Chapter 7: Economic Growth part 1

- Learn the closed economy Solow model
- See how a country’s standard of living depends on its saving and population growth rates
- Learn how to use the “Golden Rule” to find the optimal savings rate and capital stock

Importance of Economic Growth

In the poorest one-fifth of all countries,
- daily caloric intake is 1/3 lower than in the richest fifth
- the infant mortality rate is 200 per 1000 births, compared to 4 per 1000 births in the richest fifth.

- Even in rich countries, small differences can make a difference over long periods of time
- \(1.015^{50}=2.10\)
- \(1.03^{50}=4.38\)

Source: Sala-i-Martin, NBER WP 8933, 2002
1. The Solow Model

looks at determinants of economic growth and the standard of living in the long run

**MAIN ASSUMPTIONS OF THE MODEL**

1. \( K \) is no longer fixed: investment causes it to grow, depreciation causes it to shrink.
2. \( L \) is no longer fixed: population growth
3. The consumption function is simpler.
4. No \( G \) or \( T \)

**The production function**

- In aggregate terms: \( Y = F(K, L) \)

Define: \( \frac{Y}{L} = \text{output per worker} \), \( \frac{K}{L} = \text{capital per worker} \)

- Assume constant returns to scale:
  \( zY = F(zK, zL) \) for any \( z > 0 \)

- Pick \( z = \frac{1}{L} \) Then
  \( \frac{Y}{L} = F(\frac{K}{L}, 1) \)
  \( \frac{y}{l} = F(k, 1) \)
  \( y = f(k) \) where \( f(k) = F(k, 1) \)

**Building Blocks of the model**

1. NATIONAL INCOME IDENTITY
   \( Y = C + I \rightarrow y = C + I \) (in per worker terms)

2. CONSUMPTION AND SAVINGS FUNCTION
   \( C = (1-s)Y \rightarrow c = (1-s)y \)
   \( \text{saving} = sy \)

3. EQUILIBRIUM: \( y - c = sy = sf(k) = i \)
4. **WHAT IS INVESTMENT?**
Assume a fraction \( \delta \) of capital depreciates every period.

Investment = Change in capital stock + depreciation

\[
i = \Delta k + \delta k
\]

\[
\Delta k = s f(k) - \delta k
\]

Determines behavior of \( k \) over time, which determines all other endogenous variables.

E.g., income per person: \( y = f(k) \)

consumption per person: \( c = (1-s) f(k) \)

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**The steady state**

\[
\Delta k = s f(k) - \delta k
\]

If investment is just enough to cover depreciation \( [s f(k) = \delta k] \),
then \( \Delta k = 0 \).

\( k' \): **steady state capital stock**
A numerical example

1. Production function (aggregate):
   \[ Y = F(K, L) = \sqrt{K} \times L = K^{1/2} L^{1/2} \]

2. Rewrite \( y = Y/L \) and \( k = K/L \) to get
   \[ y = f(k) = k^{1/2} \]

3. Assume:
   - \( s = 0.3 \)
   - \( \delta = 0.1 \)
   - initial value of \( k = 4.0 \)

### Approaching the Steady State: A Numerical Example

Assumptions: \( y = \sqrt{k}; \ s = 0.3; \ \delta = 0.1; \) initial \( k = 4.0 \)

<table>
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<tr>
<th>Year</th>
<th>( k )</th>
<th>( y )</th>
<th>( c )</th>
<th>( i )</th>
<th>( \delta k )</th>
<th>( \delta k )</th>
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<td>2.100</td>
<td>0.900</td>
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</table>

### Exercise: solve for the steady state

Continue to assume
\( s = 0.3, \ \delta = 0.1, \) and \( y = k^{1/2} \)

Use the equation of motion
\[ \Delta k = sf(k) - \delta k \]
to solve for the steady-state values of \( k, y, \) and \( c. \)
**Solution to exercise:**

\[ \Delta k = 0 \]  
def. of steady state

\[ sf(k^*) = \delta k^* \]  
eq'n of motion with \( \Delta k = 0 \)

\[ 0.3\sqrt{k^*} = 0.1k^* \]  
using assumed values

\[ 3 = \frac{k^*}{\sqrt{k^*}} = \sqrt{k^*} \]

Solve to get:  \( k^* = 9 \) and \( y^* = \sqrt{k^*} = 3 \)

Finally,  \( c^* = (1 - s)y^* = 0.7 \times 3 = 2.1 \)

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**An increase in the saving rate**

**Prediction:**

- Higher \( s \) \( \Rightarrow \) higher \( k^* \).
- And since \( y = f(k) \),  
  higher \( k^* \) \( \Rightarrow \) higher \( y^* \).
- Countries with higher rates of saving and investment will have higher levels of capital and income per worker in the long run.
- Supported by data
2. The Golden Rule: introduction

- Different $s$ lead to different steady states. How do we know which is the "best" steady state?
- → "best" steady state has highest possible consumption per person: $c^* = (1 - s)f(k^*)$
- An increase in $s$
  - leads to higher $k^*$ and $y^*$, which may raise $c^*$
  - reduces consumption's share of income $(1 - s)$, which may lower $c^*$
- So, how do we find the $s$ and $k^*$ that maximize $c^*$?

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The Golden Rule Capital Stock

$k^*_\text{gold}$ = the Golden Rule level of capital, the steady state value of $k$ that maximizes consumption.

Express $c^*$ in terms of $k^*$:

$c^* = y^* - i^* = f(k^*) - i^* = f(k^*) - \delta k^*$

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The Golden Rule Capital Stock

$c^* = f(k^*) - \delta k^*$

biggest where

MPK = $\delta$

steady-state capital per worker, $K^*$
The transition to the Golden Rule Steady State

- The economy does NOT have a tendency to move toward the Golden Rule steady state.
- Achieving the Golden Rule requires that policymakers adjust \( s \).
- This adjustment leads to a new steady state with higher consumption.
- But what happens to consumption during the transition to the Golden Rule?

Starting with too much capital

If \( k' > k_{\text{gold}} \)
then increasing \( c' \) requires a fall in \( s \).
In the transition to the Golden Rule, consumption is higher at all points in time.

Starting with too little capital

If \( k' < k_{\text{gold}} \)
then increasing \( c' \) requires an increase in \( s \).
Future generations enjoy higher consumption, but the current one experiences an initial drop in consumption.
3. Population Growth

- Assume that the population—and labor force—grow at rate \( n \). \( (n \) is exogenous)\n
\[
\frac{\Delta L}{L} = n
\]

Then:\n
\( (\delta + n)k \) = break-even investment.\n
\( \delta k \) : to replace capital as it wears out\n\( n k \) : to equip new workers with capital

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Population growth

\[
\Delta k = s f(k) - (\delta + n)k
\]

An increase in \( n \) leads to a lower steady-state \( k \).

Countries with higher population growth rates will have lower levels \( k \) and \( y \) in the long run.

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The Golden Rule with Population Growth

To find the Golden Rule capital stock, we again express \( c^* \) in terms of \( k^* \):

\[
c^* = y^* - \bar{r} = f(k^*) - (\delta + n)k^*
\]

\( c^* \) is maximized when \( \text{MPK} = \delta + n \)

or: \( \text{MPK} - \delta = n \)