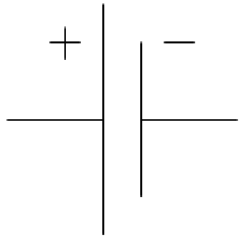
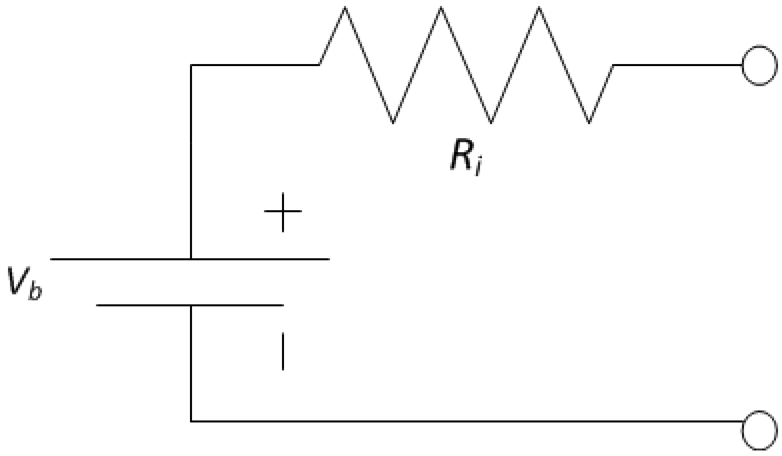
Passive Circuits

Passive Circuits

- Batteries
  - Voltages range from ~1.3V to 180V
  - Internal resistance $I_{max} = \frac{V_b}{R_i}$





Voltage at terminals

$$V_{out} = V_b - IR_i$$

An ideal battery has zero internal resistance.

Passive Circuits

- Kirchhoff's Current Law

Charge is not allowed to pile up at nodes, or

The total current entering any node is equal to the total current leaving that node.

$$\sum_{nodes} I_{node} = 0$$

Passive Circuits

- Kirchhoff's Voltage Law

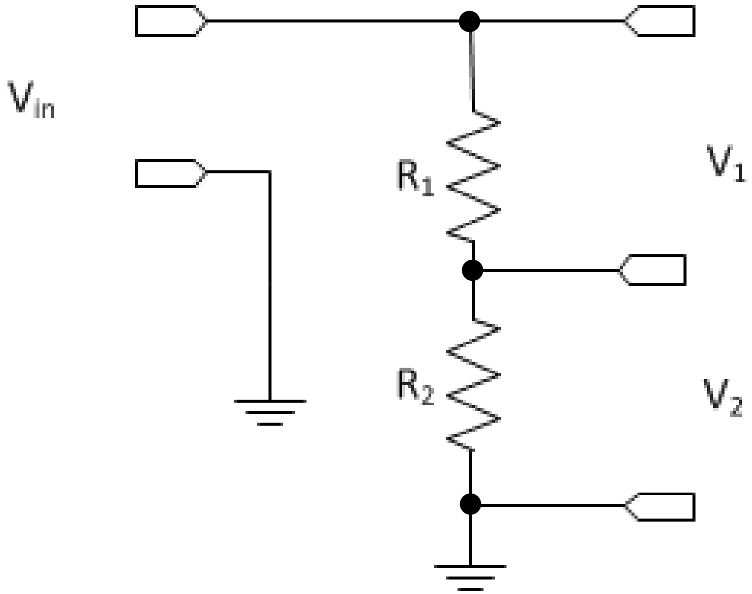
The sum of all of the source voltages in a closed path (loop) is equal to the sum of the voltage drops in that loop.

