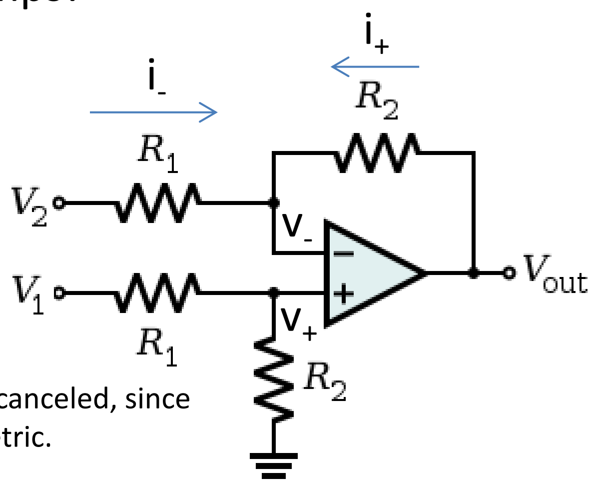
We did this circuit two weeks ago – a "difference" amp

Operational Amplifiers

Differential Amp

Op Amps!



Input bias current canceled, since inputs look symmetric.

Remember, $v_+ = \frac{R_2}{R_1+R_2} V_1$

$v_- = v_+$

So $V_{out} = \frac{R_1+R_2}{R_1} v_- - \frac{R_2}{R_1} V_2$

Becomes $V_{out} = \frac{R_1+R_2}{R_1} v_+ - \frac{R_2}{R_1} V_2$

$$V_{out} = \frac{R_1+R_2}{R_1} \frac{R_2}{R_1+R_2} V_1 - \frac{R_2}{R_1} V_2$$

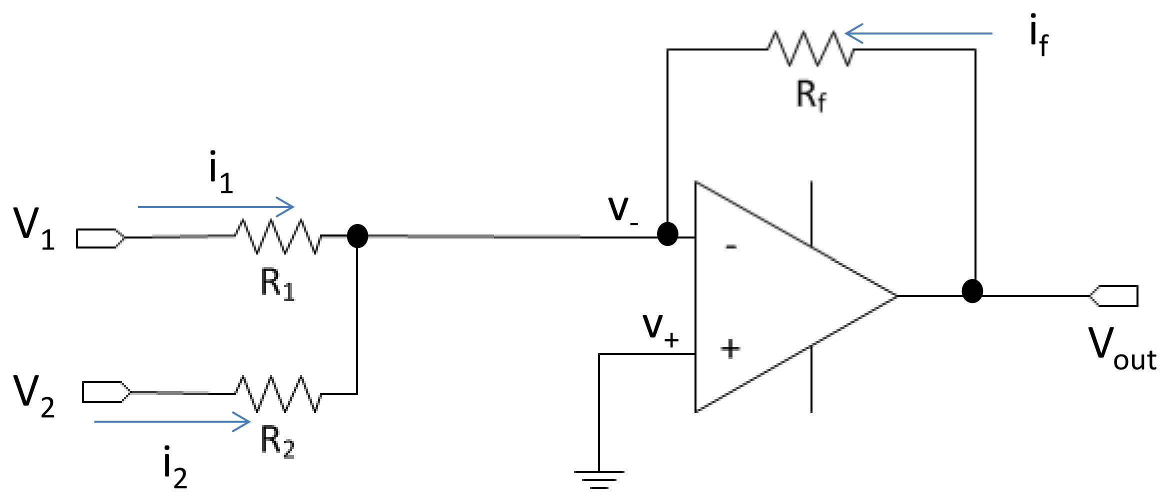
$$V_{out} = \frac{R_2}{R_1}(V_1 - V_2)$$

← Subtraction

There's a whole lot of "other" signal processing math functions we can use op-amps for.

Operational Amplifiers

Math Functions – Inverting Summing Amp



$v_+ = v_- = 0$,

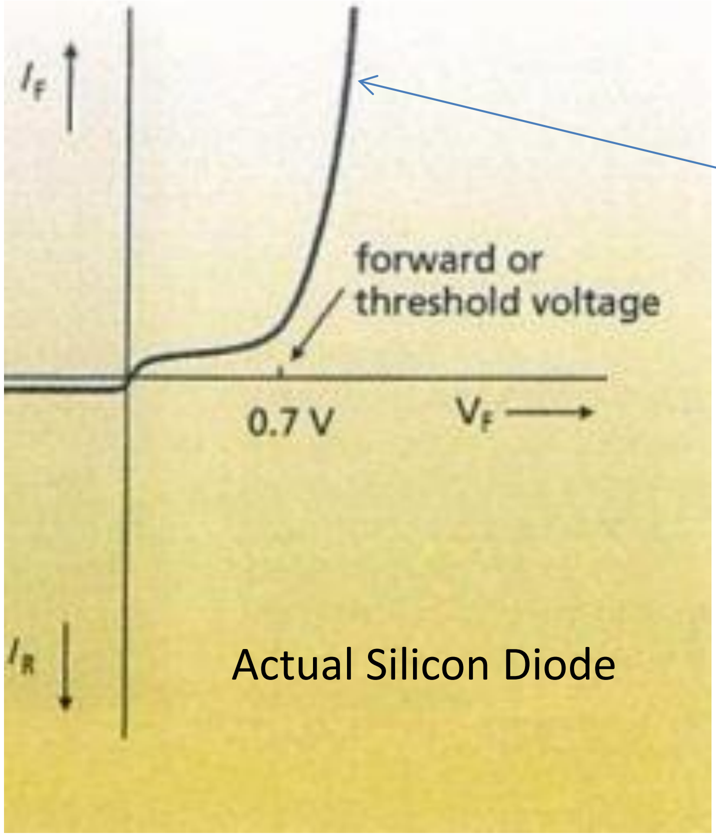
So $V_{out} = i_f R_f$ & $V_1 = i_1 R_1$ & $V_2 = i_2 R_2$

We also know $i_f + i_1 + i_2 = 0$

So $0 = \frac{V_{out}}{R_f} + \frac{V_1}{R_1} + \frac{V_2}{R_2} \rightarrow V_{out} = - R_f \left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$ Weighted (inverted) sum of V_1 & V_2

