Online Appendix to
"A theory of LBO activity based on repeated debt-equity conflicts"

by

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The Online Appendix contains the proofs of all lemmas, corollaries, and supplementary results. The numbering of the equations continues the numbering in the main paper.

Proof of Lemma 1

First, suppose that the target takes debt $D \leq (1-\lambda) X_B$. Then, shareholders can credibly promise that diversion will not occur. Then, creditors get D in both states and are willing to invest $\frac{D}{1+r}$. The value of the firm's equity at the beginning of the period is thus

$$\frac{q_T (X_G + g(D)) + (1-q_T) X_B}{1+r}. \tag{32}$$

Because $g(D)$ is strictly increasing for $D < D^*$, the optimal debt in this range is $(1-\lambda) X_B$.

Second, suppose that the target takes debt $D > (1-\lambda) X_B$. Creditors anticipate being expropriated and getting zero in the bad state and hence are willing to invest only $\frac{q_T \min(D, X_G + g(D))}{1+r}$. The value of the firm's equity at the initial date is thus

$$\begin{aligned} & \frac{q_T (X_G + g(D) - D)^+ + (1-q_T) \lambda X_B}{1+r} + \frac{q_T \min(D, X_G + g(D))}{1+r} \\ = \; & \frac{q_T (X_G + g(D)) + (1-q_T) X_B}{1+r} - \frac{(1-q_T)(1-\lambda) X_B}{1+r}. \end{aligned} \tag{33}$$

Compared with Eq. (32), equity value is reduced by the expected deadweight loss from diversion, $\frac{(1-q_T)(1-\lambda)X_B}{1+r}$. Effectively, shareholders bear the full cost of diversion by paying a high interest rate on debt. It follows that the optimal debt in this range is D^*.

It follows that the target chooses between taking high debt D^* and diverting value from creditors in the bad state and taking low debt $(1-\lambda) X_B$ and not diverting value. Its equity value in these two scenarios is Eq. (33) for $D = D^*$ and Eq. (32) for $D = (1-\lambda) X_B$, respectively. The trade-off is between the benefits of higher leverage and the deadweight loss from diversion. Given Assumption 2, the latter policy is optimal, so the target's value if it is non-PE-owned is given by V_0 in the statement of the lemma.

Proof of Lemma 2

Suppose that each PE firm can commit to not diverting value. Consider how much cash creditors are willing to invest if PE firm i takes debt D_i. Creditors get $Y_{G,i} \equiv \min(D_i, X_G + g(D_i))$ in the good state and $Y_{B,i} \equiv \min(D_i, X_B)$ in the bad state. Hence, they are willing to invest

$$D_{0,i} = \frac{q_i Y_{G,i} + (1-q_i) Y_{B,i}}{1+r}, \tag{34}$$

and the implied promised interest rate on debt is $\frac{D_i}{D_{0,i}} - 1$. The interest rate equals r if debt is riskless, $D_i < X_B$, and is strictly greater than r otherwise.

If the PE firm pays price P for the target, it must invest $P - \frac{q_i Y_{G,i} + (1-q_i) Y_{B,i}}{1+r}$ out of its own capital. The net payoff for the PE firm from undertaking the deal is, therefore,

$$\frac{q_i (X_G + g(D_i) - D_i)^+ + (1-q_i)(X_B - D_i)^+}{1+r} - \left(P - \frac{q_i Y_{G,i} + (1-q_i) Y_{B,i}}{1+r}\right), \tag{35}$$

which can be rewritten as

$$V_0 + \frac{(q_i - q_T)\Delta_X + q_i g(D_i) - q_T g_0}{1+r} - P, \tag{36}$$

where V_0 is the value of the independent target given by Eq. (2). Intuitively, because the credit market is competitive, the PE firm captures the expected added value from the deal minus the premium over the current price, $P - V_0$.

It follows that the PE firm's maximum willingness to pay for the target is given by $V_0 + \frac{(q_i - q_T)\Delta_X + d_g(q_i, D_i)}{1+r}$, so it bids in the auction if and only if $(q_i - q_T)\Delta_X + d_g(q_i, D_i) \geq 0$. The price paid by the acquirer is therefore $P = V_0$ if the other bidder does not bid and $P = V_0 + \frac{(q_j - q_T)\Delta_X + d_g(q_j, D_j)}{1+r}$ if the other bidder bids. Thus, the payoff of PE firm i conditional on realizations q_i, q_j is given by Eq. (3). Given Eq. (36), each PE firm finds it optimal to take debt D^* because it maximizes $g(D)$. Thus, the target is acquired if and only if $(\max(q_1, q_2) - q_T)\Delta_X + \max(q_1, q_2) g(D^*) - q_T g_0 > 0$. It follows that all deals with positive value take place irrespective of economy-wide factors r and γ. In addition, regardless of bidders' perceived skills χ_1, χ_2, the bidder with the highest value q_i from operational improvements wins the auction.

Proof of Lemma 3

Suppose that PE firm i takes debt D_i. First, if $D_i > (1-\lambda) X_B$, then creditors anticipate diversion, and the PE firm's payoff (36) is reduced by the expected deadweight loss from diversion, $\frac{(1-q_i)(1-\lambda)X_B}{1+r}$. Thus, its net payoff from the deal given price P is

$$V_0 + \frac{(q_i - q_T)\Delta_X + q_i g(D_i) - q_T g_0}{1+r} - \frac{(1-q_i)(1-\lambda)X_B}{1+r} - P. \tag{37}$$

Given Eq. (37), D^* is optimal among all debt levels for which diversion occurs.

Second, if $D_i \leq (1-\lambda) X_B$, the PE firm can credibly promise not to divert value. It thus does not bear the loss from diversion ex ante, and its payoff given price P and debt D_i is given by Eq. (36). In this range, $(1-\lambda) X_B$ is the optimal debt level. Given Assumption 2, the PE firm prefers this debt policy to taking debt D^* and diverting value.

