to isolate the effect of uninternalized pecuniary externalities on the equilibrium allocation and welfare.

In line with the literature that mostly considered small open economies where international lenders cannot be taxed, we start by considering optimal regulation of borrowers taking as given the supply of credit. In particular, we assume that the planner can only prescribe an allocation to the borrower while he is not able to control the behavior of the lender. This implies that in choosing the allocation the planner is constrained to respect the optimality conditions of lenders. In fact, these optimality conditions will represent the
supply schedules for loans, housing and consumption goods that the planner uses in order to compute market prices associated to each allocation prescribed to the borrower.

The planner problem is to choose an allocation \( \{ c_t, c'_t, h_t, h'_t, b_t \} \) and prices \( \{ R_t, q_t \} \) to solve

\[
\max E_1 \sum_{t=1}^{3} \beta^t \log (c_t)
\]

subject to:

\[
c_t + q_t h_t - b_t \leq \omega_t + A_t h^c_{t-1} + q_t h_{t-1} - b_{t-1} R_{t-1}
\]

\[
b_t \leq m q_t h_t
\]

\[
c'_t + q_t h'_t + b'_t \leq \omega'_t + A_t h'^c_{t-1} + q_t h'_{t-1} - b'_{t-1} R_{t-1}
\]

\[
1 = \beta E_t \left\{ \frac{c'_t}{c'_{t+1}} \right\} R_t
\]

\[
1 = \beta E_t \left\{ \frac{c'_t A_{t+1} F' (h'_t) + q_{t+1}}{q_t} \right\}
\]

\[
E_1 \sum_{t=1}^{3} \beta^t \log (c'_t) \geq v^{CE} (h_{-1}, h'_{-1}, b_{-1} R_{-1})
\]

where \( v^{CE} (h_{-1}, h'_{-1}, b_{-1} R_{-1}) \) is the indirect utility function of the lender in a competitive equilibrium with an initial distribution of wealth given by \( (h_{-1}, h'_{-1}, b_{-1} R_{-1}) \). The last constraint hence ensures that the lender gets at least as much utility as in a laissez-faire equilibrium.

**Definition 2** The competitive equilibrium is constrained inefficient if the value obtained by the borrower in the Planner’s allocation, the optimized objective (11), is higher than his indirect utility function in a competitive equilibrium with the same initial distribution of wealth, \( v^{CE} (h_{-1}, h'_{-1}, b_{-1} R_{-1}) \).

While the analysis of the constrained efficient allocation and its comparison with the equilibrium clarifies the types of policies that are desirable in our framework, in practice it might be difficult to directly regulate consumption and portfolio choices of borrowers. Hence we now turn to study implementation of the planner’s allocation in a decentralized equilibrium in which the only instruments available are taxes on borrowing.
3 Ramsey Problem and Equivalence Results

At the heart of the constrained efficient planner (henceforth CEP) problem is the assumption that the interest rate and house prices are determined by the lender, and that such prices cannot be affected by the planner. Such restriction shapes the policies of the constrained efficient planner to a large extent.

Proposition 1 In the constrained efficient planners problem, if all prices \((q, R)\) can be set arbitrarily, then the planner achieves the first-best solution. This result holds regardless of whether the borrowing constraint is present or not.

The intuition for this result is that if the planner could choose the prices \((q, R)\) then it is effectively choosing directly the allocations of the lender (as it does for the saver). Such equilibrium concept is equivalent to the first best where only the technological constraints need to be satisfied. Also once prices are set directly by the planner, the borrowing constraint becomes irrelevant. The planner can choose asset prices \(q\) to sidestep the borrowing constraint, and then can set \(R\) to redistribute wealth among savers and lenders.

While the theorem states the powerful incentives to influence prices, the implementation of such policies would possibly require tax instruments that are unfeasible in a market economy and for this reason it is unrealistic to assume that the pair \((q, R)\) can be set directly by the planner.

The next theorem states the opposite result. If the planner does not internalize the effect of its policies on prices then the solution would coincide with the competitive equilibrium. This result is intuitive since the margin of improvement from the planner relies in correcting the externality that prices generate on the collateral constraint.

Proposition 2 In the constrained efficient planners problem, if the planner does not internalize the effects on prices, and if there are no externalities other than those on prices, then the solution coincides with the competitive equilibrium. This result holds as long as the minimum amount of utility given to the saver is consistent with the competitive equilibrium.