$$\sum_{loop} V_{loop} = 0$$

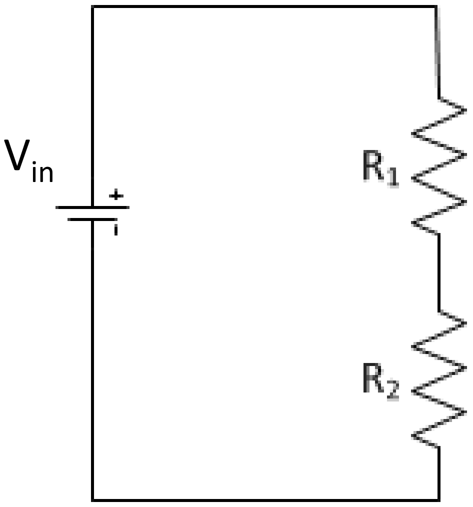
Passive Circuits

- Resistor Networks

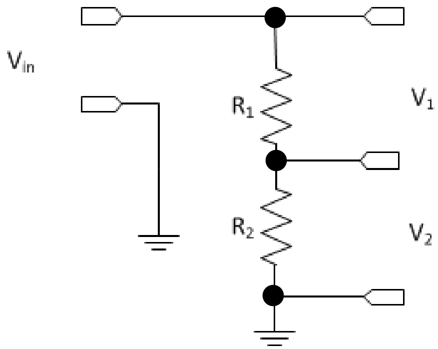
Voltage Divider



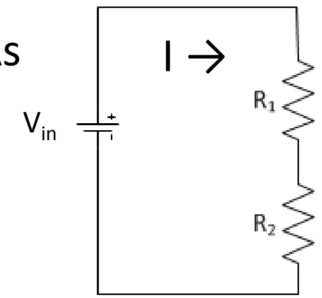
Same As



Passive Circuits



Same As



$R_{total} = R_1 + R_2$

$V_1 = I\,R_1$
$V_2 = I\,R_2$

$V_{in} = V_1 + V_2 = I\,R_{total} = I\,(R_1 + R_2)$

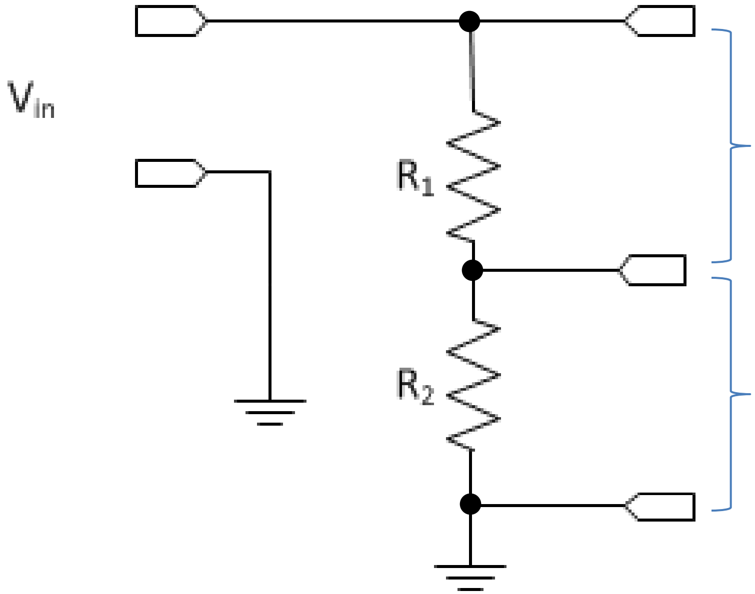
So $I = \dfrac{V_{in}}{R_1 + R_2}$

And $V_1 = V_{in}\dfrac{R_1}{R_1 + R_2}$ & $V_2 = V_{in}\dfrac{R_2}{R_1 + R_2}$

Passive Circuits

- Resistor Networks

Voltage Divider



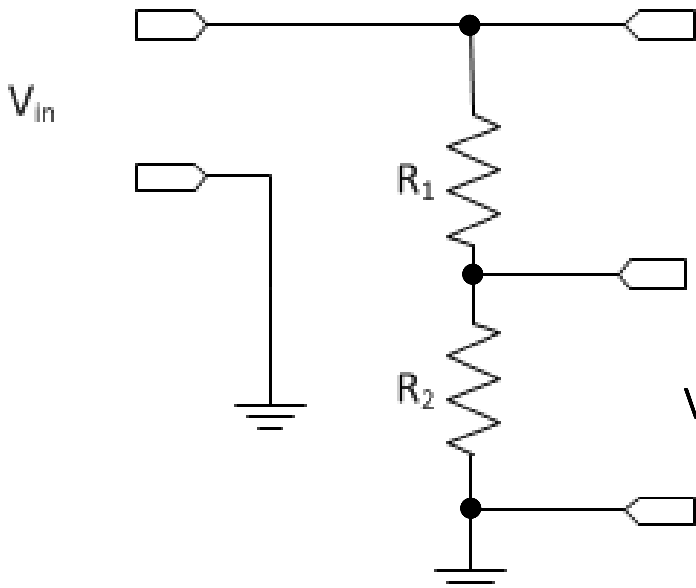
$$V_1 = V_{in} \frac{R_1}{R_1 + R_2}$$

$$V_2 = V_{in} \frac{R_2}{R_1 + R_2}$$

Passive Circuits

- Resistor Networks

Voltage Divider



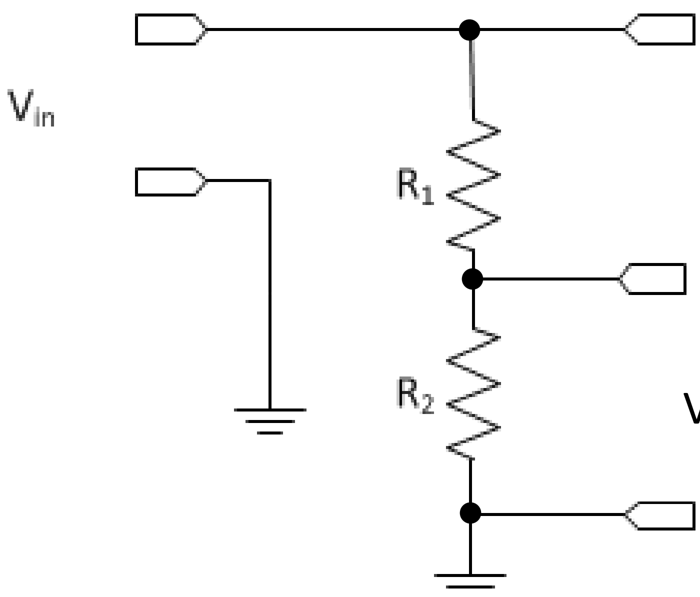
$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

Note: There's an infinite number of valid solutions for R_1 & R_2.

Passive Circuits

- Resistor Networks

Voltage Divider



$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

Suppose V_{in} = 10 volts, and I want an output voltage of 2 volts.

Even with the constraint that $\frac{R_2}{R_1 + R_2} = \frac{1}{5}$, there's an infinite number of valid possible values of R_1 & R_2.

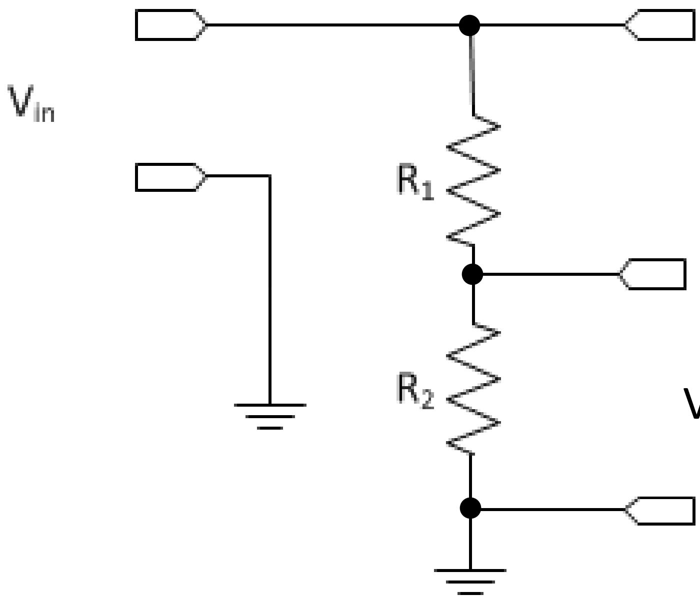
Whadda ya do?

This is dilemma is very common in electronics.

Passive Circuits

- Resistor Networks

Voltage Divider



$$V_{out} = V_{in}\frac{R_2}{R_1 + R_2}$$

Make an arbitrary decision.

Secondary considerations – we might want to be energy efficient. Power loss in the divider is

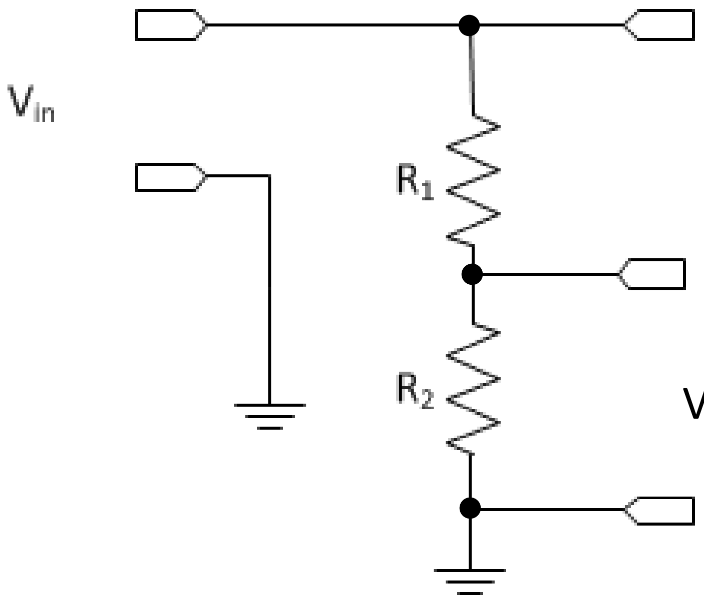
$$P_{loss} = \frac{V_{in}^{\ 2}}{R_1 + R_2}$$

So you can increase energy efficiency by increasing (R_1 + R_2).

Passive Circuits

- Resistor Networks

Voltage Divider



$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

Make an arbitrary decision.

Secondary considerations –

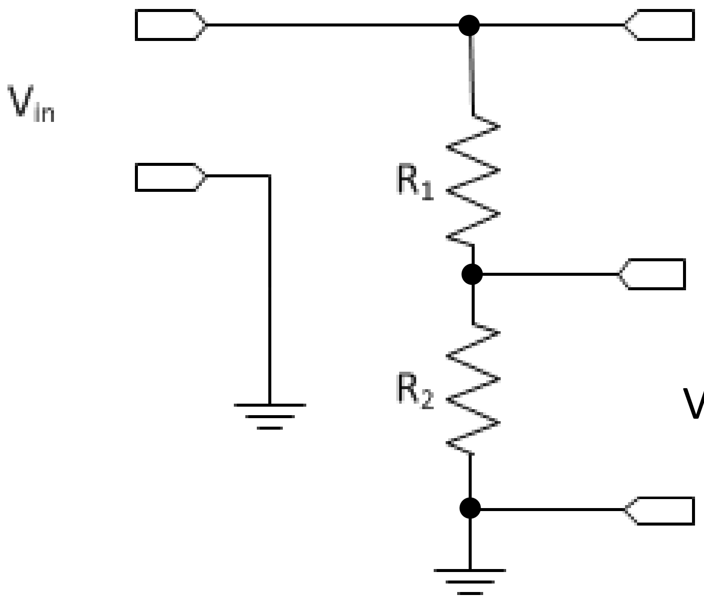
So you can increase energy efficiency by increasing (R_1 + R_2).

BUT… You need R_1 & R_2 to be low enough that whatever circuit you attach to V_{out} does not itself draw enough current to void our voltage divider equation. (i.e. “short” R_2)

Passive Circuits

- Resistor Networks

Voltage Divider



$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

Make an arbitrary decision.