Proof of Lemma 4

We start by proving that all symmetric equilibria take the following form: $e = 0$, $D \geq (1-\lambda) X_B$.

Suppose, first, that there exists an equilibrium with $e = 0$ but $D < (1-\lambda) X_B$. Consider a deviation to $D' = (1-\lambda) X_B$ in the current period. Because both D and D' do not exceed $(1-\lambda) X_B$, creditors believe that the firm will not divert value. Thus, for both D and D', the PE firm's payoff in the current period is given by Eq. (36), which is strictly higher for D' as $g(D)$ is increasing in this range. The firm's payoff in future periods does not decrease from this deviation because it can follow the same strategy of taking debt $D < (1-\lambda) X_B$ every period. Thus, such a deviation is strictly profitable. Second, suppose there exists an equilibrium with $e = 1$ and some D. By Assumption 2, the PE firm is strictly better off deviating to taking debt $(1-\lambda) X_B$ and not diverting value. Indeed, because $g(D)$ achieves its maximum at D^*, Assumption 2 implies that $g(D) - g((1-\lambda) X_B) < \frac{1-q}{q}(1-\lambda) X_B$ for any D. Based on the specified off-equilibrium beliefs, the PE firm's payoff in future periods does not decrease either. Hence, such a deviation is strictly profitable.

We next prove the existence of the N-equilibrium. Consider the N-equilibrium with

$D = (1-\lambda) X_B$. Given this debt level, it is not optimal to divert value, and the only possible deviation is to a different debt level $D \neq (1-\lambda) X_B$. Deviation to $D < (1-\lambda) X_B$ is suboptimal because $g(D)$ increases for $D \leq D^*$. Based on the specified off-equilibrium beliefs, creditors believe that if a PE firm deviates to $D > (1-\lambda) X_B$, it will divert value in the bad state both now and in any future period when it takes $D > (1-\lambda) X_B$. Hence, the best deviation would be to debt D^*, which is suboptimal by Assumption 2 and because the payoff from future deals does not increase.

We next find conditions for the existence of the C-equilibrium and U-equilibrium: Consider two possible deviations of the PE firm from the equilibrium $D, e = 0$: (1) taking the equilibrium debt, D, but diverting value in the bad state and (2) taking a different level of debt. First, consider a deviation to diversion. If the PE firm chooses to stick to its equilibrium strategy of not diverting value, its expected payoff is

$$(X_B - D)^+ + \frac{\gamma}{r}\mathbb{E}\left[\left[(q_i - q_T)\Delta_X + d_g(q_i, D) - ((q_j - q_T)\Delta_X + d_g(q_j, D))^+\right]^+\right]. \quad (38)$$

The first term is the payoff from the current deal, and the second term is the expected discounted payoff from future deals, as follows from Eq. (3) and the fact that, in each period, the PE firm finds a target with probability γ. If the PE firm deviates and diverts value in the bad state, creditors switch to believing that this PE firm will always divert value in the bad state if the debt level is greater than $(1-\lambda) X_B$. Therefore, if the PE firm diverts value today, the best it can do in the future any time it acquires a target is take debt $(1-\lambda) X_B$ and not divert value. Thus, the payoff from the deviation is

$$\lambda X_B + \frac{\gamma}{r}\mathbb{E}\left[\left[(q_i - q_T)\Delta_X + d_g(q_i, D_0) - ((q_j - q_T)\Delta_X + d_g(q_j, D))^+\right]^+\right]. \quad (39)$$

Such a deviation is suboptimal if and only if (39) $\leq$ (38), which is equivalent to condition (5) due to the property $x^+ - (x-a)^+ = [x]_0^a$.

Second, consider a deviation to a different level of debt $D' \neq D$. As shown previously, any deviation to $D < (1-\lambda) X_B$ is dominated by a deviation to $D = (1-\lambda) X_B$, and any deviation to $D > (1-\lambda) X_B$ is dominated by a deviation to D^*. It is thus sufficient to ensure that deviations to $(1-\lambda) X_B$ and D^* are suboptimal. Upon deviating to $D' \neq D$ today, the best the PE firm can do in future periods is to take debt $(1-\lambda) X_B$ and not divert value. In addition, by Assumption 2, the expected current period payoff from choosing

$(1-\lambda) X_B$ is higher than the expected current period payoff from choosing D^*. Hence, a deviation to D^* is dominated by a deviation to $(1-\lambda) X_B$, and it is sufficient to ensure that a deviation to $(1-\lambda) X_B$ is suboptimal. The expected payoff from playing the equilibrium strategy is proportional to the second term in Eq. (38), and the expected payoff from the deviation to $(1-\lambda) X_B$ is proportional to the second term in Eq. (39). Hence, this deviation is suboptimal if and only if the right-hand side of condition (5) is non-negative. Because $D > (1-\lambda) X_B$, the left-hand side of condition (5) is non-negative, so condition (5) guarantees that a deviation to $D' \neq D$ is suboptimal.

Combining the two cases, condition (5) is the necessary and sufficient condition for the equilibrium with debt $D > (1-\lambda) X_B$ and no diversion to be supported.