The two theorems above show that in our setup the ability to influence prices determines the extent to which a planner can improve on the market. Should a planner be able to
completely determine prices, then the first best can be achieved. If the planner does not internalize the effects of its policy on prices, then the market and planner’s solutions coincide. The degree to which a planner finds herself closer to the former or the latter depends to a large extent on the set of available policy instruments. To examine more closely the implementation of policy we now set the problem of a Ramsey Planner that can set taxes on borrowing and housing both on the lender and the saver. We then examine the effects of restricting the set of available taxes.

We assume that the tax revenues of both borrowing and housing taxes are immediately rebated to households as lump-sum transfers. Under this scenario the budget constraint of borrowers and households remains unchanged. But with the new set of taxes the FOCs of borrowers and households become:

\[
- \phi_1 \left( \frac{1}{c_1} q_1 (1 + \tau_{h1}) - \beta \left( \pi A_{2L} \gamma h_{1}^{\gamma - 1} + q_{2L} \right) c_{2L} + (1 - \pi) A_{2H} \gamma h_{1}^{\gamma - 1} + q_{2H} \right) - \lambda_1 m_1 (1 + \tau_{m_1}) q_1 = 0
\]

(21)

\[
- \phi_{2L} \left( \frac{1}{c_{2L}} q_{2L} (1 + \tau_{h2L}) - \beta \frac{1}{c_{3L}} A_{3L} \gamma h_{2L}^{\gamma - 1} - \lambda_2 m_2 (1 + \tau_{m_2}) q_2 \right) = 0
\]

(22)

\[
- \phi_{2H} \left( \frac{1}{c_{2H}} q_{2H} (1 + \tau_{h2H}) - \beta \frac{1}{c_{3H}} A_{3H} \gamma h_{2H}^{\gamma - 1} - \lambda_2 m_2 (1 + \tau_{m_2}) q_2 \right) = 0
\]

(23)

\[
- \phi' \left( \frac{1}{c'_1} q_1 (1 + \tau'_{h1}) - \beta' \left( \pi A'_{2L} \gamma (1 - h_1) (1 - m_1) + q_{2L} \right) c'_{2L} + (1 - \pi) A'_{2H} \gamma (1 - h_1) (1 - m_1) + q_{2H} \right) = 0
\]

(24)

\[
- \phi'_{2L} \left( \frac{1}{c'_{2L}} q_{2L} (1 + \tau'_{h2L}) - \beta' \frac{1}{c'_{3L}} A'_{3L} \gamma (1 - h_2) (1 - m_2) \right) = 0
\]

(25)

\[
- \phi'_{2H} \left( \frac{1}{c'_{2H}} q_{2H} (1 + \tau'_{h2H}) - \beta' \frac{1}{c'_{3H}} A'_{3H} \gamma (1 - h_2) (1 - m_2) \right) = 0
\]

(26)

\[
- \theta_1 \left( \frac{1}{c_1} (1 - \tau_{b1}) - \beta R_1 \left( \pi \frac{1}{c_{2L}} + (1 - \pi) \frac{1}{c_{2H}} \right) - \lambda_1 \right) = 0
\]

(27)

\[
- \theta_{2L} \left( \frac{1}{c_{2L}} (1 - \tau_{b2L}) - \beta \frac{1}{c_{3L}} R_{2L} - \lambda_{2L} \right) = 0
\]

(28)

\[
- \theta_{2H} \left( \frac{1}{c_{2H}} (1 - \tau_{b2H}) - \beta \frac{1}{c_{3H}} R_{2H} - \lambda_{2H} \right) = 0
\]

(29)

\[
- \theta'_{1} \left( \frac{1}{c'_1} (1 - \tau'_{b1}) - \beta' R_1 \left( \pi \frac{1}{c'_{2L}} + (1 - \pi) \frac{1}{c'_{2H}} \right) \right) = 0
\]

(30)

\[
- \theta'_{2L} \left( \frac{1}{c'_{2L}} (1 - \tau'_{b2L}) - \beta' \frac{1}{c'_{3L}} R_{2L} \right) = 0
\]

(31)

\[
- \theta'_{2H} \left( \frac{1}{c'_{2H}} (1 - \tau'_{b2H}) - \beta' \frac{1}{c'_{3H}} R_{2H} \right) = 0
\]