Operational Amplifiers

Math Functions – Logarithmic Amp



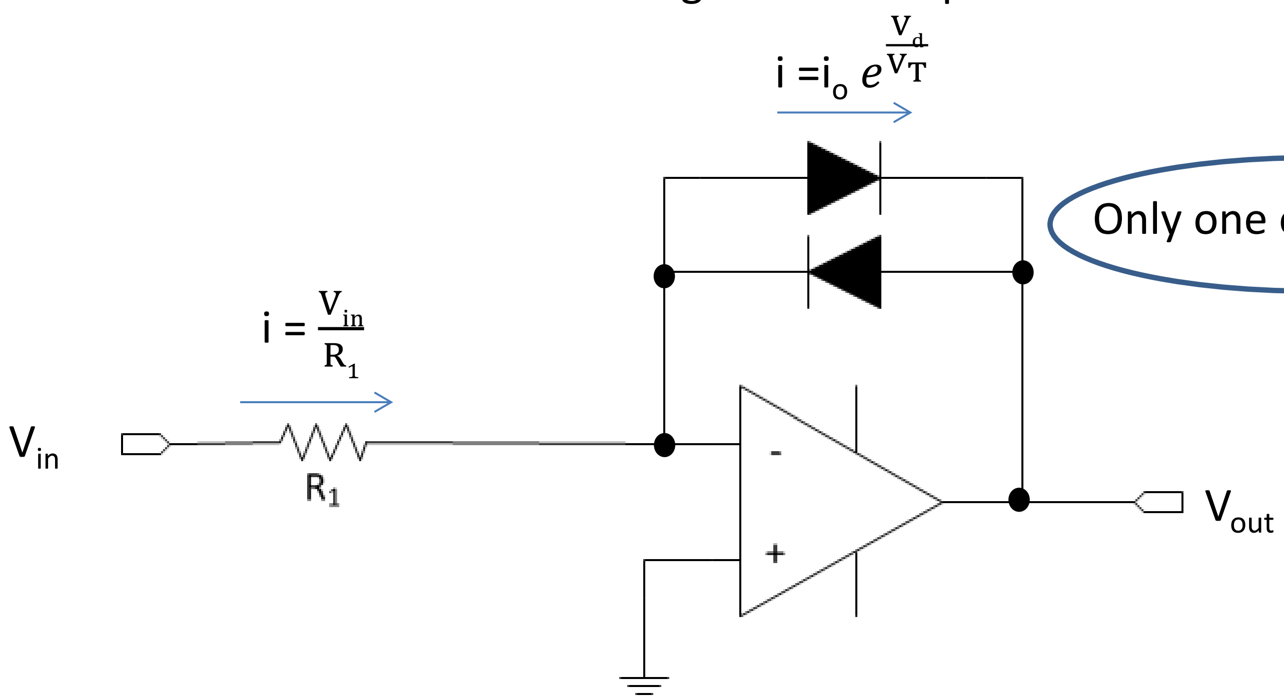
$$I \sim I_o e^{\frac{V}{V_T}}$$

For Silicon Diodes,

V_T ~ 0.6-0.7 volts

Operational Amplifiers

Math Functions – Logarithmic Amp



Only one on at a time

$v_+ = v_- = 0,$

At input, $i = \frac{V_{in}}{R_1}$

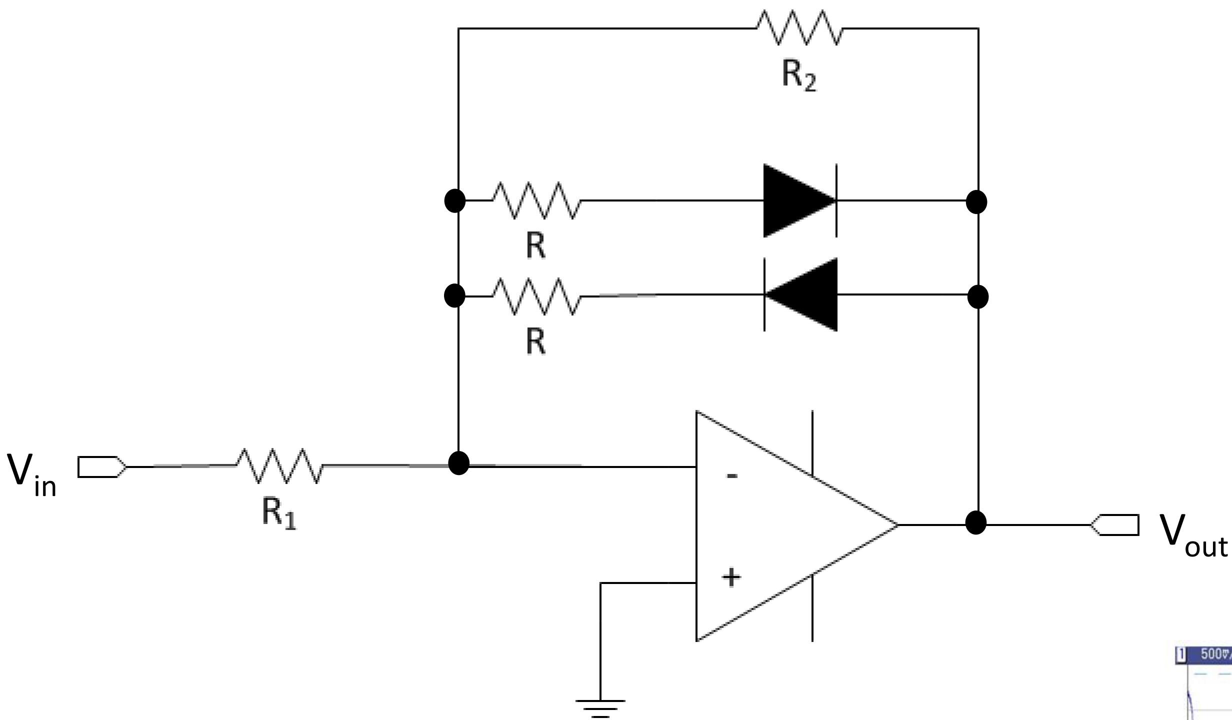
At output $i = i_o\, e^{\frac{V_d}{V_T}}$

So $\frac{V_{in}}{R_1} = i_o\, e^{\frac{V_d}{V_T}}$

Or, since $V_{out} = - V_d$, $V_{out} = - V_T \ln\left(\frac{V_{in}}{R_1 i_o}\right)$

Operational Amplifiers

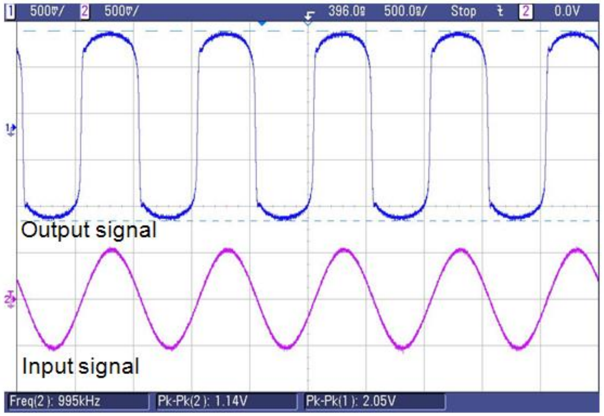
Math Functions – Logarithmic Amp (more generally)



R's limit current.
R2 allows it to function when neither diode is on.