Last, we prove that if an equilibrium with $D > D^*$ and $e = 0$ exists, then the U-equilibrium exists as well (and hence the efficiency refinement implies that the equilibrium with $D > D^*$ and $e = 0$ is never selected). To see this, we prove that if condition (5) is satisfied for $D > D^*$, it is also satisfied for $D = D^*$. Indeed, the left-hand side of condition (5) weakly increases in D. Because the right-hand side only depends on D through $g(D)$ and because $g(D) < g(D^*)$ for $D > D^*$, it is sufficient to show that the right-hand side of condition (5) increases in $g(D)$ in the region $g(D) \geq g_0$. To prove this, denote $R(\cdot)$ the right-hand side of condition (5) as a function of $g(D)$:

$$R(g) = \frac{\gamma}{r} \mathbb{E}\left[(q_1 - q_T)\Delta_X + q_1 g - q_T g_0 - ((q_2 - q_T)\Delta_X + q_2 g - q_T g_0)^+\right]_0^{q_1(g-g_0)}, \tag{40}$$

where the expectation is taken over realizations of q_1, q_2. Denoting $a = q_T(\Delta_X + g_0)$,

$$\begin{aligned} \tfrac{r}{\gamma} R(g) &= \mathbb{E}\left[q_1(\Delta_X + g) - a - (q_2(\Delta_X + g) - a)^+\right]_0^{q_1(g-g_0)} \\ &= \mathbb{E}\begin{pmatrix} \left[q_1(\Delta_X + g) - a - (q_2(\Delta_X + g) - a)^+\right]^+ \\ - \left[q_1(\Delta_X + g_0) - a - (q_2(\Delta_X + g) - a)^+\right]^+ \end{pmatrix}. \end{aligned} \tag{41}$$

Denote the function under the expectation by $h(g, q_1, q_2)$. There are two cases:

1. If $q_2(\Delta_X + g) - a \leq 0 \Leftrightarrow q_2 \leq \frac{a}{g+\Delta_X}$, then $h(g, q_1, q_2) = [q_1(\Delta_X + g) - a]^+ - [q_1(\Delta_X + g_0) - a]^+$, which equals zero if $q_1(\Delta_X + g) - a < 0$, equals $q_1(g - g_0)$ if $q_1(\Delta_X + g_0) - a > 0$, and equals $q_1(\Delta_X + g) - a$ if $q_1(\Delta_X + g) - a > 0 > q_1(\Delta_X + g_0) - a$.

2. If $q_2(\Delta_X + g) - a > 0 \Leftrightarrow q_2 > \frac{a}{g+\Delta_X}$, then $h(g, q_1, q_2) = [(q_1 - q_2)(g + \Delta_X)]^+ - [q_1 g_0 - q_2 g + (q_1 - q_2)\Delta_X]^+$, which equals zero if $q_1 - q_2 < 0$, equals $q_1(g - g_0)$ if $q_1 g_0 -$

$q_2 g + (q_1 - q_2)\,\Delta_X > 0$, and equals $(q_1 - q_2)\,(g + \Delta_X)$ otherwise.

Rewriting $R\,(g)$ in the integral form, we get

$$\begin{aligned}
\tfrac{r}{\gamma} R\,(g) &= \int_{-\infty}^{\frac{a}{g+\Delta_X}} \left[\begin{array}{c} \int_{\frac{a}{g+\Delta_X}}^{\frac{a}{g_0+\Delta_X}} \left[q_1\,(\Delta_X + g) - a\right] f\,(q_1)\,dq_1 \\ + \int_{\frac{a}{g_0+\Delta_X}}^{\infty} \left[q_1\,(g - g_0)\right] f\,(q_1)\,dq_1 \end{array} \right] f\,(q_2)\,dq_2 \\
&+ \int_{\frac{a}{g+\Delta_X}}^{\infty} \left[\begin{array}{c} \int_{q_2}^{\frac{g+\Delta_X}{g_0+\Delta_X} q_2} \left[(q_1 - q_2)\,(g + \Delta_X)\right] f\,(q_1)\,dq_1 \\ + \int_{\frac{g+\Delta_X}{g_0+\Delta_X} q_2}^{\infty} \left[q_1\,(g - g_0)\right] f\,(q_1)\,dq_1 \end{array} \right] f\,(q_2)\,dq_2,
\end{aligned} \tag{42}$$

where $f\,(q)$ is the density of q, which equals zero for $q \notin [0,1]$. Taking the derivative and simplifying, $\frac{r}{\gamma} R'\,(g)$ equals

$$\begin{aligned}
&\int_{\frac{a}{g+\Delta_X}}^{\infty} q_1 f\,(q_1)\,dq_1 \int_{-\infty}^{\frac{a}{g+\Delta_X}} f\,(q_2)\,dq_2 \\
&+ \int_{\frac{a}{g+\Delta_X}}^{\infty} \left[\int_{q_2}^{\infty} q_1 f\,(q_1)\,dq_1 - q_2 \int_{q_2}^{\frac{g+\Delta_X}{g_0+\Delta_X} q_2} f\,(q_1)\,dq_1 \right] f\,(q_2)\,dq_2.
\end{aligned} \tag{43}$$

The first term is positive, and the second term is positive because

$$\int_{q_2}^{\infty} q_1 f\,(q_1)\,dq_1 - q_2 \int_{q_2}^{\frac{g+\Delta_X}{g_0+\Delta_X} q_2} f\,(q_1)\,dq_1 > q_2 \int_{q_2}^{\infty} f\,(q_1)\,dq_1 - q_2 \int_{q_2}^{\frac{g+\Delta_X}{g_0+\Delta_X} q_2} f\,(q_1)\,dq_1, \tag{44}$$

which is positive. Hence, $R'\,(g) > 0$, which completes the proof.