(32)
where $\tau_b, \tau_h, \tau_b', \tau_h'$ are the taxes on borrowing and housing, and $\phi, \phi', \theta, \theta'$ are the associated Lagrange multipliers in the maximization problem. The restriction on the savers utility is unchanged but the planner also needs to respect the Kuhn-Tucker conditions from the problem of savers and borrowers.

\[(b_1 \leq m_1 (1 + \tau_{m_1}) q_1 h_1) \text{ and } \lambda_1 (b_1 - m_1 (1 + \tau_{m_1}) q_1 h_1) = 0 \quad (33)\]
\[(b_{2L} \leq m_{2L} (1 + \tau_{m_{2L}}) q_{2L} h_{2L}) \text{ and } \lambda_{2L} (b_{2L} - m_{2L} (1 + \tau_{m_{2L}}) q_{2L} h_{2L}) = 0 \quad (34)\]
\[(b_{2H} \leq m_{2H} (1 + \tau_{m_{2H}}) q_{2H} h_{2H}) \text{ and } \lambda_{2H} (b_{2H} - m_{2H} (1 + \tau_{m_{2H}}) q_{2H} h_{2H}) = 0 \quad (35)\]
\[\lambda_1 \geq 0, \lambda_{2L} \geq 0, \lambda_{2H} \geq 0 \quad (36)\]

where $\tau_m$ are direct policy instruments on the loan-to-value ratio. For direct comparison with the constrained efficient planner we consider that $\tau_m = 0$ unless otherwise noted. The appendix shows the Lagrangean formulation of this problem.

**Proposition 3** If $\tau_b' = \tau_h' = 0$, then any Ramsey taxation optimization cannot improve on the constrained efficient planner allocation.

This result is also intuitive. In the constrained efficient planner’s problem, the $R$ and $q$ functions are given by the first order conditions of the saver. If the Ramsey planner cannot affect those incentives then it faces the same constraints as the CEP. Hence, the Ramsey planner facing the constraints $\tau_b' = \tau_h' = 0$ cannot improve on the CEP. Note that in such circumstance the Ramsey planner can do worse. The CEP chooses directly the allocations of the borrower. If the Ramsey planner has enough instruments such that $\phi_1 = \phi_{2L} = \phi_{2H} = 0$, i.e. the FOCs of the borrower are not binding, then the two solutions coincide. But the Ramsey planner will achieve a lower welfare as long as it faces restrictions on the tax instruments that affect the borrower’s decisions.

**Proposition 4** If the planner has enough tax instruments such that $\phi_1 = \phi_{2L} = \phi_{2H} = 0$, and the Ramsey planner retains the option to tax the saver—i.e. choose $\tau_b'$ and $\tau_h'$—then the welfare from the Ramsey planner’s allocation is at least as high as that from the constrained efficient planner.

The proof of this result follows from the fact that if the Ramsey planner constraints itself to choose $\tau_b' = \tau_h' = 0$ then the solution would coincide with the constrained efficient planner.
Corollary 5 The solution to the Ramsey planner without lump-sum taxes but with the choice of a restricted set \( \{ \tau_b, \tau_h, \tau'_b, \tau'_h \} \) may deliver both higher and lower welfare than the constrained efficient planner.

This result follows from the combination of Propositions 4 and 5. As long as the Ramsey planner can affect the saver then it faces an advantage relative to the CEP. However, the constrained efficient planner can choose enough tax instruments to completely circumvent the FOCs of the borrower. A Ramsey Planner that has some tax instruments but may not completely avoid the FOCs of borrowers — say because taxes on borrowing are time- and state-invariant — faces a disadvantage relative to the CEP.

The theoretical results in this section show the relation between the Ramsey planner and the constrained efficient planner. The constrained efficient planner concept has been used mainly in small open economy environments where international lenders cannot be taxed. In this paper, we consider an economy where both borrowers and savers are domestic and can be taxed by the planner. In the next section we explore quantitatively how the restrictions on the available set of instruments on the Ramsey planner restrict macroprudential policy. In doing so, we will examine how the overborrowing and underborrowing regions change based on the set of available instruments, risk, wealth distribution, and political economy considerations.

4 Calibration and Findings

4.1 Calibration

We assume that \( \beta = 1 \) for all agents. In the production function we set \( \gamma = 0.5 \). Productivity is set by \( A = 1 \) in period 1 and 3. In period 2, we choose two calibrations depending on whether we introduce uncertainty. At \( t = 2 \), \( A \) can be either 1 when there is no uncertainty, or \( A_H = 1.25 \) and \( A_L = 0.75 \) with equal probability when there is uncertainty. The total endowment of \( h \) is 1 in all periods. The loan-to-value ratio is \( m = 1/2 \).