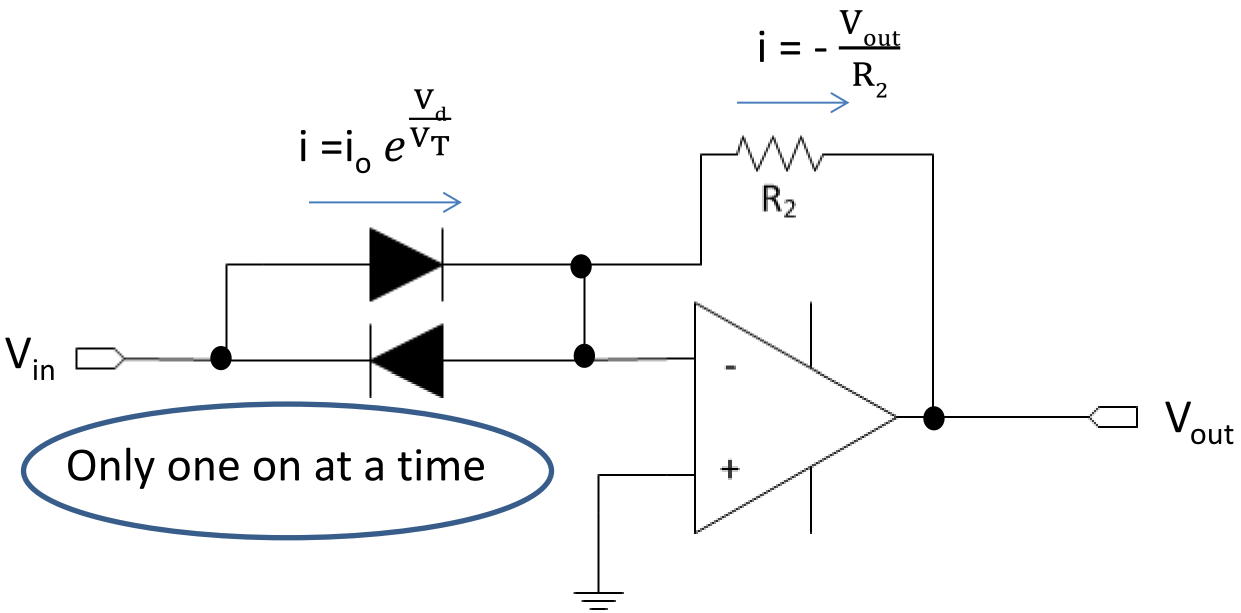
Max compression is $\frac{R_1}{R}$

Example: $R_2 \sim 10R_1$



Operational Amplifiers

Math Functions – Exponential Amp



$v_+ = v_- = 0$,

At output, $i = \frac{V_{out}}{R_2}$

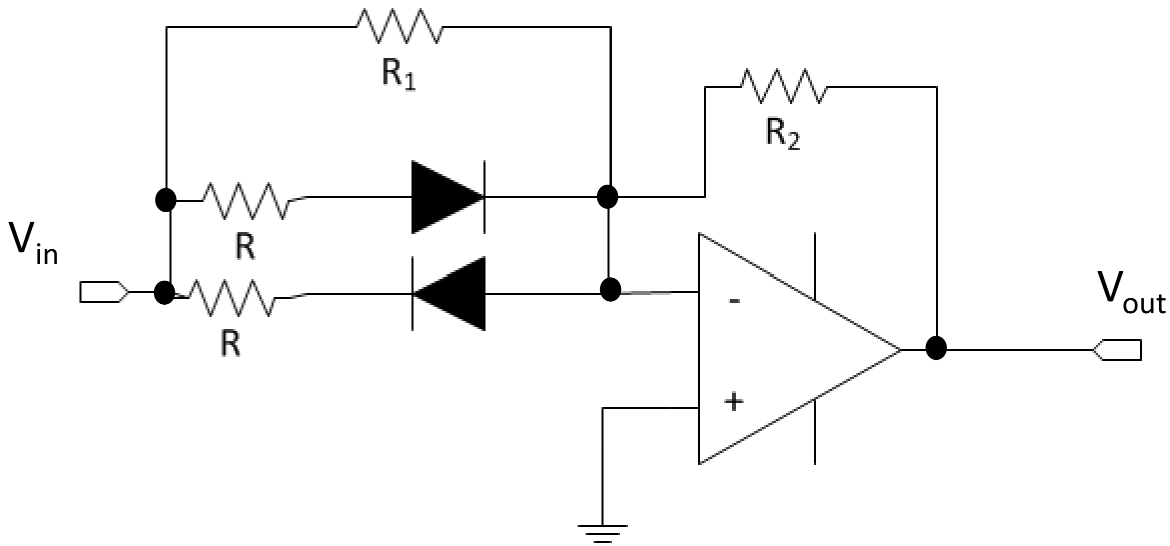
At input $i = i_o\, e^{\frac{Vd}{V_T}}$

So $\frac{V_{out}}{R_2} = i_o\, e^{\frac{V_d}{V_T}}$

Or, since $V_{in} = V_d$ $\quad V_{out} = -R_2 i_o\, e^{\frac{V_{in}}{V_T}}$

Operational Amplifiers

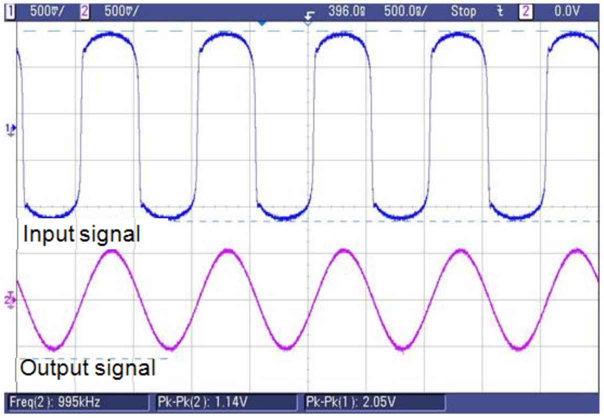
Math Functions – Exponential Amp (more generally)



R's limit current.
R1 allows it to function when neither diode is on.

Max expansion is $\frac{R_2}{R}$

Example $R_2 \sim R_1/10$

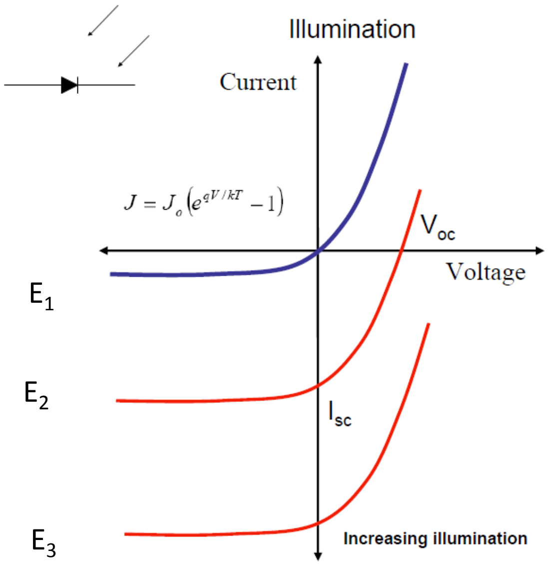


Operational Amplifiers

Math Functions – Log & Exponential Amps

Dynamic range is the ratio between the largest and smallest possible values of a changeable quantity

An example where log (& exponential) amps are used is with sensors with larger dynamic range than the data collection system can handle directly.



Light Intensity $E_1 < E_2 < E_3$
Avalanche photodiodes dynamic range can be 10^9
([Nuclear Instruments and Methods in Physics Research A 350, 595 (1994)])

Suppose sensor has a dynamic range of 10^9.

Suppose you want to record this digitally.

16-bit analog-to-digital converter has a dynamic range of 2^{16} , which is 65536.

24-bit analog-to-digital converter has a dynamic range of 2^{24} , which is ~1.7×10^7.

10^9 directly would need a 30-bit analog-to-digital converter.

Operational Amplifiers

Math Functions – Log & Exponential Amps

Dynamic range is the ratio between the largest and smallest possible values of a changeable quantity

The dynamic range of *magnetic tape* is approximately 55 dB. (decibels)

dB = 10 $\log_{10}\left(\frac{P_2}{P_1}\right)$ defined in terms of *power (H & H, page 15)*

Power ~ V^2

dB = 20 $\log_{10}\left(\frac{V_2}{V_1}\right)$ defined in terms of *voltage (or other signal)*

So a dB is a dB regardless of whether one is talking power or signal.

So magnetic tape has a dynamic range of $10^{\frac{55}{20}} \sim 10^{2.75} \sim 562$

Operational Amplifiers

Math Functions – Log & Exponential Amps

The dynamic range of *magnetic tape* is approximately 55 dB. (decibels)

So magnetic tape has a dynamic range of $10^{\frac{55}{20}} \sim 10^{2.75} \sim 562$

“Dolby A” adds approximately 10 dB to the dynamic range that will fit on magnetic tape (to ~ 1800), DBX adds 30 dB (to ~ 18,000).