Proof of Lemma 5

Fix χ and $\tilde{\chi}$, and let D^* and $D(\tilde{\chi})$ denote the equilibrium debt levels of all firms with skill χ and of the one firm with skill $\tilde{\chi}$, respectively. The analog of the no diversion condition (5) for skill $\tilde{\chi}$ is

$$\begin{aligned}
& \lambda X_B - (X_B - D(\tilde{\chi}))^+ \\
\leq\ & \rho \mathbb{E}\left[(q_1 - q_T)\,\Delta_X + d_g\,(q_1, D(\tilde{\chi})) - ((q_2 - q_T)\,\Delta_X + d_g\,(q_2, D^*))^+ \right]_0^{q_1 (g(D(\tilde{\chi})) - g_0)}.
\end{aligned} \tag{45}$$

Next, consider $\tilde{\chi}' > \tilde{\chi}$. By assumption, $F(\cdot|\tilde{\chi}')$ first-order stochastically dominates $F(\cdot|\tilde{\chi})$. Because the function under the expectation operator of condition (45) increases in q_1, then, by the properties of FOSD, the right-hand side of condition (45) is higher for $\tilde{\chi}'$

than for $\tilde{\chi}$. Hence, if condition (45) is satisfied for $\tilde{\chi}$ and debt $D(\tilde{\chi})$, it is also satisfied for $\tilde{\chi}'$ and debt $D(\tilde{\chi})$, so debt $D(\tilde{\chi})$ is also supported for $\tilde{\chi}'$. Thus, the equilibrium debt level of $\tilde{\chi}'$ is higher than $D(\tilde{\chi})$.

Proof of Corollary 1

The statement of Corollary 1 follows directly from Proposition 2 for $q_0 = q_T$.

Proof of Lemma 6

The evolution of the distribution of types satisfies the following equations:

$$\begin{aligned} \mu_H(t+1) &= (1-\varphi)\left(\mu_H(t) + \frac{1}{2}(1-p)\gamma\mu_U(t)\right), && (46)\\ \mu_U(t+1) &= (1-\varphi)\left[p\gamma + 1 - \gamma\right]\mu_U(t) + \varphi, && (47)\\ \mu_L(t+1) &= (1-\varphi)\left(\mu_L(t) + \frac{1}{2}(1-p)\gamma\mu_U(t)\right). && (48) \end{aligned}$$

Replacing $\mu_\theta(t)$ and $\mu_\theta(t+1)$ by μ_θ for $\theta \in \{H, U, L\}$ gives the statement of the lemma.

Proof of Lemma 7

Consider type $\theta \in \{H, L\}$. Because, in each period, the PE firm remains in the market only with probability φ, $V_R(\theta)$ satisfies

$$\begin{aligned} &V_R(\theta) = \gamma \int\int (z_1(q) - z^+)^+ f(q|\theta)\,\eta(z)\,dqdz + \tfrac{1-\varphi}{1+r} V_R(\theta) \\ &\Leftrightarrow V_R(\theta) = \tfrac{\gamma(1+r)}{r+\varphi} \int\int (z_1(q) - z^+)^+ f(q|\theta)\,\eta(z)\,dqdz. \end{aligned} \qquad (49)$$

By the same argument,

$$V_{NR}(\theta) = \frac{\gamma(1+r)}{r+\varphi} \int\int \left(z_0(q) - z^+\right)^+ f(q|\theta)\,\eta(z)\,dqdz. \qquad (50)$$

Using the property $y^+ - (y-a)^+ = [y]_0^a$ for $y = z_1(q) - z^+$ and $a = z_1(q) - z_0(q) = \frac{q\Delta_g}{1+r}$, we get Eq. (10).

Consider $\theta = U$. If type U has reputation for non-diversion, its expected value satisfies

$$\begin{array}{l} V_R(U) = \frac{(1-\gamma)(1-\varphi)}{1+r} V_R(U) + \gamma p \left(\int\int (z_1(q) - z^+)^+ f(q) \eta(z) \, dq dz + \frac{1-\varphi}{1+r} V_R(U) \right) \\ \quad + \gamma \frac{1-p}{2} \left(\int\int \left(z_{R(H)}(q) - z^+ \right)^+ f_H(q) \eta(z) \, dq dz + \frac{1-\varphi}{1+r} V(H) \right) \\ \quad + \gamma \frac{1-p}{2} \left(\int\int \left(z_{R(L)}(q) - z^+ \right)^+ f_L(q) \eta(z) \, dq dz + \frac{1-\varphi}{1+r} V(L) \right), \end{array} \tag{51}$$

where the first term corresponds to the case in which type U does not find a target in the next period and hence remains unknown, the second term corresponds to the realization of $q \in \left[\frac{1}{2} - d, \frac{1}{2} + d\right]$, and the third and fourth terms correspond to the realizations of $q \in \left[\underline{q}, \frac{1}{2} - d\right]$ and $q \in \left[\frac{1}{2} + d, \bar{q}\right]$, respectively.

Denote $m_R(q,z) \equiv (z_R(q) - z^+)^+$. From the formulas for $V_R(\theta)$ and $V_{NR}(\theta)$, it follows that, for $\theta \in \{H, L\}$,

$$\begin{array}{c} \frac{r+\varphi}{\gamma(1+r)} V(\theta) = \int\int m_{R(\theta)}(q,z) f(q|\theta) \eta(z) \, dq dz \\ = (1-p) \int\int m_{R(\theta)}(q,z) f_\theta(q) \eta(z) \, dq dz + p \int\int m_{R(\theta)}(q,z) f(q) \eta(z) \, dq dz. \end{array} \tag{52}$$

Expressing $\int\int m_{R(\theta)}(q,z) f_\theta(q) \eta(z) \, dq dz$ from Eq. (52) and plugging it into Eq. (51), we can rewrite $V_R(U)$ as

$$\begin{array}{c} V_R(U) = \frac{V(H)+V(L)}{2} \\ + \frac{p(1+r)}{\frac{r+\varphi}{\gamma} + (1-\varphi)(1-p)} \int\int \left(m_1(q,z) - \frac{m_{R(H)}(q,z)}{2} - \frac{m_{R(L)}(q,z)}{2} \right) f(q) \eta(z) \, dq dz. \end{array} \tag{53}$$