Secondary considerations – Thermal noise from a resistance (R):

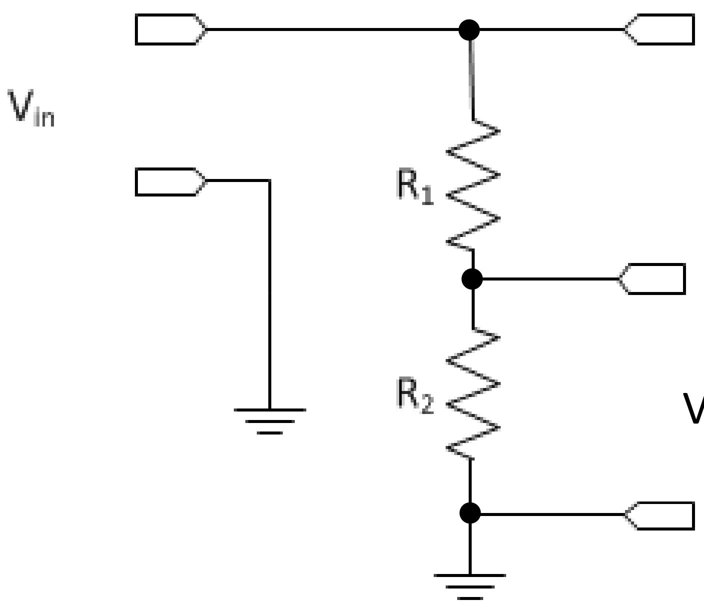
$$V_{RMS} = \sqrt{4k_B T\, R\, \Delta f}$$

Δf = frequency range

Passive Circuits

- Resistor Networks

Voltage Divider



$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

Make an arbitrary decision.

So generally one has to pick a value of $(R_1 + R_2)$ that isn't too large and isn't too small. That choice depends on what you will do with V_{out}.

Passive Circuits

- Impedance

$$Z = R + i\chi \quad \text{where } i = \sqrt{-1},\ \chi \text{ is called the reactance}$$

Engineers use *j* instead of *i*.
H & H uses *j*.

For a resistor, $Z = R$

For a capacitor of capacitance C, $\chi = \dfrac{i}{\omega C}$

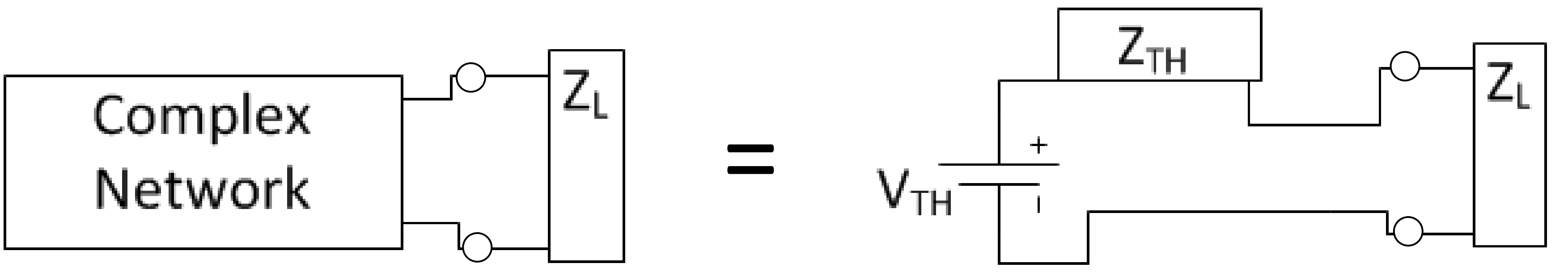
For an inductor of inductance L, $\chi = -i\omega L$

Generalized Ohm's Law: $Z(\omega) = \dfrac{V(\omega)}{I(\omega)}$

Passive Circuits

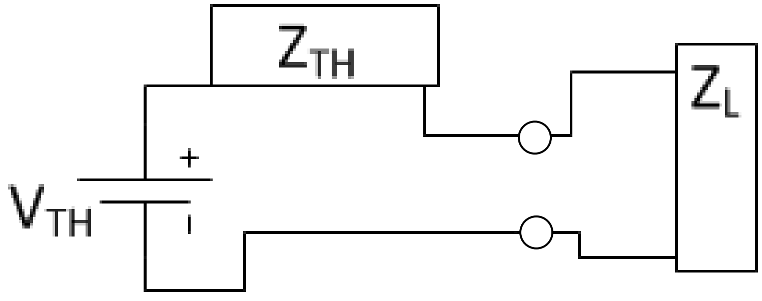
Thevenin's Theorem

We can convert any impedance Z_L, connected to two terminals of a network into an equivalent voltage source (V_{TH}) in series with an impedance Z_{TH}, connected to that Z_L.



Passive Circuits

Thevenin's Theorem

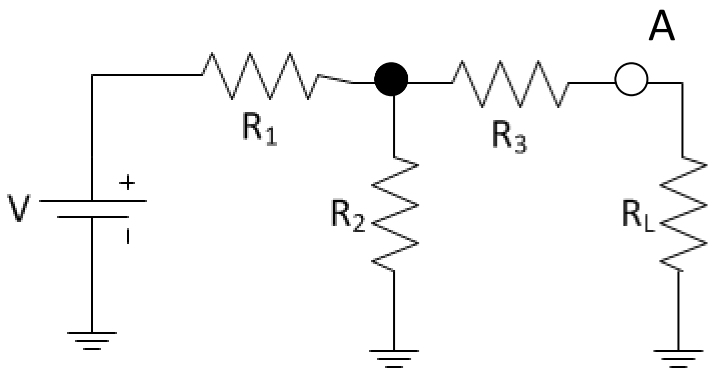


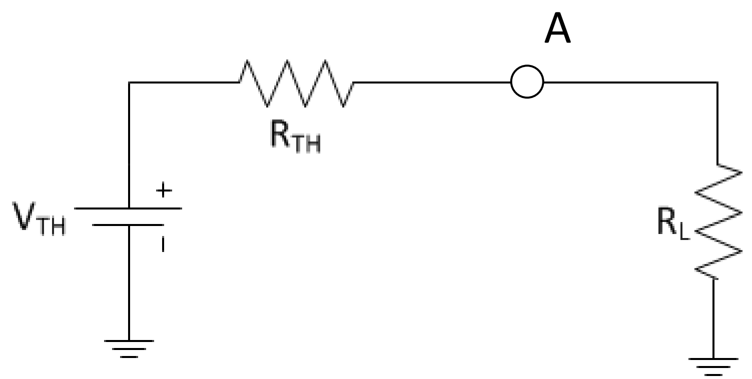
V_{TH} = open circuit voltage of terminals with Z_L not connected.

Z_{TH} = impedance looking into terminals
= "output impedance"

Passive Circuits

Thevenin's Theorem





$V_{TH} = V_A$ without R_L

Without R_L, circuit is just a voltage divider formed by R_1 & R_2.

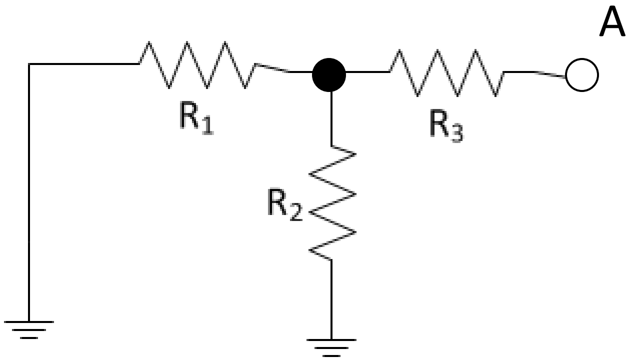
So $V_{TH} = V \dfrac{R_2}{R_1 + R_2}$

Passive Circuits

Thevenin's Theorem

What about R_{TH}?