Vinyl records also about 55 dB, but no compression tricks.

CD has a dynamic range of 96dB in theory (~63,000), but practically ~90dB (~31,600).

Some digital audio recording could theoretically have 120dB (20-bit) or 144 dB (24-bit) dynamic range, but microphones are not that good & file formats (e.g. MP3) discard data.

Human hearing ~120 dB (~ 1,000,000).

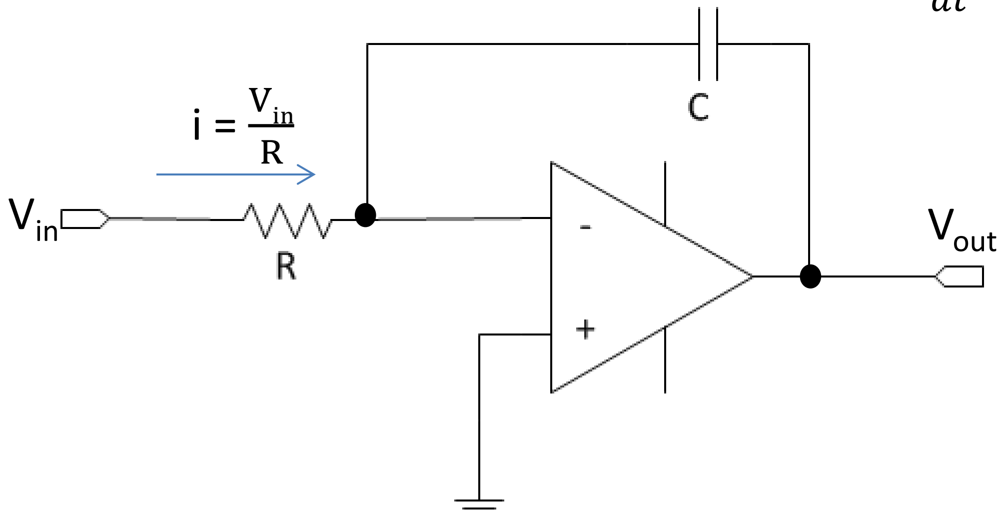
Operational Amplifiers

Math Functions – Integrator

$V = \frac{Q}{C}$ & $i = \frac{dQ}{dt}$ $\therefore$ $i = C\frac{dV}{dt}$

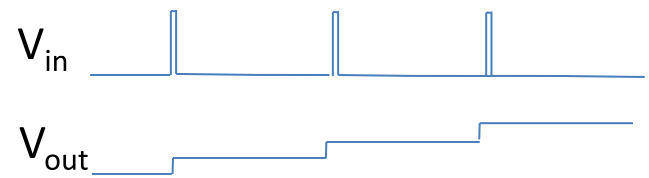
$i = -C\,\frac{dV_{out}}{dt}$

$i = \frac{V_{in}}{R}$



So $\frac{V_{in}}{R} = -C\,\frac{dV_{out}}{dt}$

$\therefore$ $V_{out} = \frac{-1}{RC}\int V_{in}(t)\,dt$



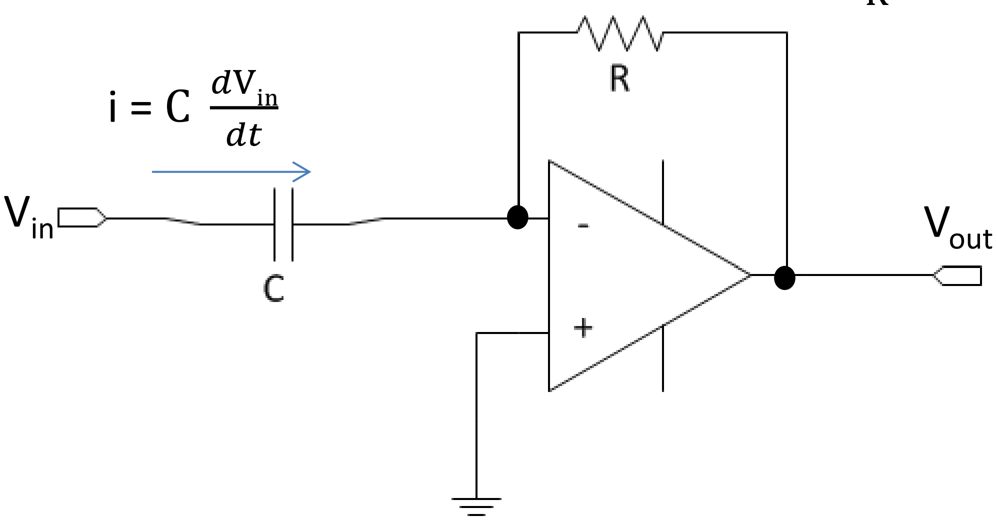
Generally works better than just a C & R, usually limited by the op-amp input bias current.

Operational Amplifiers

Math Functions – Differentiator

Same as integrator, just interchange R & C

$i = -\frac{V_{out}}{R}$



$i = C\frac{dV_{in}}{dt}$

So $\frac{V_{out}}{R} = -C\frac{dV_{in}}{dt}$

$\therefore V_{out} = -RC\frac{dV_{in}}{dt}$

Operational Amplifiers

Math Functions – Integrators & Differentiator

Using op-amp Integrators & Differentiators, you can simulate ANY differential equation. You are not limited by order or linearity constraints.

These analog computers have the advantage over digital computers of computing continuous variables, rather than discretized variables. i.e. ***Analog computers can compute using real numbers, digital computers are limited to rational numbers.***

Until just a couple of decades ago, analog computers were the only way to simulate complex fluid dynamics for aircraft/missile/rocket designs.