Similarly,

$$\begin{aligned} V_{NR}(U) &= \frac{(1-\gamma)(1-\varphi)}{1+r} V_{NR}(U) \\ &+ \gamma p \left(\int\int \left(z_0(q) - z^+ \right)^+ f(q) \eta(z) \, dq dz + \frac{1-\varphi}{1+r} V_{NR}(U) \right) \\ &+ \gamma \frac{1-p}{2} \left(\int\int \left(z_0(q) - z^+ \right)^+ f_H(q) \eta(z) \, dq dz + \frac{1-\varphi}{1+r} V_{NR}(H) \right) \\ &+ \gamma \frac{1-p}{2} \left(\int\int \left(z_0(q) - z^+ \right)^+ f_L(q) \eta(z) \, dq dz + \frac{1-\varphi}{1+r} V_{NR}(L) \right), \end{aligned} \tag{54}$$

and repeating similar arguments, we get $V_{NR}(U) = \frac{V_{NR}(H)+V_{NR}(L)}{2}$. The difference $V_R(U) -$

$V_{NR}(U)$ thus satisfies

$$V_R(U) - V_{NR}(U) = \tfrac{V(H)-V_{NR}(H)+V(L)-V_{NR}(L)}{2} \\ + \tfrac{p(1+r)}{\frac{r+\varphi}{\gamma}+(1-\varphi)(1-p)} \int\int \left(m_1(q,z) - \tfrac{1}{2}m_{R(H)}(q,z) - \tfrac{1}{2}m_{R(L)}(q,z)\right) f(q)\,\eta(z)\,dqdz. \tag{55}$$

Using the property $y^+ - (y-a)^+ = [y]_0^a$ with $a = \frac{q\Delta_g - q\Delta_g 1_{R(\theta)=1}}{1+r} = \frac{q\Delta_g 1_{R(\theta)=0}}{1+r}$, we can re-write the integrand to get Eq. (11).

Supplementary Lemma A.1

We next prove a supplementary Lemma A.1, which characterizes the necessary and sufficient conditions for the existence of each equilibrium in the model of Section 4.

Lemma A.1. *Consider the model of Section 4. The N-equilibrium always exists. The H-equilibrium exists if and only if* $\lambda X_B - (X_B - D^*)^+ \leq V_R^H(H) - V_{NR}^H(H)$, *where*

$$V_R^H(H) - V_{NR}^H(H) = \frac{\gamma(1+r)}{r+\varphi}\mathbb{E}\left[\left[z_1(q_1) - \left(z_{1\{\theta_2=H\}}(q_2)\right)^+\right]_0^{\frac{q_1\Delta_g}{1+r}} \mid \chi_1 = H\right]. \tag{56}$$

The HU-equilibrium exists if and only if $\lambda X_B - (X_B - D^*)^+ \leq V_R^{HU}(U) - V_{NR}^{HU}(U)$, *where*

$$V_R^{HU}(U) - V_{NR}^{HU}(U) = \tfrac{1}{2}\tfrac{\gamma(1+r)}{r+\varphi}\mathbb{E}\left[\left[z_1(q_1) - \left(z_{1\{\theta_2\neq L\}}(q_2)\right)^+\right]_0^{\frac{q_1\Delta_g}{1+r}} \mid \chi_1 = H\right] \\ + \tfrac{1}{2}\tfrac{p(1+r)}{\frac{r+\varphi}{\gamma}+(1-\varphi)(1-p)}\mathbb{E}\left[\left[z_1(q_1) - \left(z_{1\{\theta_2\neq L\}}(q_2)\right)^+\right]_0^{\frac{q_1\Delta_g}{1+r}} \mid q_1 \in \left[\tfrac{1}{2}-d, \tfrac{1}{2}+d\right]\right]. \tag{57}$$

The HUL-equilibrium exists if and only if $\lambda X_B - (X_B - D^*)^+ \leq V_R^{HUL}(L) - V_{NR}^{HUL}(L)$, *where*

$$V_R^{HUL}(L) - V_{NR}^{HUL}(L) = \frac{\gamma(1+r)}{r+\varphi}\mathbb{E}\left[\left[z_1(q_1) - (z_1(q_2))^+\right]_0^{\frac{q_1\Delta_g}{1+r}} \mid \chi_1 = L\right]. \tag{58}$$

The right-hand sides of Eqs. (56), (57), and (58) decrease in r. *As* r *decreases, the most efficient equilibrium first switches from the N-equilibrium to the H-equilibrium, and then switches to either the HU- or to the HUL-equilibrium.*

Proof of Lemma A.1

1. In the N-equilibrium, if a PE firm deviates from taking debt $(1-\lambda)X_B$, creditors believe that the firm will divert value if $D > (1-\lambda)X_B$. Given these beliefs and Assumption 2, deviation is not optimal, and hence the N-equilibrium always exists.

2. Eq. (56) follows directly from Eq. (10).

3. Using Eq. (11), in the HU-equilibrium, we can rewrite $V_R^{HU}(U) - V_{NR}^{HU}(U)$ as

$$\begin{aligned} &V_R^{HU}(U) - V_{NR}^{HU}(U) = \tfrac{V_R^{HU}(H) - V_{NR}^{HU}(H)}{2} \\ &+\tfrac{1}{2}\tfrac{p(1+r)}{\frac{r+\varphi}{\gamma}+(1-\varphi)(1-p)} \int\int \left[z_1(q) - z^+\right]_0^{\frac{q\Delta_g}{1+r}} f(q)\,\eta_{HU}(z)\,dqdz, \end{aligned} \tag{59}$$

and using Eq. (10), $V_R^{HU}(U) - V_{NR}^{HU}(U) < V_R^{HU}(H) - V_{NR}^{HU}(H)$ is equivalent to

$$\begin{aligned} &\tfrac{p(1+r)}{\frac{r+\varphi}{\gamma}+(1-\varphi)(1-p)} \int\int \left[z_1(q) - z^+\right]_0^{\frac{q\Delta_g}{1+r}} f(q)\,\eta_{HU}(z)\,dqdz \\ &< V_R^{HU}(H) - V_{NR}^{HU}(H) = \tfrac{\gamma(1+r)}{r+\varphi} \int\int \left[z_1(q) - z^+\right]_0^{\frac{q\Delta_g}{1+r}} f(q|H)\,\eta_{HU}(z)\,dqdz. \end{aligned} \tag{60}$$