Remember, ideal batteries (ideal voltage sources) have zero internal resistance. So, "looking at terminal A with R_L not connected" looks like:



So

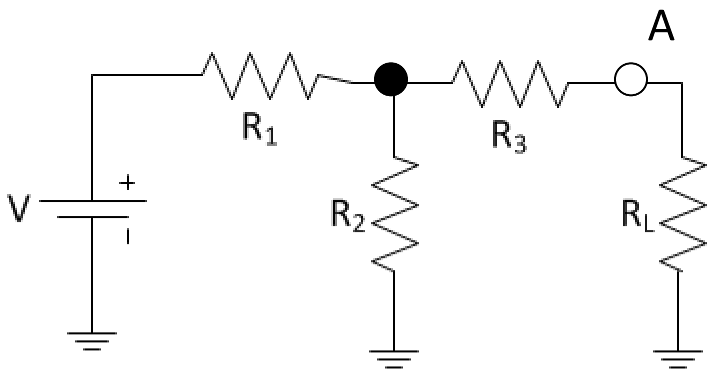
$R_{TH} = R_3 + R_1||R_2$

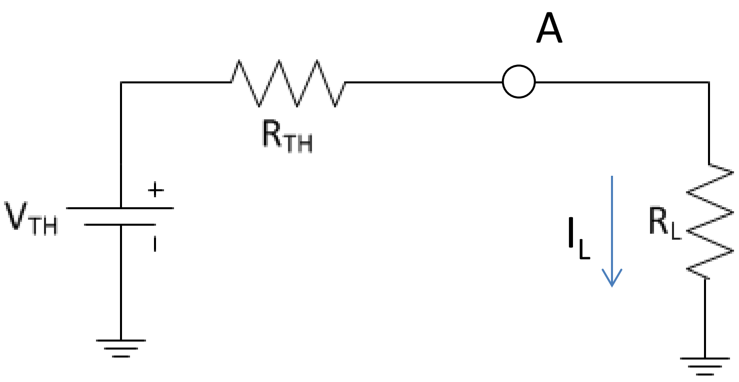
Shorthand for "R_1 in parallel with R_2".

$$\frac{1}{R_1||R_2} = \frac{1}{R_1} + \frac{1}{R_2}$$

Passive Circuits

Thevenin's Theorem





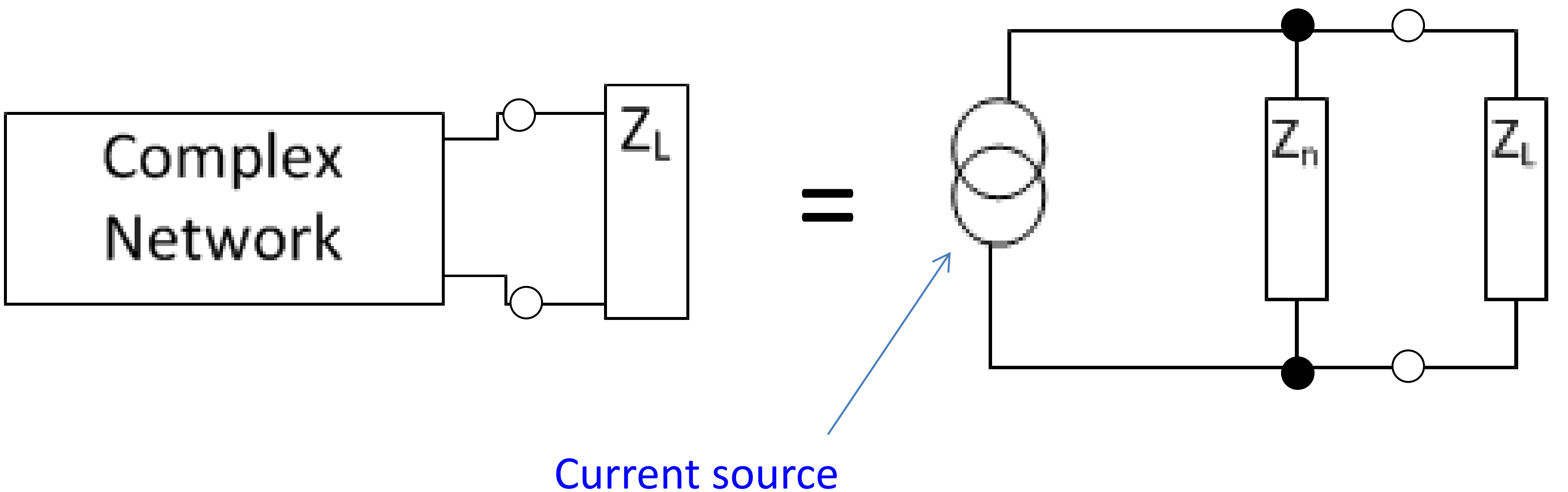
Short the output (R_L =0)

$I_L = \frac{V_{TH}}{R_L + R_{TH}}$

Passive Circuits

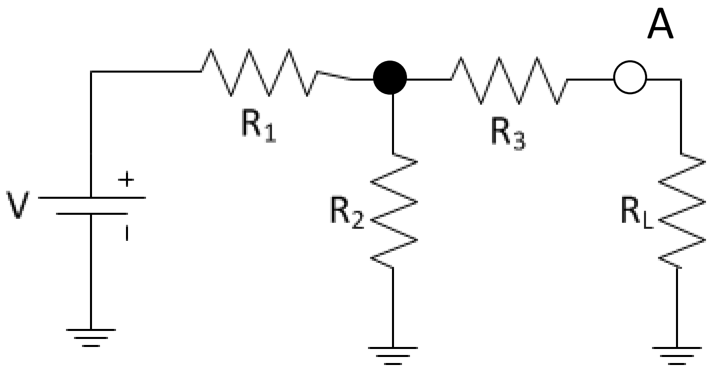
Norton's Theorem

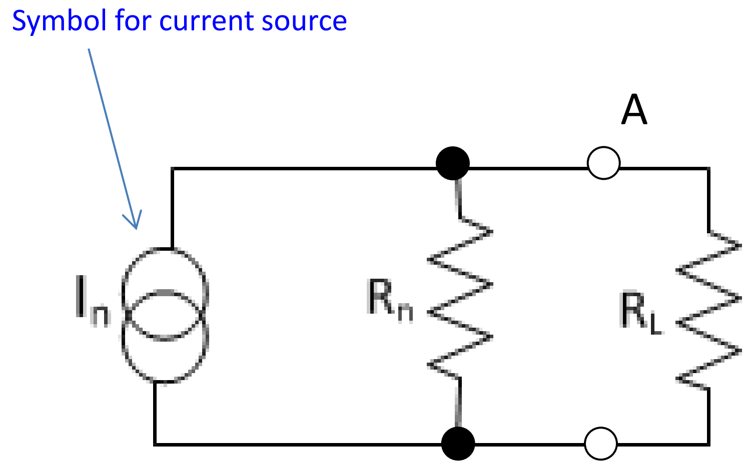
The current through any impedance Z_L, connected to two terminals of a network is the same as if Z_L were connected in parallel with another impedance Z_n and a current source I_n.



Passive Circuits

Norton's Theorem





Current through R_L is I_L. Current spits each path in proportion of its resistance to the total resistance.

$$I_L = I_n \frac{R_n}{R_n + R_L}$$ (current divider)

Open circuit (R_L gone) requires $V = I_n R_n$

Passive Circuits

We know $V_{TH} = V = I_n R_n$

In both pictures of this circuit, we know $I_L = I_L$

Thevenin | Norton

$$I_L = \frac{V_{TH}}{R_L + R_{TH}} = I_n \frac{R_n}{R_n + R_L}$$

Plug in $V_{TH} = V = I_n R_n$

$$I_L = \frac{I_n R_n}{R_L + R_{TH}} = I_n \frac{R_n}{R_n + R_L}$$

Results in

$$R_L + R_{TH} = R_n + R_L$$

$$R_{TH} = R_n$$

Since both R_{TH} & R_n represent the resistance "looking at" the circuit at the same point, they better be the same.