Operational Amplifiers

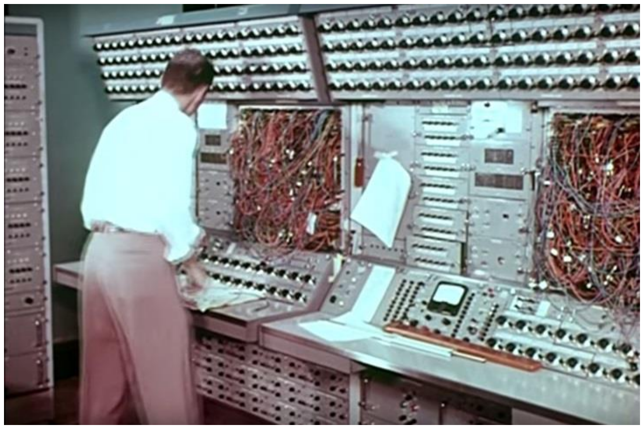
Math Functions – Analog Computers

1960 EC-1 Module

1970 Compumedic
Analog Computer
Model 6F13 Module

Operational Amplifiers

Math Functions – Analog Computers



X-15 simulator analog computer

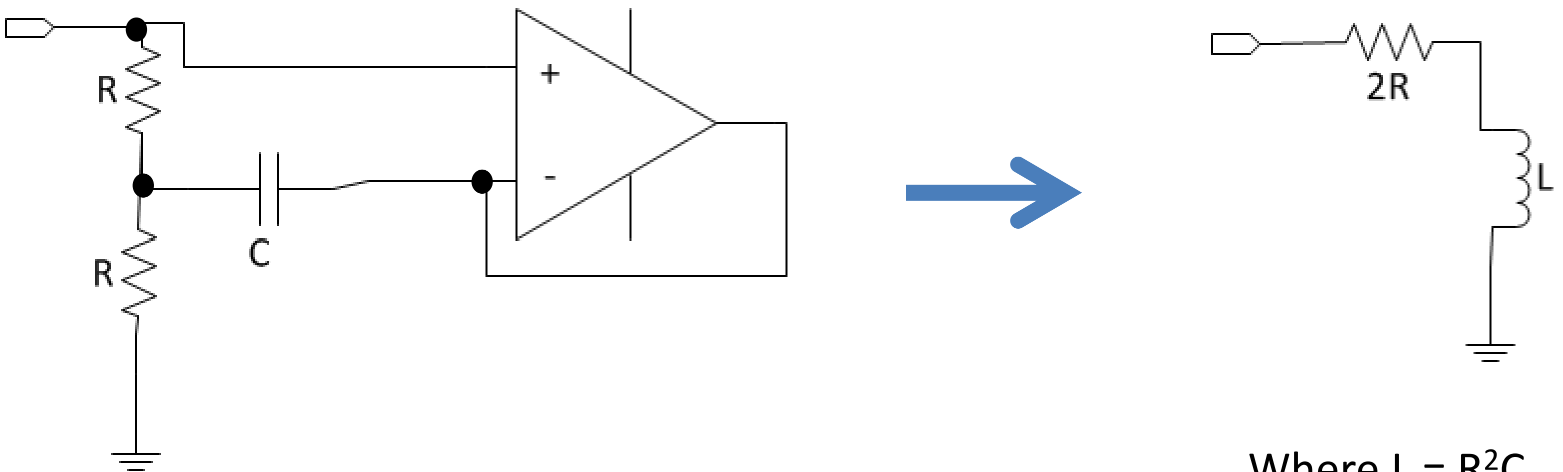
X-15 – Still holds highest speed ever recorded by a manned, powered aircraft (1967) @Mach 6.72 at 102,100 feet [4,520 miles per hour (7,274 km/h)]

More on X-15 analog computer stuff: https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19680019932.pdf

Operational Amplifiers

Simulate other components

Humongous inductor – Simulate making a “Gyrator”



Where $L = R^2C$

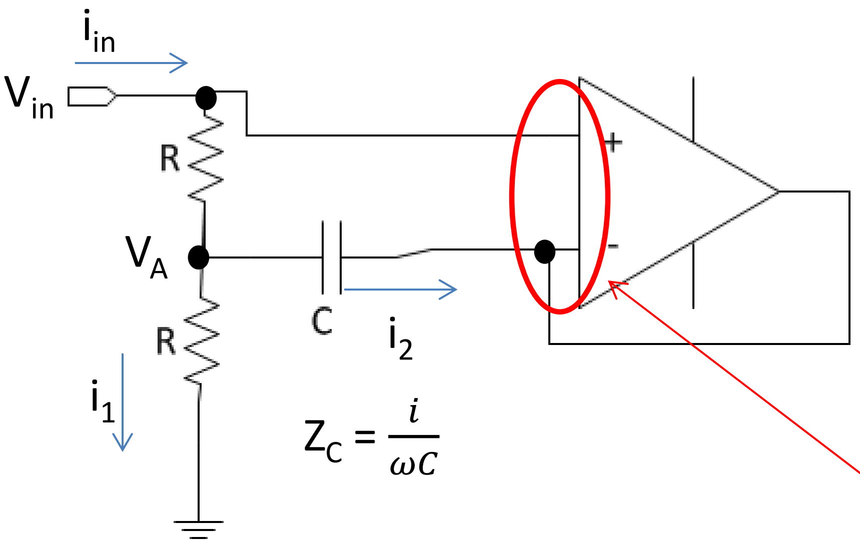
Use this when you need a really really big inductor.

e.g. R= 100Ω, C = 100μf, results in L = 1 Henry

Operational Amplifiers

Simulate other components

Humongous inductor – Simulate making a "Gyrator"



$Z_C = \frac{i}{\omega C}$

How does this work?

$i_{in} = i_1 + i_2$

$i_{in} = \frac{V_{in} - VA}{R}$

$i_1 = \frac{V_A}{R}$

What's going on here?

$v_+ = v_-$ & $v_+ = V_{in}$ $\therefore v_- = V_{in}$

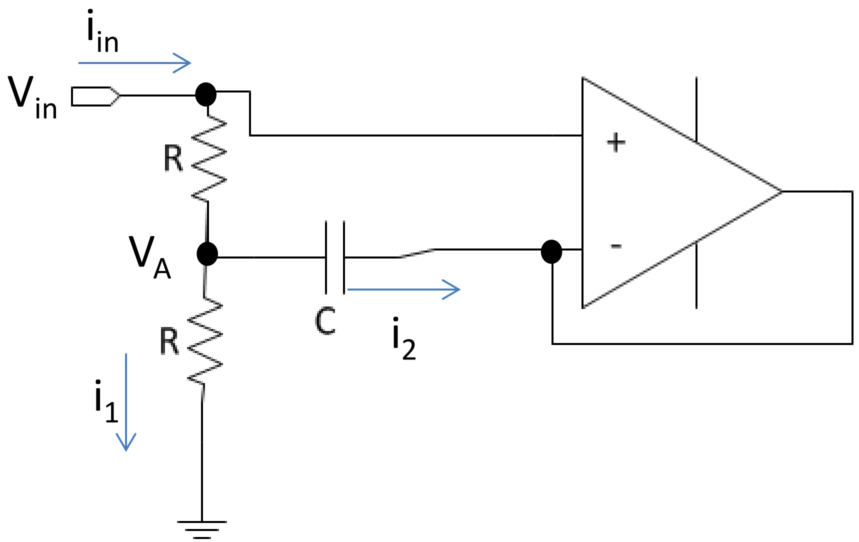
So… $i_2 = \frac{V_A - Vin}{Z_c} = i\omega C\,(V_{in} - V_A)$

Remember, $Z_c = \frac{-1}{i\omega C}$

Operational Amplifiers

Simulate other components

Humongous inductor – Simulate making a "Gyrator"



$i_{in} = i_1 + i_2$

$i_{in} = \frac{V_A}{R} + i\omega C\,(V_{in} - V_A)$

Since $i_{in} = \frac{V_{in} - VA}{R}$

$i_{in} = \frac{V_{in}}{R} + i\omega CR\, i_{in} - i_{in}$

So $V_{in} = i_{in}(2R - i\omega CR^2)$

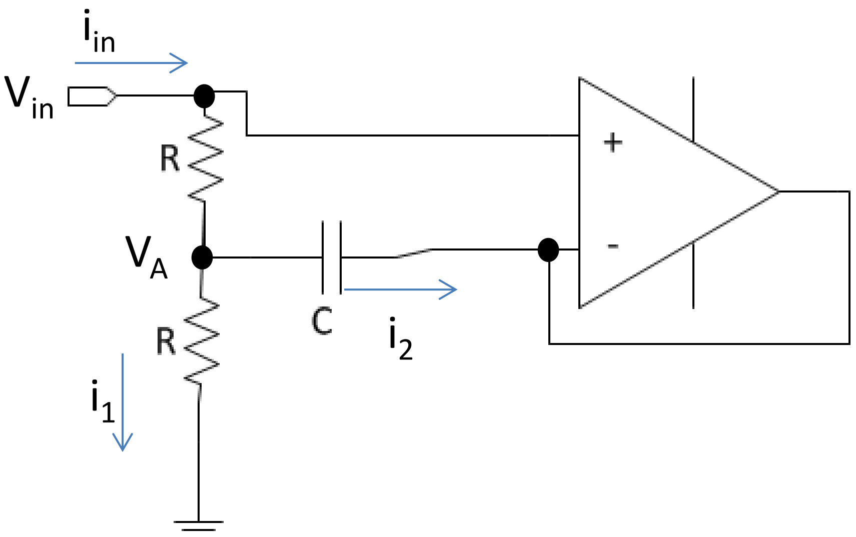
Now $Z_{in} = \frac{V_{in}}{i_{in}}$

So $Z_{in} = (2R - i\omega CR^2)$

Operational Amplifiers

Simulate other components

Humongous inductor – Simulate making a "Gyrator"