Because $\left[z_1(q) - z^+\right]_0^{\frac{q\Delta_g}{1+r}}$ increases in q, the distribution $f(q|H)$ first-order stochastically dominates $f(q)$, and because $\frac{\gamma(1+r)}{r+\varphi} > \frac{p(1+r)}{\frac{r+\varphi}{\gamma}+(1-\varphi)(1-p)}$, this inequality holds. Hence, if type U can sustain no diversion, then type H can sustain no diversion as well. Thus, the HU-equilibrium exists if and only if $\lambda X_B - (X_B - D^*)^+ \leq V_R^{HU}(U) - V_{NR}^{HU}(U)$. Combining Eqs. (59) and (10) gives Eq. (57).

4. As shown in the proof of Proposition 3, in the HUL-equilibrium, $V_R^{HUL}(L) - V_{NR}^{HUL}(L) < V_R^{HUL}(U) - V_{NR}^{HUL}(U) < V_R^{HUL}(H) - V_{NR}^{HUL}(H)$. Hence, the HUL-equilibrium exists if and only if $\lambda X_B - (X_B - D^*)^+ \leq V_R^{HUL}(L) - V_{NR}^{HUL}(L)$. Using Eq. (10), we derive Eq. (58).

5. Note also that

$$\begin{aligned} &(1+r)\left[z_1(q_1) - (z_R(q_2))^+\right]_0^{\frac{q_1\Delta_g}{1+r}} \\ =\; &\left[(q_1 - q_T)(\Delta_X + g_0) + q_1\Delta_g - ((q_2 - q_T)(\Delta_X + g_0) + q_2\Delta_g 1_{R=1})^+\right]_0^{q_1\Delta_g}, \end{aligned} \tag{61}$$

which does not depend on r. Hence, the right-hand sides of Eqs. (56), (57), and (58) decrease in r.

6. Finally, note that the right-hand side of Eq. (56) is smaller than both the right-hand

side of Eq. (57) and the right-hand side of Eq. (58). It is smaller than the right-hand side of Eq. (57) because $\frac{p}{\frac{r+\varphi}{\gamma}+(1-\varphi)(1-p)} \leq \frac{\gamma}{r+\varphi}$, $z_{1\{\theta_2 \neq L\}}(q_2) \geq z_{1\{\theta_2=H\}}(q_2)$, and the distribution $f(\cdot|H)$ first-order stochastically dominates $f(\cdot)$. It is smaller than the right-hand side of Eq. (58) because $z_1(q_2) \geq z_{1\{\theta_2=H\}}(q_2)$ and the distribution $f(\cdot|H)$ first-order stochastically dominates $f(\cdot|L)$. Hence, as the discount rate decreases, the most efficient equilibrium first switches from the N-equilibrium to the H-equilibrium, and then switches to either the HU- or to the HUL-equilibrium.

Proof of Lemma 8

For brevity, it is convenient to introduce the following notations:

$$Q_L = \left[\underline{q}, \frac{1}{2} - d\right];\ Q_M = \left[\frac{1}{2} - d, \frac{1}{2} + d\right];\ Q_H = \left[\frac{1}{2} + d, \bar{q}\right]. \tag{62}$$

Let $W_R^{\varrho}(\theta)$ denote the expected value to type $\theta \in \{H, L, U\}$ in the ϱ-equilibrium with club deals without diversion and upon diversion, respectively, $\varrho \in \{HUL, HU, H\}$.

Step 1: Sustainability of the HUL-equilibrium.

Consider the HUL-equilibrium and the incentives of type θ to divert value from creditors once $s = B$ is realized. If it does not divert value, its expected payoff from future deals, $W_R^{HUL}(\theta)$, is the same as in the basic model, $V_R^{HUL}(\theta)$, because club deals do not occur on the equilibrium path. If the bidder diverts value, it can team up with the other bidder in the future. For types $\theta \in \{L, H\}$:

$$\begin{aligned} W_{NR}^{HUL}(\theta) &= \gamma\left(\mathbb{E}\left[\left[z_0(q_1) - (z_1(q_2))^+\right]^+ \mid \chi_1 = \theta\right] + \mathbb{E}\left[\frac{1}{2}S_{club}|\chi_1 = \theta\right]\right) \\ +\frac{1-\varphi}{1+r}W_{NR}^{HUL}(\theta) &\Leftrightarrow W_{NR}^{HUL}(\theta) = V_{NR}^{HUL}(\theta) + \frac{\gamma(1+r)}{r+\varphi}\mathbb{E}\left[\frac{1}{2}S_{club}|\chi_1 = \theta\right]. \end{aligned} \tag{63}$$

Thus, type $\theta \in \{H, L\}$ does not divert value if and only if

$$\begin{aligned} &\lambda X_B - (X_B - D^*)^+ \leq W_R^{HUL}(\theta) - W_{NR}^{HUL}(\theta) \\ &= V_R^{HUL}(\theta) - V_{NR}^{HUL}(\theta) - \tfrac{\gamma(1+r)}{r+\varphi}\mathbb{E}\left[\tfrac{1}{2}S_{club}|\chi_1 = \theta\right]. \end{aligned} \tag{64}$$