AC Circuits & Circuits

Standard physics notation
Steady "Sine" Wave Oscillation:

$$V = A\, e^{-i\omega t} = A\,(\cos\omega t\; - i\;\sin\omega t)$$

Standard stuff: $\omega = 2\pi f$ f = frequency [f] = Hz   [ω] = rad/sec

We always only observe the real part

AC Circuits & Circuits

Resistors:

V = IR

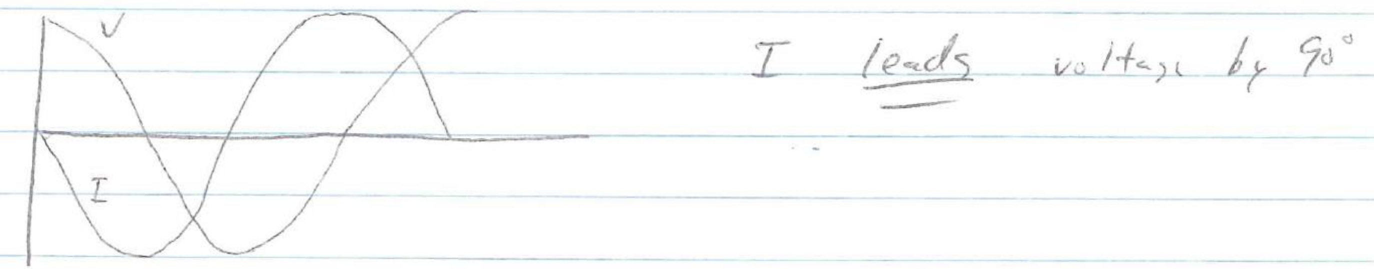
I = $\frac{A}{R}e^{-i\omega t}$ $\rightarrow$ (real) $\frac{A}{R}\cos\omega t$ I & V are in phase

Capacitors:

V = $\frac{Q}{C}$ I = $\frac{dQ}{dt}$ = $C\frac{dV}{dt}$

So I = $-i\omega CA\ e^{-i\omega t} = -i\omega CA\ (\cos\omega t\ - i\ \sin\omega t)$

Real part of I: $-\ \omega CA \sin\omega t$



AC Circuits & Circuits

Capacitors (*cont*):

Define the capacitance reactance (χ_c) so we can write an "ohms law":

$V = I\,\chi_c$

We just saw that $I = -i\omega CA\, e^{-i\omega t}$ for a voltage $A\, e^{-i\omega t}$,

So $I = -i\omega CV$ which means

$$V = -i\omega CV\,\chi_c$$

So

$$\chi_c = \frac{1}{-i\omega C} = \frac{-1}{i\omega C}$$

The impedance of a capacitor decreases with increasing capacitance & increasing frequency.

As frequency goes up, capacitors start to look like short circuits.

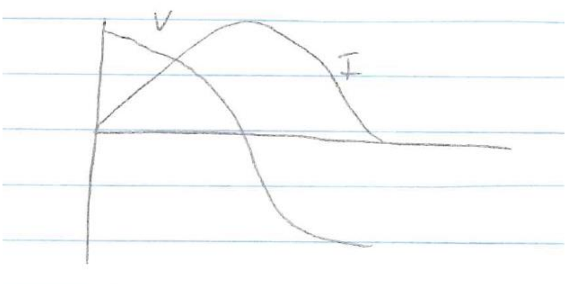
AC Circuits & Circuits

Inductors:

V = L $\frac{dI}{dt}$ which means I = $\frac{1}{L}\int V\,dt$

So I = $\frac{A}{-i\omega L}e^{-i\omega t}$

Real part of I: $= \frac{A}{\omega L}\sin\omega t$



Current lags voltage by 90°

V = I χ_c

V = $\frac{V}{-i\omega L}\chi_c$

So $\chi_c = -i\omega L$

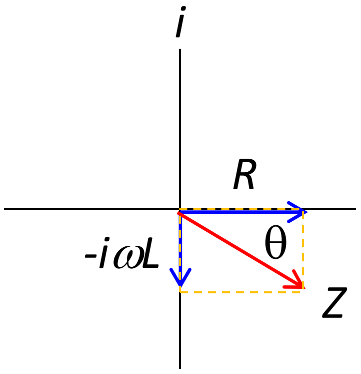
The impedance of a inductor increases with increasing inductoance & increasing frequency.

As frequency goes up, inductors start to look like open circuits.

AC Circuits & Circuits

Impedance Plot:

Z of a R,L,C circuit is the vector sum of the amplitudes. Phase angle is the angle between current & voltage.



R + L in series

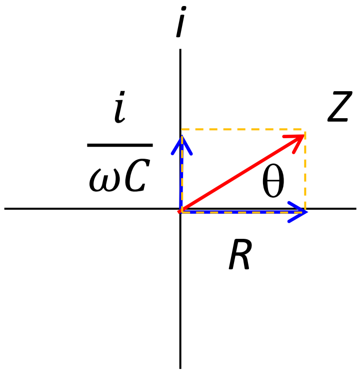
$$Z = R - i\omega L$$

$$|Z| = \sqrt{R^2 + \omega^2 L^2}$$

$$\theta = \tan^{-1}\frac{-\omega L}{R}$$

AC Circuits & Circuits

R + C in series



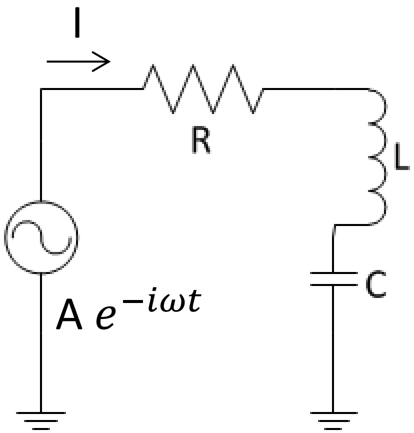
$$Z = R - \frac{i}{\omega C}$$

$$|Z| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

$$\theta = \tan^{-1}\frac{1}{R\omega C}$$

AC Circuits & Circuits

Resonance Circuits R, C, L series



$$0 = V_{total} - V_R - V_c - V_L$$

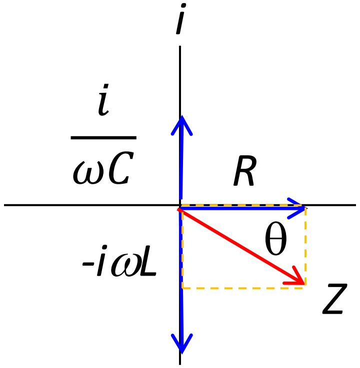
$$0 = A\, e^{-i\omega t} - IR - L\frac{dI}{dt} - \frac{Q}{C}$$

$$A\, e^{-i\omega t} = IR + L\frac{dI}{dt} + \frac{Q}{C}$$

We could solve this the usual (algebraic) way, but there's another way.