For a resistor, $Z = R$ (No ω dependence)

For a capacitor of capacitance C, $Z = \frac{i}{\omega C}$

Proportional to $1/\omega$

For an inductor of inductance L, $Z = -i\omega L$

Proportional to ω

No ω dependence

Proportional to ω

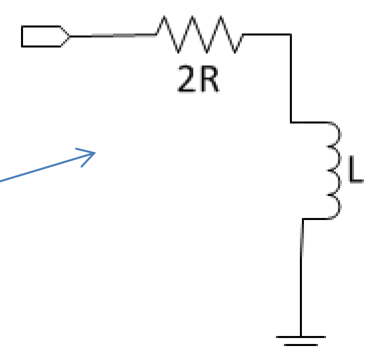
"L" = CR^2

$$Z_{in} = (2R - i\omega CR^2)$$

Resistor-like

Inductor-like

And since they add, they are in series.



Operational Amplifiers

Other Functions

Comparators

An ideal comparator is just an op-amp with an enormous gain so that

$V_{out} = \infty$ if $v_+ > v_-$

$V_{out} = -\infty$ if $v_+ < v_-$

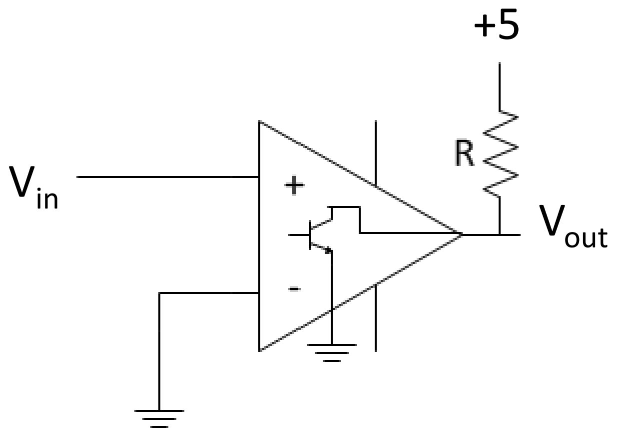
Even if $|v_+ - v_-| \sim \varepsilon$

Operational Amplifiers

Other Functions

Comparators

There are special op-amps for this purpose which either short the output to ground or have $\infty\Omega$ to ground.



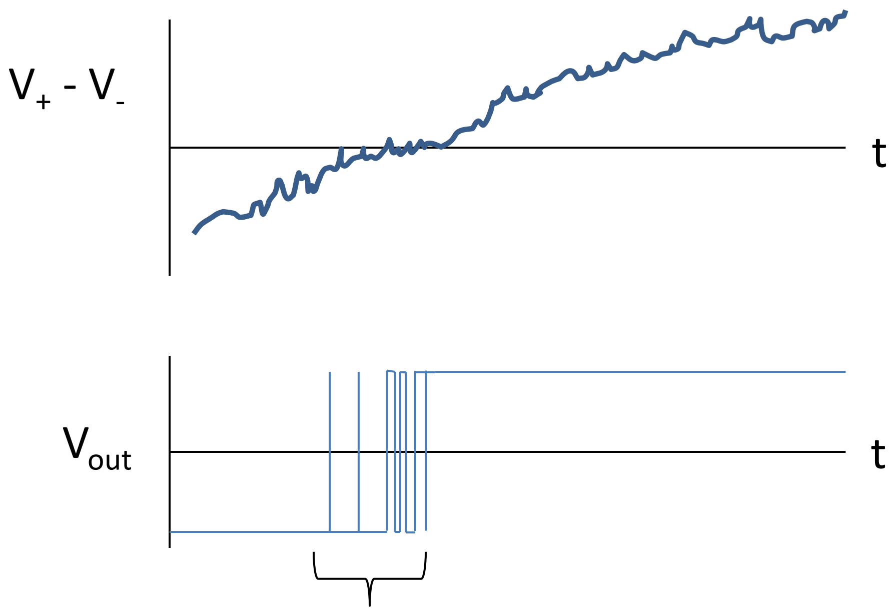
This way you bias the output to suit your needs separate from the input.

Operational Amplifiers

Other Functions

Comparators

As you will probably noticed from the lab where you will make a comparator, noise an such can make the circuit switch if $|V_+ - V_-|$ is small.



We usually don't want this "jitter".

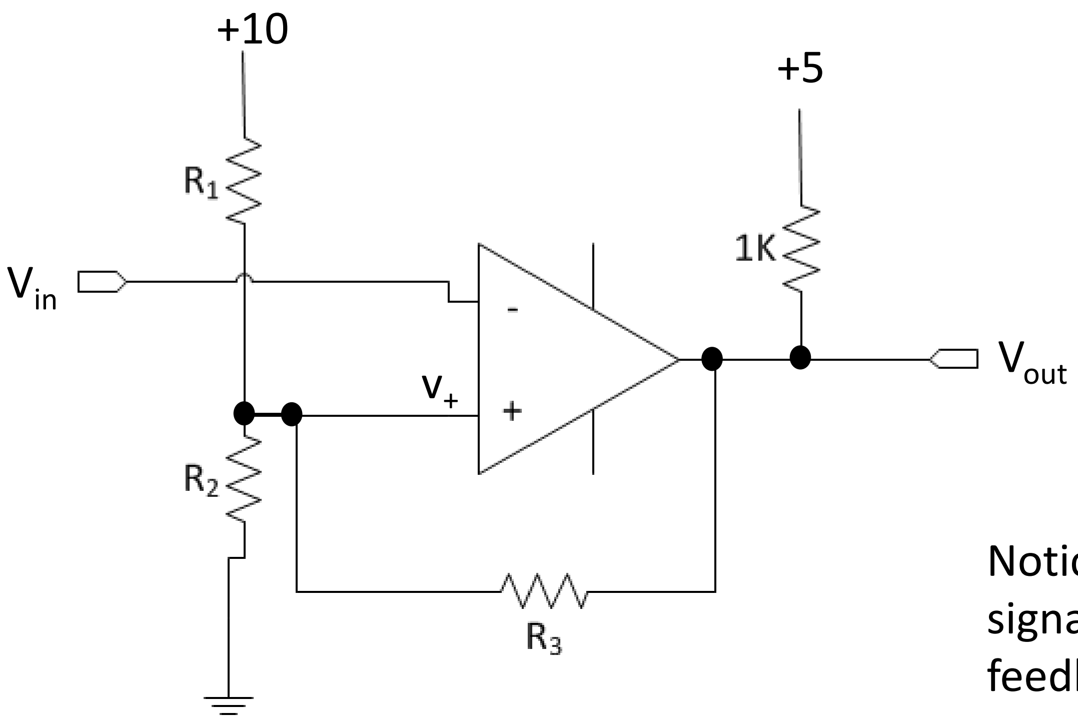
We can eliminate the jitter by adding a small amount of hysteresis to the threshold.