We assume borrowers have an initial endowment of \( h_0 \) and \( \omega_1 \) in the first period, whereas savers have \( 1 - h_0 \) and \( 1 - \omega_1 \).

4.2 Findings

Optimal Policy, No uncertainty: To highlight the sources of inefficiencies in our economy, it is worthwhile to start with the benchmark case in which there is no uncertainty. In
this case, the competitive equilibrium is in general constrained Pareto optimal if \( m \) is large enough or if the initial allocations are similar across agents.

However, when \( m \) is low enough and the initial allocations are such that relatively poor agents want to borrow, the competitive equilibrium is constrained inefficient and features underborrowing. Optimal policy calls in this case for a relaxation of borrowing constraint. In practice, this can occur via a subsidy on borrowing that is financed by lump-sum taxation in the same period. When constraints are slack, laissez faire is in general optimal.

These cases are plotted in Figure 1, which plots market allocations relative to the Ramsey planner as a function of the initial wealth share of the borrower, given by his initial endowment share of \( h_0 \) and \( \omega_1 \). To avoid cluttering, we only compare the market allocations with those implemented by a planner who uses taxes on borrowing. However, the Ramsey allocations are very similar to those chosen by a constrained planner.

Panel 1 plots market borrowing. As the endowment share of one agent becomes closer to one half, agents become identical so that optimal borrowing approaches zero. The lower the initial endowment of the borrower, the larger his incentive to borrow is. For low values of the endowment, the borrower hits the borrowing constraint in periods 1 and 2, as shown by the grey shaded region. Panel 2 plots market borrowing relative to the borrowing chosen by a planner who can use taxes. The planner can increase utility of the borrower (without making the saver worse off), as shown in panel 3. He can do so by allowing the borrower to increase his leverage in period 2 by giving a subsidy to borrowing financed by lump-sum taxes (panels 4 to 6).

**Optimal Policy with Uncertainty:** The notion that borrowing must be subsidized when there is no uncertainty changes when we consider uncertainty. Here policy becomes macro-prudential, in the sense that there is a region of the parameter space when laissez faire is suboptimal even when constraints are not binding today.

Figure 2 illustrates this second case. When the initial endowment is very small (grey region), there is still underborrowing as the constraint always binds regardless of the future states of the world. In the region where the constraint always binds, optimal policy gives an average subsidy to borrowing — to undo credit frictions. In addition, optimal policy taxes borrowing in booms, and subsidizes borrowing in recessions. It does so because repayments are not indexed to the state of the economy, but indexing them makes everybody better off.

For intermediate values of the endowment (the green region), the borrowing constraint becomes slack today, but may bind tomorrow in a recession. In this case, there is overborrowing (panel 2) and optimal policy becomes macroprudential: the planner taxes borrowing
on average — to avoid overborrowing if constraints are expected to bind in the future. Then, if crisis hit and constraints become tight, policy subsidizes borrowing.

5 Conclusions

We examine optimal credit market regulation in an heterogeneous agents economy with borrowing constraints. In this setup, the optimal choice of macroprudential policy depends on redistribution concerns between borrowers and savers. Optimal policy also depends on whether the Pareto Planner can solve the enforcement problem faced by the markets and affect both savers’ and borrowers’ incentives. We examine both the cases in which the Ramsey Planner can choose taxes that only affect the demand for credit as well as when it can affect the supply.

While the standard macroprudential motive would make it optimal for the Planner to reduce total borrowing, the competitive equilibrium of our model can also feature underborrowing when the borrower’s initial wealth is low enough. The presence of both overborrowing and under-borrowing reflects two roles of macro-prudential policy. The first role relates to the need to restrict borrowing in times when a crisis is not present but is eminent such that the severity of the crises is mitigated. The second role relates to the inefficiency of limiting the flow of funds between agents that want to trade. Borrowing may be optimal but too low if the collateral constraint is too tight after a crisis as well as if the marginal value of funds held by borrowers is much larger than that of savers.