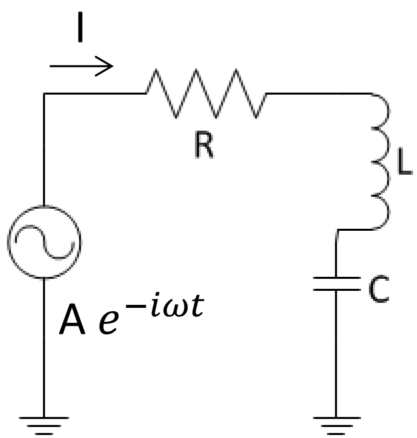
$$Z = Z_R + Z_L + Z_C \quad \rightarrow \quad Z = R - i\omega L + \frac{i}{\omega C}$$

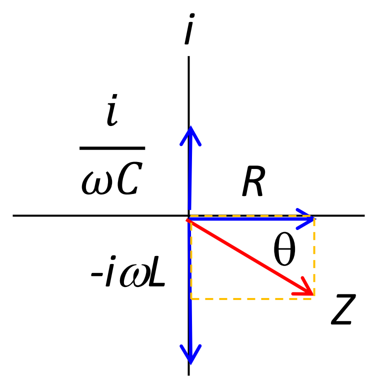
$$|Z| = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$$



AC Circuits & Circuits

Resonance Circuits R, C, L series





$Z = R - i\omega L + \frac{i}{\omega C}$

Ohm's Law $I = \frac{V}{Z}$

$I = \frac{A\,e^{-i\omega t}}{Z}$

The current is a maximum when |Z| is a minimum.

Why? $|I| = \left|\frac{A\,e^{-i\omega t}}{Z}\right| = \frac{A}{|Z|}$

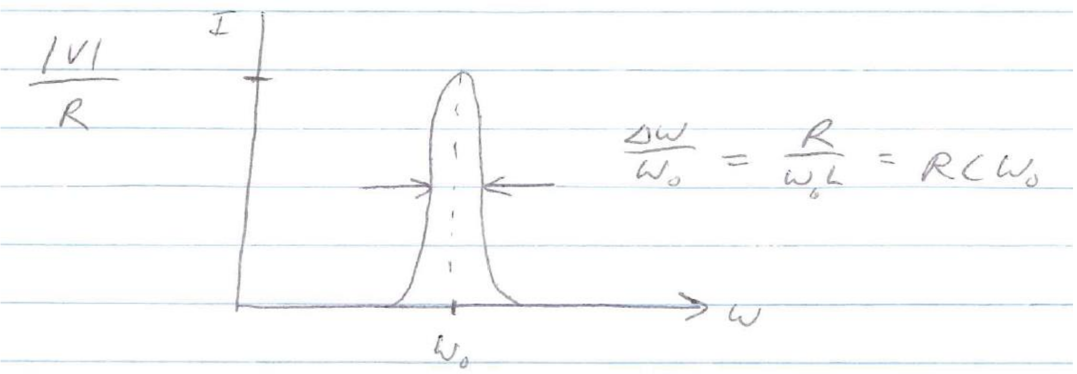
For fixed L, R & C, this occurs if we can make

$Z = R - i\omega L + \frac{i}{\omega C} = R$ or $\omega L = \frac{1}{\omega C}$ or $\omega = \omega_o \equiv \frac{1}{\sqrt{LC}}$

AC Circuits & Circuits

Resonance Circuits R, C, L series

$$\omega_o \equiv \frac{1}{\sqrt{LC}}$$

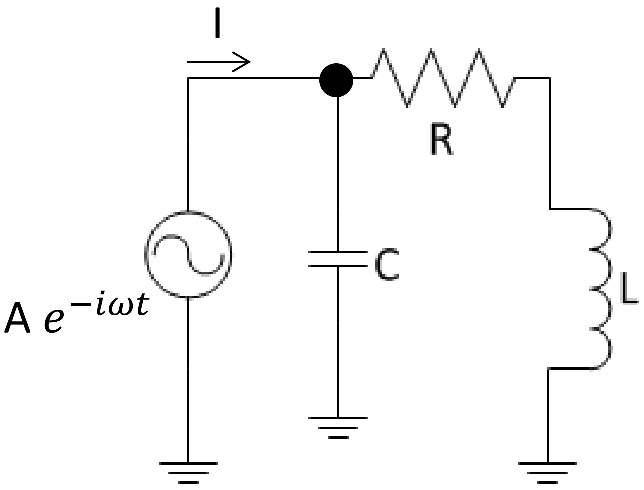


The "Q" or "quality factor" of any resonating thing = $\frac{\omega_o}{\Delta\omega}$

For this case, Q = $\frac{\omega_o}{\Delta\omega}$ = $\frac{\omega_o L}{R}$ = $\frac{1}{\omega_o RC}$

AC Circuits & Circuits

Resonance Circuits R, C, L parallel



$$\frac{1}{Z_{||}} = \frac{1}{Z_C} + \frac{1}{Z_R + Z_L}$$

$$Z_R = R,\ Z_c = \chi_c = \frac{i}{\omega C}\ and\ Z_L = \chi_L = -i\omega L$$

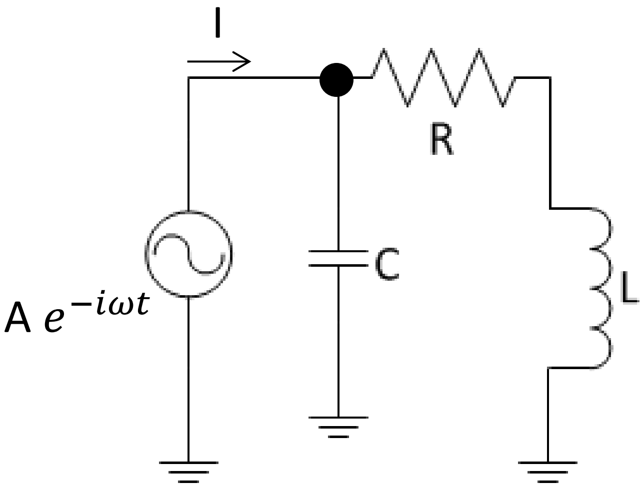
So

$$\frac{1}{Z_{||}} = -i\omega C + \frac{1}{R - i\omega L}$$

$$Z_{||} = \frac{R - i\omega L}{1 - i\omega CR - \omega^2 LC}$$

AC Circuits & Circuits

Resonance Circuits R, C, L parallel



$$Z_{||} = \frac{R - i\omega L}{1 - i\omega CR - \omega^2 LC}$$

$$|Z_{||}| = \sqrt{\frac{R^2 + \omega^2 L^2}{\omega^2 R^2 C^2 + (1 - \omega^2 LC)^2}}$$

Again, $|Z_{||}|$ is a maximum when

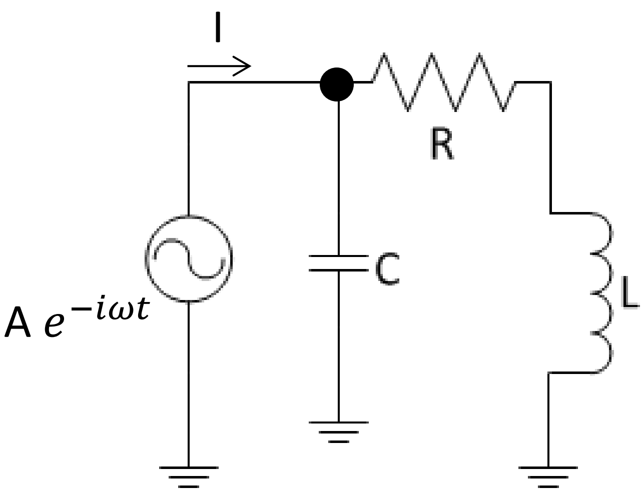
$$\omega = \omega_o \equiv \frac{1}{\sqrt{LC}}$$

When R << ωL: $\frac{1}{Z_{||}} = -i\omega C + \frac{1}{-i\omega L} = i(1 - \omega^2 LC)/\omega CL$

$$|Z_{||}| \sim \sqrt{\frac{\omega^2 L^2}{(1 - \omega^2 LC)^2}} = \frac{\omega L}{1 - \omega^2 LC} \rightarrow \infty \text{ as } \omega \rightarrow \omega_o$$