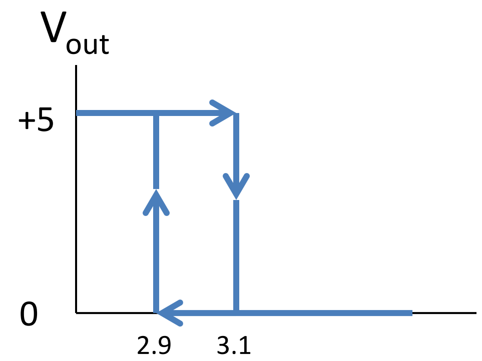
Operational Amplifiers

Other Functions

Comparators

Suppose we want to compare Vin to find when it is 3 ± 0.1 volts?



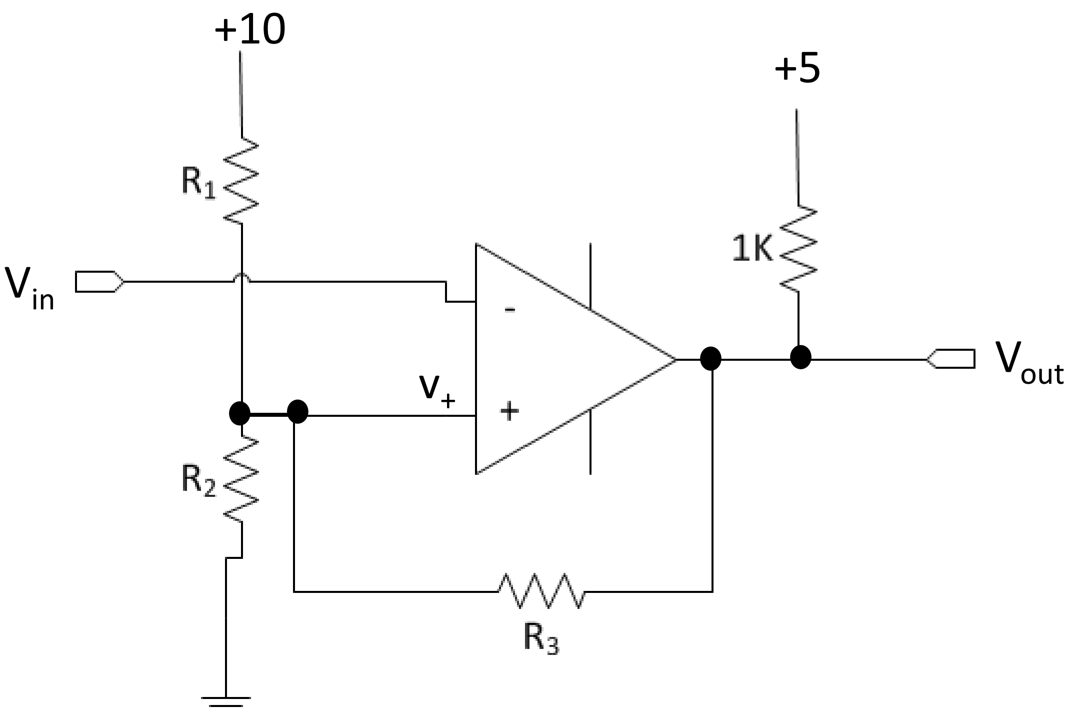


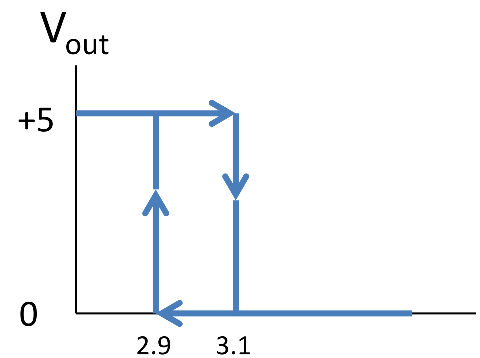
Notice we are putting the signal into the – input and feedback into the + input.

Operational Amplifiers

Other Functions

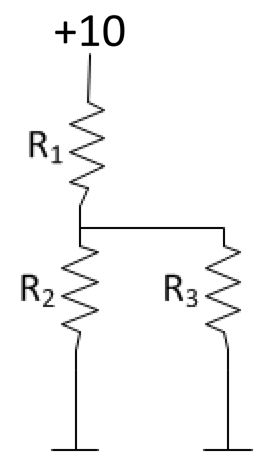
Comparators





Suppose at t=0, we assume $V_{in} > 2.9$ volts. $\therefore V_{out} = 0$

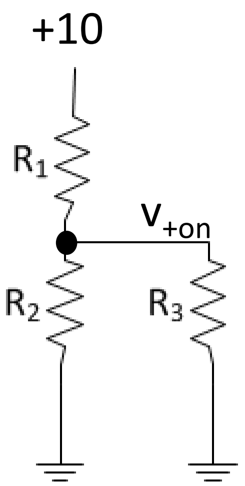
So the resistor network looks like:



Operational Amplifiers

Other Functions

Comparators



So $v_{+on} = 10\,\frac{R_2||R_3}{R_1+(R_2||R_3)}$

Set v_{+on} = 2.9 volts

$\therefore\ R1 + (R_2||R_3) = \frac{10}{2.9}(R_2||R_3)$

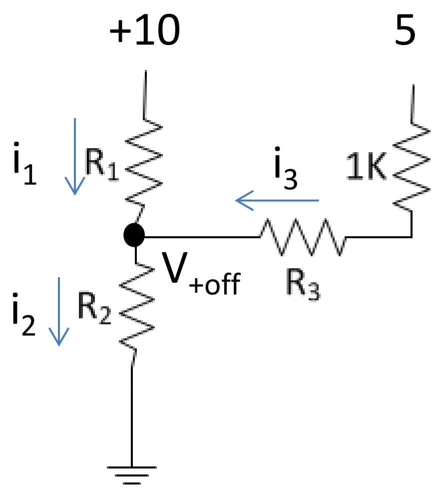
Operational Amplifiers

Other Functions

Comparators

Now we look at the off threshold. There, $V_{out} \cong 5$ volts.

So the resistor network looks like:



We want V_{+off} to be 3.1 volts.

So $i_1R_1 = (10 - V_{+off}) = 6.9$ volts

$i_2R_2 = 6.9$ volts

& $(R_3 + 1K)\, i_3 = (5 - V_{+off}) = 1.9$ volts

We know $i_2 = i_1 + i_3$ from current conservation.

So $3.1\, R_1\, (R_3 + 1K) = 6.9\, (R_3 + 1K)R_2 + R_1R_2 1.9$

Operational Amplifiers

Other Functions

Comparators

Two equations, three unknowns.

$$R1 + (R_2||R_3) = \frac{10}{2.9}(R_2||R_3)$$

$$3.1\ R_1\ (R_3 + 1K) = 6.9\ (R_3 + 1K)R_2 + R_1R_21.9$$

What do we do?