Our results clarify that if there is no uncertainty and borrowing is constrained, the economy features under-borrowing. The introduction of uncertainty brings a second element into macroprudential policy. Once uncertainty is present, the economy features both under-borrowing and over-borrowing. When borrower’s wealth is low enough, the under-borrowing motive prevails. If the economy is in a situation where a crisis may occur next period and the collateral constraint may become binding, then overborrowing is present and the planner taxes borrowing.

By explicitly modelling both borrowers and savers, our paper examines the general equilibrium effects, including the effects on interest rates and asset prices, that are otherwise disregarded in small open economy models with international lenders. Hence, the choice of the loan-to-value ratio and the taxes on assets and borrowing induce distributional effects. We find that the optimal choice of taxes as well as the presence of over-borrowing critically depends on only taking into account the welfare of borrowers while disregarding that of
savers.

We find that if the status quo loan-to-value ratios are inefficiently low, then increasing them is welfare improving for both agents. However, if the loan-to-value ratios are inefficiently high, then reducing them is welfare improving for borrowers but not for lenders. The reason is that a reduction in the loan-to-value ratio is equivalent to a coordination mechanism by borrowers to achieve a more favorable interest rate. This result also shows that, from a distributional and political economy perspective, the loan-to-value ratio is easier to increase than to decrease.

In terms of policy implementation, we find that time-invariant or state-invariant policies cannot always implement optimal policy well. This result is intuitive since we find that once borrowers and savers are taken into account, there is both a region of over-borrowing and under-borrowing over which the optimal tax levels are quite different. On the other hand, if the levels of wealth or the goals of the planner make it such that only over-borrowing is present then imposing state-invariant taxes is less constraining.
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Appendix A  Ramsey Planner

The Lagrangean for a generic Ramsey Planner with:

$$\max \log c_1 + \beta \pi \log c_{2L} + \beta (1 - \pi) \log c_{2H} + \beta^2 \pi \log c_{3L} + \beta^2 (1 - \pi) \log c_{3H}$$ (A.1)

$$- \mu_1 (c_1 + q_1 (h_1 - h_0) - \omega - b_1)$$

$$- \mu_2L (c_{2L} + q_{2L}h_{2L} - A_{2L}h_1^{1} - b_{2L} + R_1b_1 - q_{2L}h_1)$$

$$- \mu_2H (c_{2H} + q_{2H}h_{2H} - A_{2H}h_1^{1} - b_{2H} + R_1b_1 - q_{2H}h_1)$$

$$- \mu_3L (c_{3L} + R_{2L}b_{2L} - A_{3L}h_2^{1}) - \mu_3H (c_{3H} + R_{2H}b_{2H} - A_{3H}h_2^{1})$$

$$- \mu'_1 (c'_1 - q_1 (h_1 - h_0) - \omega + b_1)$$

$$- \mu'_2L (c'_{2L} - q_{2L}h_{2L} - A'_{2L} (1 - h_1)^{\gamma} + b_{2L} - R_1b_1 + q_{2L}h_1)$$

$$- \mu'_2H (c'_{2H} - q_{2H}h_{2H} - A'_{2H} (1 - h_1)^{\gamma} + b_{2H} - R_1b_1 + q_{2H}h_1)$$

$$- \mu'_3L (c'_{3L} - R_{2L}b_{2L} - A'_{3L} (1 - h_2)^{\gamma}) - \mu'_3H (c'_{3H} - R_{2H}b_{2H} - A'_{3H} (1 - h_2)^{\gamma})$$

$$- \phi_1 \left( \frac{1}{c_1} q_1 (1 + \tau h_1) - \beta \left( \frac{A_{2L} \gamma h_1^{\gamma-1} + q_{2L}}{c_{2L}} + (1 - \pi) \frac{A_{2H} \gamma h_1^{\gamma-1} + q_{2H}}{c_{2H}} \right) - \lambda_1 m_1 (1 + \tau m_1) q_1 \right)$$

$$- \phi_2L \left( \frac{1}{c_{2L}} q_{2L} (1 + \tau h_{2L}) - \beta \frac{1}{c_{3L}} A_{3L} \gamma h_{2L}^{\gamma-1} - \lambda_2 m_{2L} (1 + \tau m_{2L}) q_{2L} \right)$$

$$- \phi_2H \left( \frac{1}{c_{2H}} q_{2H} (1 + \tau h_{2H}) - \beta \frac{1}{c_{3H}} A_{3H} \gamma h_{2H}^{\gamma-1} - \lambda_2 m_{2H} (1 + \tau m_{2H}) q_{2H} \right)$$