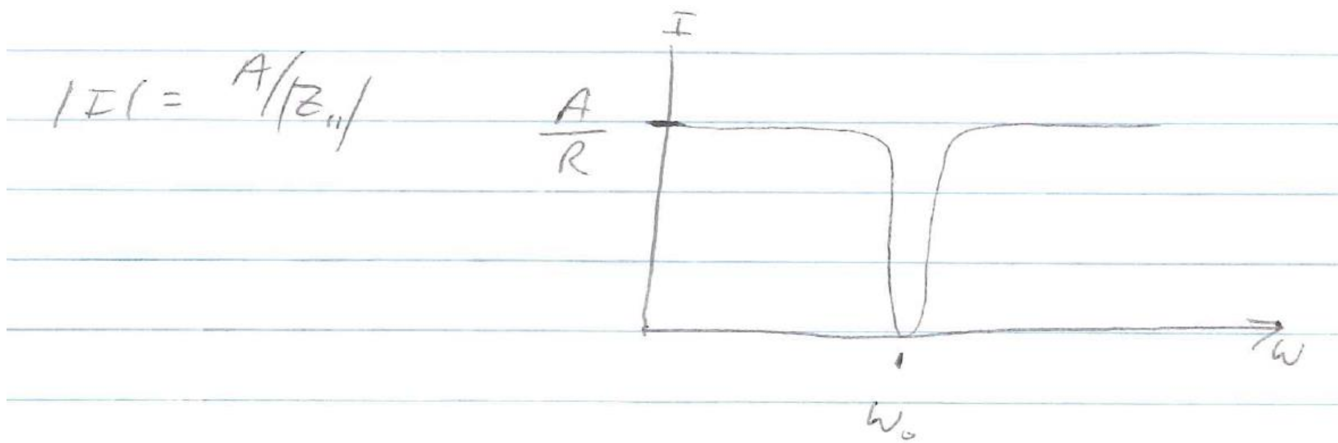
AC Circuits & Circuits

Resonance Circuits R, C, L parallel



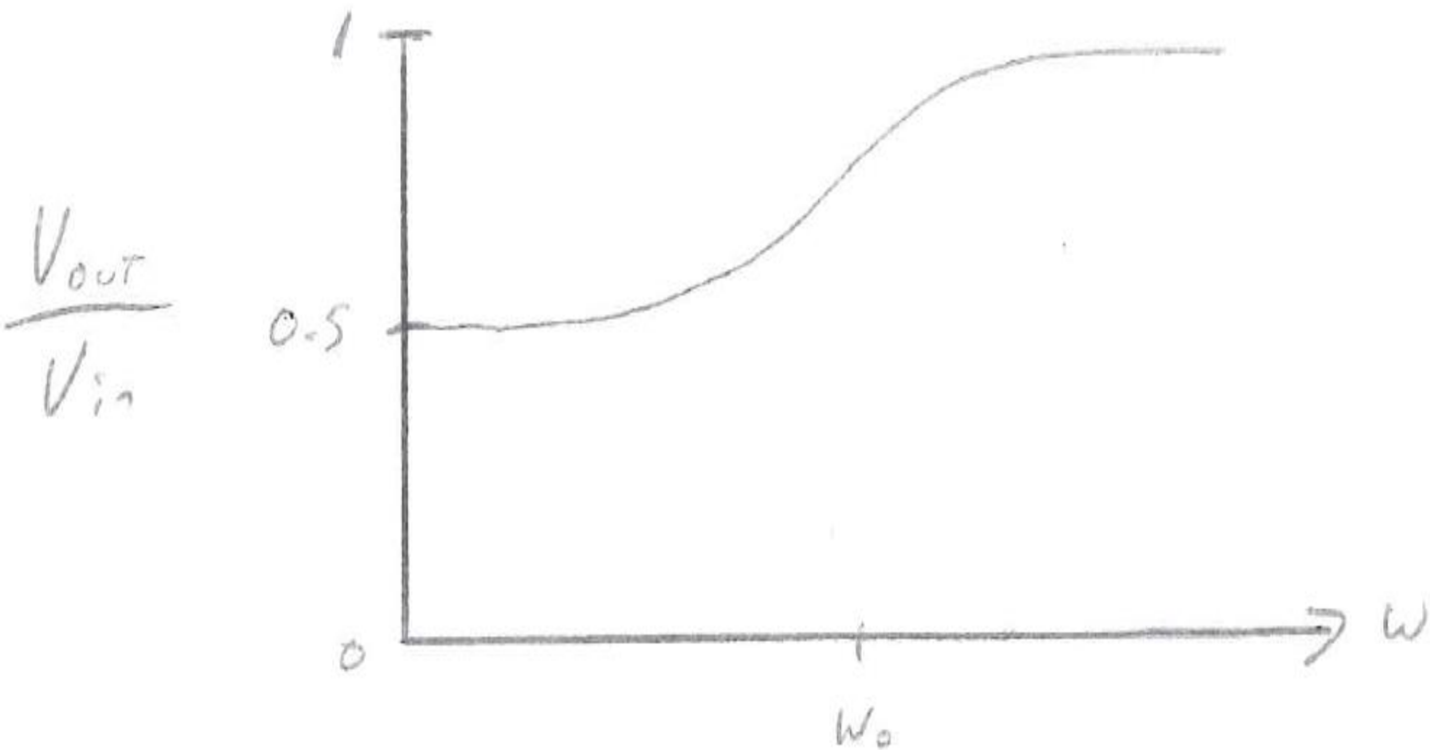
When R << ωL:

$$|Z_{||}| \sim \sqrt{\frac{\omega^2 L^2}{(1-\omega^2 LC)^2}} = \frac{\omega L}{1-\omega^2 LC} \to \infty \text{ as } \omega \to \omega_o$$



AC Circuits & Circuits

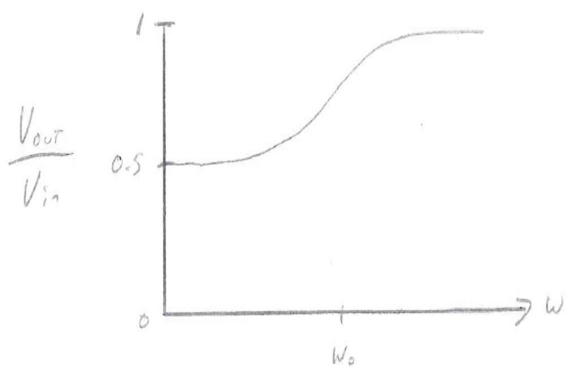
Filters



$\omega_o = \frac{1}{\tau}$ where τ is some time constant (like an “RC” time constant) that you set.

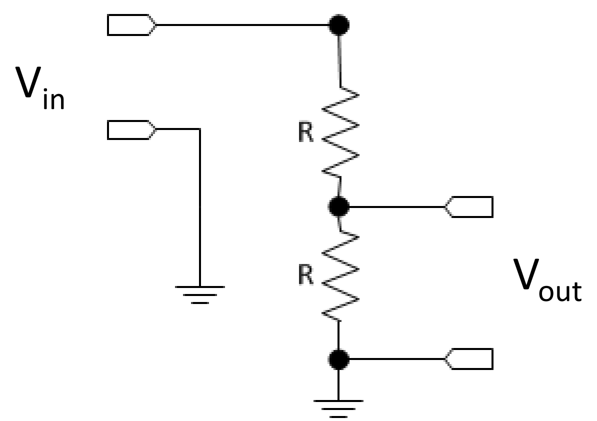
AC Circuits & Circuits

Filters



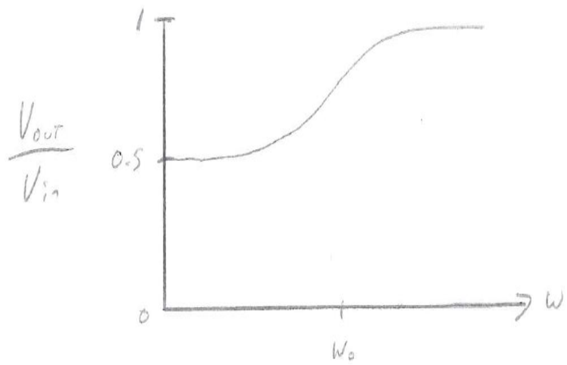
At ω =0, we know a filter with this behavior must just divide to voltage by ½.