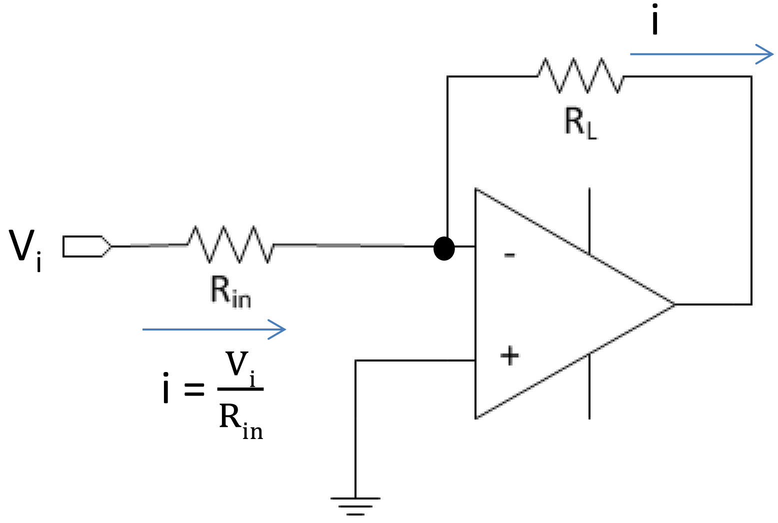
Make a design decision: Pick R_2 to be 10K

Then $R_1 \approx 22K$ & $R_3 \approx 183K$

Operational Amplifiers

Other Functions

Constant current source



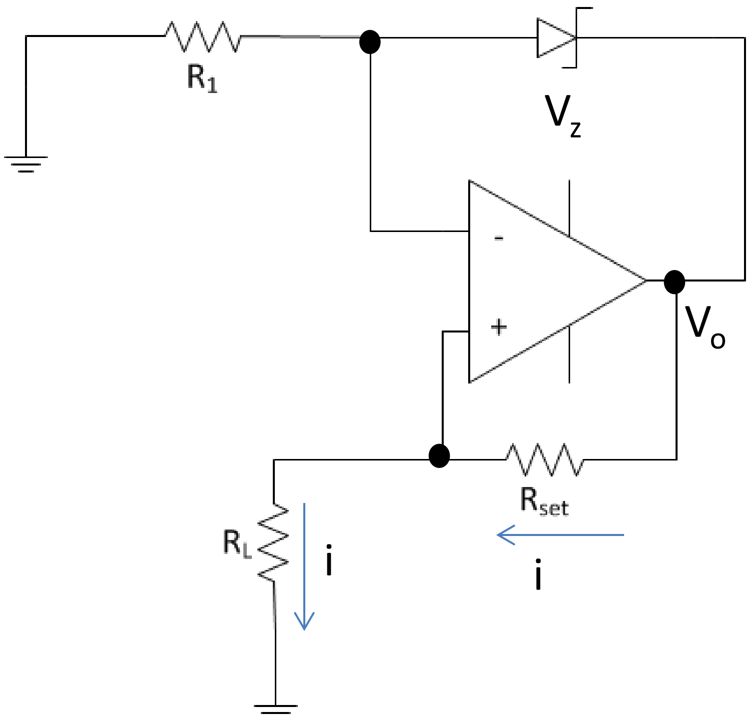
Note that R_L can vary all over, and i through R_L stays at $\frac{V_i}{R_{in}}$.

Of course, this is only as good as V_i & R_{in}. If they drift, so does i.

Operational Amplifiers

Other Functions

Better constant current source



$i = \frac{V_z}{R_{set}}$

Why?

$i = \frac{V_o - V_+}{R_{set}}$

But $V_- = V_+$

So $i = \frac{V_o - V_-}{R_{set}}$

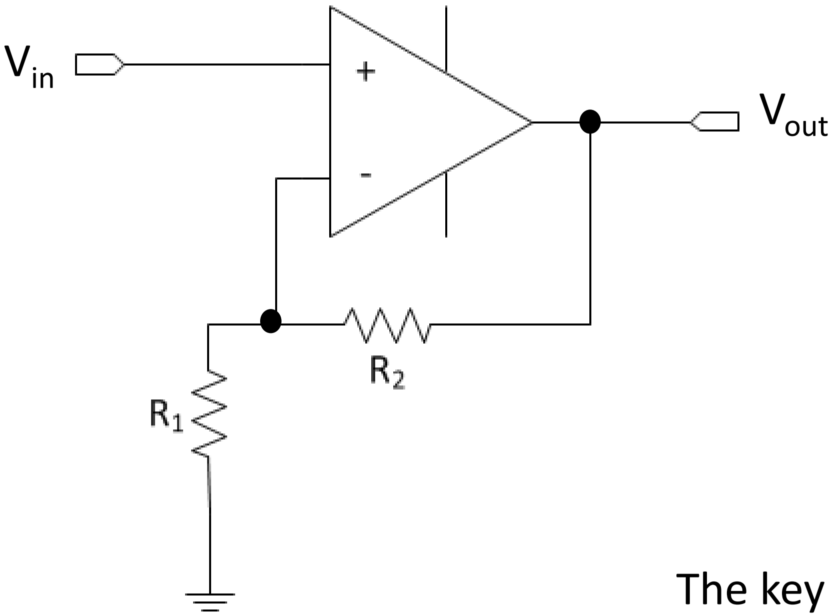
But $V_- = V_o - V_z$

So $i = \frac{V_z}{R_{set}}$

Operational Amplifiers

Other Functions

Constant voltage source



Start with a non-inverting amp.

$V_{in} = V_-$

$V_- = \frac{R_2}{R_1+R_2} V_{out}$

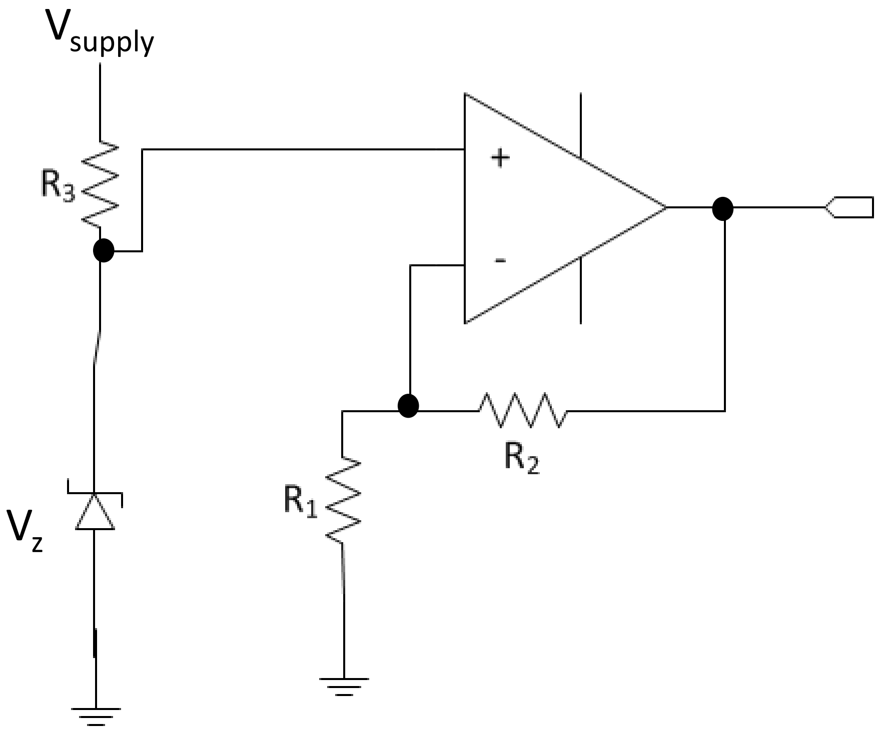
$V_{out} = V_- \left(1 + \frac{R_1}{R_2}\right)$

The key is how stable you can make V_{in}.

Operational Amplifiers

Other Functions

Constant voltage source



You already know how to make a fixed voltage using zener diodes.

$$V_{out} = V_z\left(1 + \frac{R_1}{R_2}\right)$$

Is there an advantage to using the zener + op-amp instead of just the zener?

Operational Amplifiers

Other Functions

Constant voltage source

Is there an advantage to using the zener + op-amp instead of just the zener?

Yes!

1. We do not want the load (which would be in parallel with the zener) to shift us on the zener I-V curve.
2. We can scale the voltage to anything we want from the zenor (even use a pot so the scaling is adjustable).