Using Eqs. (12) and (10), we can rewrite the difference $W_R^{HUL}(\theta) - W_{NR}^{HUL}(\theta)$ as $\frac{1}{2}\left(V_R^{HUL}(\theta) - V_{NR}^{HUL}(\theta)\right)$. As shown before, $V_R^{HUL}(L) - V_{NR}^{HUL}(L) < V_R^{HUL}(H) - V_{NR}^{HUL}(H)$, and hence $W_R^{HUL}(L) - W_{NR}^{HUL}(L) < W_R^{HUL}(H) - W_{NR}^{HUL}(H)$. For the bidder with unknown skill, $\theta = U$:

$$\begin{array}{l} W_{NR}^{HUL}(U) = \frac{(1-\gamma)(1-\varphi)}{1+r} W_{NR}^{HUL}(U) \\ +\gamma p \left(\int\int (z_0(q) - z^+)^+ f(q)\,\eta_{HUL}(z)\,dqdz + \mathbb{E}\left[\frac{1}{2} S_{club} | q_1 \in Q_M\right] + \frac{1-\varphi}{1+r} W_{NR}^{HUL}(U) \right) \\ +\gamma \frac{1-p}{2} \left(\int\int (z_0(q) - z^+)^+ f_H(q)\,\eta_{HUL}(z)\,dqdz + \mathbb{E}\left[\frac{1}{2} S_{club} | q_1 \in Q_H\right] + \frac{1-\varphi}{1+r} W_{NR}^{HUL}(H) \right) \\ +\gamma \frac{1-p}{2} \left(\int\int (z_0(q) - z^+)^+ f_L(q)\,\eta_{HUL}(z)\,dqdz + \mathbb{E}\left[\frac{1}{2} S_{club} | q_1 \in Q_L\right] + \frac{1-\varphi}{1+r} W_{NR}^{HUL}(L) \right), \end{array} \tag{65}$$

which is equivalent to

$$\begin{aligned} & \frac{r + \varphi + \gamma(1-p)(1-\varphi)}{1+r} W_{NR}^{HUL}(U) \\ = \; & \frac{\gamma}{2} \left(\int\int \left(z_0(q) - z^+\right)^+ f(q|H)\,\eta_{HUL}(z)\,dqdz + \mathbb{E}\left[\frac{1}{2} S_{club} | \chi_1 = H\right] \right) \\ & + \frac{\gamma}{2} \left(\int\int \left(z_0(q) - z^+\right)^+ f(q|L)\,\eta_{HUL}(z)\,dqdz + \mathbb{E}\left[\frac{1}{2} S_{club} | \chi_1 = L\right] \right) \\ & + \gamma(1-p) \frac{1-\varphi}{1+r} \frac{W_{NR}^{HUL}(H) + W_{NR}^{HUL}(L)}{2}. \end{aligned} \tag{66}$$

Using Eq. (63) and the expression for $W_{NR}^{HUL}(\theta)$, $\theta \in \{L, H\}$, we obtain

$$W_{NR}^{HUL}(U) = \frac{W_{NR}^{HUL}(H) + W_{NR}^{HUL}(L)}{2}. \tag{67}$$

Because $W_R^{HUL}(U) = V_R^{HUL}(U) = \frac{1}{2}\left(W_R^{HUL}(H) + W_R^{HUL}(L)\right)$ and $W_R^{HUL}(L) - W_{NR}^{HUL}(L) < W_R^{HUL}(H) - W_{NR}^{HUL}(H)$, we conclude that

$$W_R^{HUL}(L) - W_{NR}^{HUL}(L) < W_R^{HUL}(U) - W_{NR}^{HUL}(U) < W_R^{HUL}(H) - W_{NR}^{HUL}(H). \tag{68}$$

Therefore, the HUL equilibrium exists if and only if type $\theta = L$ has no incentive to divert, that is, if and only if condition (13) holds.

Step 2: Sustainability of the H-equilibrium.

Consider the H-equilibrium and the incentives of type H to divert value from creditors once $s = B$ is realized. Without diversion, its expected payoff from future deals, $W_R^H(H)$, is

$$V_R^H(H) + \frac{\gamma(1+r)}{2(r+\varphi)} \left(\begin{array}{c} \left(\mu_L + \frac{\mu_U}{2}\right) \mathbb{E}\left[S_{club} | \chi_1 = L, \chi_2 = H\right] \\ + \frac{\mu_U}{2} p \mathbb{E}\left[S_{club} | q_1 \in Q_M, \chi_2 = H\right] \end{array} \right). \tag{69}$$

The first component is the expected payoff from the English auction. The second component reflects the expected additional surplus from club formation. It can occur if the other bidder is of low-skill (probability $\mu_L + \frac{\mu_U}{2}$) or if the other bidder is of high-skill but its type is unknown at the time of club formation (probability $\frac{\mu_U}{2}p$). Next, suppose that type H deviates and diverts value in the bad state. Then, creditors switch to charging the bidder high interest rates on debt, and the only way for the bidder to obtain cheap financing is to team up with a type-H bidder. This happens if the bidder was known to be high-skill at the beginning of the period or if the bidder's skill was unknown but was revealed to be high because $q \in Q_H$ was realized. Hence, the bidder's expected payoff, $W_{NR}^{H}(H)$, is

$$V_{NR}^{H}(H) + \frac{\gamma(1+r)}{2(r+\varphi)} \begin{pmatrix} \mu_H \mathbb{E}\left[S_{club}|\chi_1 = H, \chi_2 = H\right] \\ +\frac{\mu_U}{2}(1-p)\,\mathbb{E}\left[S_{club}|\chi_1 = H, q_2 \in Q_H\right] \end{pmatrix}. \tag{70}$$