$$- \phi'_1 \left( \frac{1}{c'_1} q_1 (1 + \tau' h_1) - \beta' \left( \frac{A'_{2L} \gamma (1 - h_1)^{\gamma-1} + q_{2L}}{c'_{2L}} + (1 - \pi) \frac{A'_{2H} \gamma (1 - h_1)^{\gamma-1} + q_{2H}}{c'_{2H}} \right) \right)$$

$$- \phi'_2L \left( \frac{q_{2L} (1 + \tau' h_{2L})}{c'_{2L}} - \beta' \frac{A'_{3L} \gamma (1 - h_{2L})^{\gamma-1}}{c'_{3L}} \right) - \phi'_2H \left( \frac{q_{2H} (1 + \tau' h_{2H})}{c'_{2H}} - \beta' \frac{A'_{3H} \gamma (1 - h_{2H})^{\gamma-1}}{c'_{3H}} \right)$$

$$- \theta_1 \left( \frac{1}{c_1} (1 - \tau h_1) - \beta R_1 \left( \frac{1}{c_{2L}} + (1 - \pi) \frac{1}{c_{2H}} \right) - \lambda_1 \right)$$

$$- \theta_2L \left( \frac{1}{c_{2L}} (1 - \tau h_{2L}) - \beta \frac{1}{c_{3L}} R_{2L} - \lambda_2 \right) - \theta_2H \left( \frac{1}{c_{2H}} (1 - \tau h_{2H}) - \beta \frac{1}{c_{3H}} R_{2H} - \lambda_2 \right)$$

$$- \theta'_1 \left( \frac{1}{c'_1} (1 - \tau' h_1) - \beta' R_1 \left( \frac{1}{c'_{2L}} + (1 - \pi) \frac{1}{c'_{2H}} \right) \right)$$

$$- \theta'_2L \left( \frac{1}{c'_{2L}} (1 - \tau' h_{2L}) - \beta' \frac{1}{c'_{3L}} R_{2L} \right) - \theta'_2H \left( \frac{1}{c'_{2H}} (1 - \tau' h_{2H}) - \beta' \frac{1}{c'_{3H}} R_{2H} \right)$$

$$- \mu_p (\bar{\nu} - (\log c'_1 + \beta' \pi \log c'_{2L} + \beta' (1 - \pi) \log c'_{2H} + \beta'^2 \pi \log c'_{3L} + \beta'^2 (1 - \pi) \log c'_{3H}))$$

where $\tau_b$, $\tau_h$, $\tau'_b$, $\tau'_h$ are the taxes on borrowing and housing. The planner also needs to
respect the Kuhn-Tucker conditions from the problem of savers and borrowers.

\[ (b_1 \leq m_1 (1 + \tau_{m_1}) q_1 h_1) \quad \text{and} \quad \lambda_1 (b_1 - m_1 (1 + \tau_{m_1}) q_1 h_1) = 0 \] \tag{A.2}

\[ (b_{2L} \leq m_{2L} (1 + \tau_{m_{2L}}) q_{2L} h_{2L}) \quad \text{and} \quad \lambda_{2L} (b_{2L} - m_{2L} (1 + \tau_{m_{2L}}) q_{2L} h_{2L}) = 0 \] \tag{A.3}

\[ (b_{2H} \leq m_{2H} (1 + \tau_{m_{2H}}) q_{2H} h_{2H}) \quad \text{and} \quad \lambda_{2H} (b_{2H} - m_{2H} (1 + \tau_{m_{2H}}) q_{2H} h_{2H}) = 0 \] \tag{A.4}

\[ \lambda_1 \geq 0, \lambda_{2L} \geq 0, \lambda_{2H} \geq 0 \] \tag{A.5}
Each panel plots specific outcomes as a share of the initial wealth share of one of the two agents. The economy is symmetric when the wealth share of one of the two agents is one half.

Notes: Absent uncertainty, the economy always features underborrowing. When the constraint is binding (grey shaded area), the planner can improve outcomes by subsidizing borrowing in the first period and taxing it in the second.
Each panel plots specific outcomes as a share of the initial wealth share of one of the two agents. The economy is symmetric when the wealth share of one of the two agents is one half.

Notes: With uncertainty, the economy features overborrowing in the green region. This is the region where the constraint is not binding in t1 and in t2 when the state of the economy is high, but is binding in t2 when the state of the economy is low.