So at ω =0, we know it must look something like this:



AC Circuits & Circuits

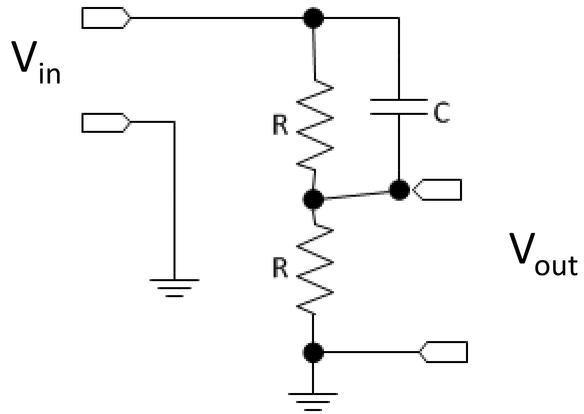
Filters



At $\omega = \infty$, we know the signal must bypass the top R so that the divider does not divide anymore.

We know the impedance of capacitors is ∞ at $\omega = 0$, & 0 at $\omega = \infty$.

So we can do something like this:

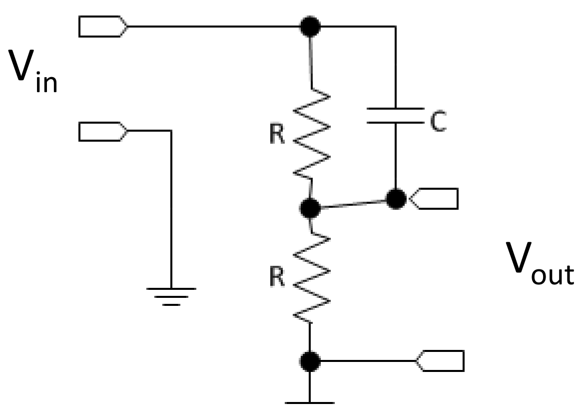


AC Circuits & Circuits

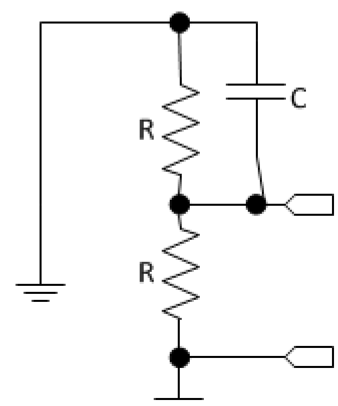
Filters

So what is ω_o ? Use Thevenin's Theorem.

Ideal voltage sources have zero impedance. (They put out up to infinite current to hold voltage steady – no internal resistance.)

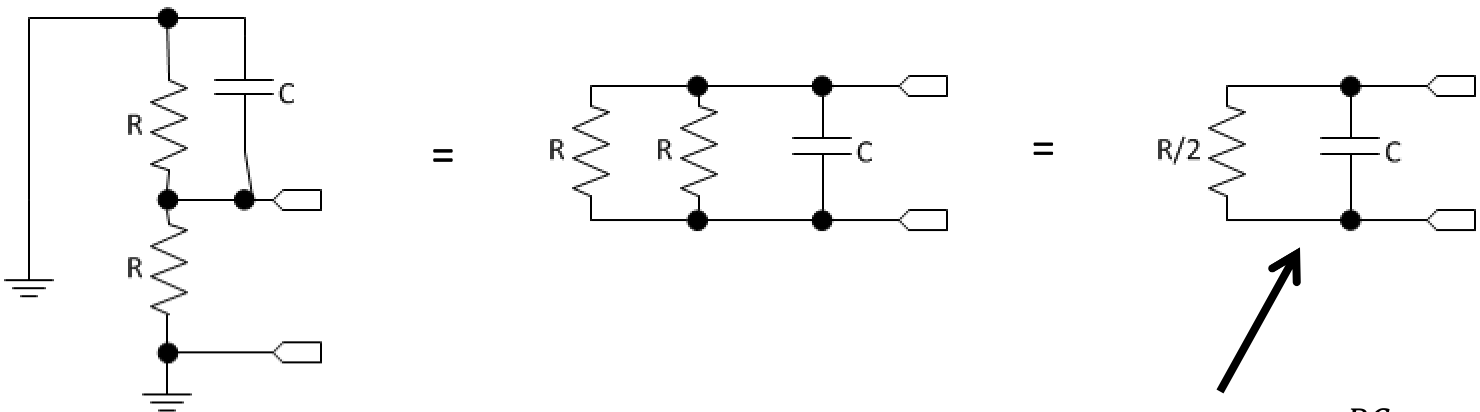


Looks like:



AC Circuits & Circuits

Filters

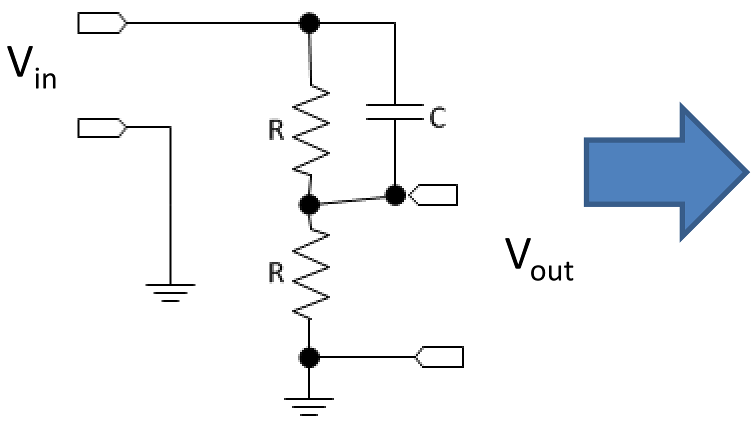


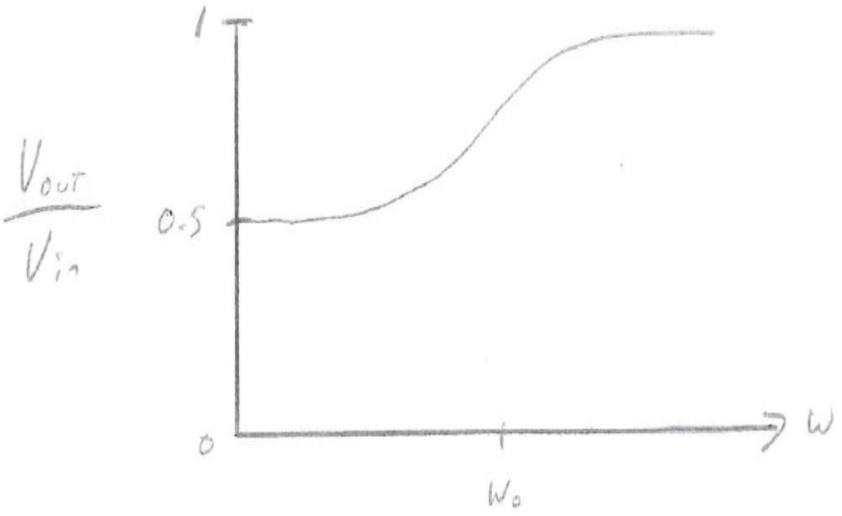
This you know has an RC time constant τ of $\frac{RC}{2}$

$\omega_o = \frac{1}{\tau}$ so $\omega_o = \frac{2}{RC}$

AC Circuits & Circuits

Filters





$$\omega_o = \frac{1}{\tau} \text{ so } \omega_o = \frac{2}{RC}$$