Combining Eq. (69) with Eq. (70) and using the fact that $E\left[\frac{1}{2}S_{club}|\chi_1 = L, \chi_2 = H\right] = p^2 E\left[\frac{1}{2}S_{club}|q_1 \in Q_M, q_2 \in Q_M\right]$, we conclude that the H-equilibrium is sustainable if and only if

$$\begin{aligned} &\lambda X_B - (X_B - D^*)^+ \leq V_R^H(H) - V_{NR}^H(H) \\ &+\tfrac{\gamma(1+r)}{r+\varphi}\left((\mu_L + \mu_U)\,\mathbb{E}\left[\tfrac{1}{2}S_{club}|\chi_1 = L, \chi_2 = H\right] - \mu_H \mathbb{E}\left[\tfrac{1}{2}S_{club}|\chi_1 = H, \chi_2 = H\right]\right. \\ &\left.-\tfrac{\mu_U}{2}(1-p)^2\,\mathbb{E}\left[\tfrac{1}{2}S_{club}|q_1 \in Q_H, q_2 \in Q_H\right]\right). \end{aligned} \tag{71}$$

Conditional on $\chi_1 = H, q_2 \in Q_H$, a club can form only if $q_1 > q_2$, which requires $q_1 \in Q_H$. This implies that condition (71) is equivalent to condition (14).

Step 3: Sustainability of the HU-equilibrium.

First, consider type H and its incentive to divert value from creditors upon realization of state $s = B$. Without diversion, its expected payoff from future deals, $W_R^{HU}(H)$, is

$$V_R^{HU}(H) + \frac{\gamma(1+r)}{r+\varphi} \begin{pmatrix} \mu_L \mathbb{E}\left[\frac{1}{2}S_{club}|\chi_1 = L, \chi_2 = H\right] \\ +\frac{\mu_U(1-p)}{2}\mathbb{E}\left[\frac{1}{2}S_{club}|q_1 \in Q_L, \chi_2 = H\right] \end{pmatrix}. \tag{72}$$

The first component is the expected payoff from the English auction. The second component reflects the expected additional surplus from club formation. The type-H bidder forms a club if it matches with the low type, which happens either if the other bidder is known to be low-skill at the start of the period or if the other bidder is of unknown skill at the start of the period and $q \in Q_L$ is realized. Because conditional on $\chi_2 = H$, the realization of q is never below $\frac{1}{2} - d$, $E\left[\frac{1}{2}S_{club}|q_1 \in Q_L, \chi_2 = H\right] = 0$. Therefore,

$$W_R^{HU}(H) = V_R^{HU}(H) + \frac{\gamma(1+r)}{r+\varphi}\mu_L \mathbb{E}\left[\frac{1}{2}S_{club}|\chi_1 = L, \chi_2 = H\right]. \tag{73}$$

Next, suppose that type H deviates and diverts value in the bad state. It can form a club if it meets a bidder with a reputation for non-diversion, i.e., either a bidder with known high-skill (probability μ_H) or a bidder of unknown skill at the start of the period (probability μ_U) that gets a realization of q not below $\frac{1}{2} - d$. Thus, the bidder's expected payoff can be written as

$$\begin{aligned} W_{NR}^{HU}(H) = V_{NR}^{HU}(H) + \tfrac{\gamma(1+r)}{r+\varphi}&((\mu_H + \mu_U)\, p\mathbb{E}\left[\tfrac{1}{2}S_{club}|\chi_1 = H, q_2 \in Q_M\right] \\ &+ \left(\mu_H + \tfrac{\mu_U}{2}\right)(1-p)\,\mathbb{E}\left[\tfrac{1}{2}S_{club}|\chi_1 = H, q_2 \in Q_H\right]). \end{aligned} \tag{74}$$

Taking the difference and simplifying, $W_R^{HU}(H) - W_{NR}^{HU}(H)$ equals

$$\begin{aligned} &V_R^{HU}(H) - V_{NR}^{HU}(H) - \tfrac{\gamma(1+r)}{r+\varphi}(\mu_U p^2 \mathbb{E}\left[\tfrac{1}{2}S_{club}|q_1 \in Q_M, q_2 \in Q_M\right] \\ &+ (\mu_H + \mu_U)\, p(1-p)\,\mathbb{E}\left[\tfrac{1}{2}S_{club}|q_1 \in Q_H, q_2 \in Q_M\right] \\ &+ \left(\mu_H + \tfrac{\mu_U}{2}\right)(1-p)^2\,\mathbb{E}\left[\tfrac{1}{2}S_{club}|q_1 \in Q_H, q_2 \in Q_H\right]). \end{aligned} \tag{75}$$

Second, consider a bidder of unknown skill and its incentive to divert value from creditors upon realization of state $s = B$. A bidder can have unknown skill after the realization of q only if it was of unknown skill at the beginning of the period and $q \in Q_M$. If the bidder does not divert value, its expected payoff from future deals, $W_R^{HU}(U)$, equals

$$\begin{aligned} &W_R^{HU}(U) = \tfrac{(1-\gamma)(1-\varphi)}{1+r}W_R^{HU}(U) + \gamma p[\textstyle\int\int (z_1(q) - z^+)^+ f(q)\,\eta_{HU}(z)\,dqdz \\ &+\mu_L p\mathbb{E}[\tfrac{1}{2}S_{club}|q_1 \in Q_M, q_2 \in Q_M] + \tfrac{1-\varphi}{1+r}W_R^{HU}(U)] \\ &+\gamma\tfrac{1-p}{2}[(\textstyle\int\int (z_1(q) - z^+)^+ f_H(q)\,\eta_{HU}(z)\,dqdz + \tfrac{1-\varphi}{1+r}W_R^{HU}(H)) \\ &+\gamma\tfrac{1-p}{2}\textstyle\int\int (z_0(q) - z^+)^+ f_L(q)\,\eta_{HU}(z)\,dqdz + \tfrac{1-\varphi}{1+r}W^{HU}(L)]. \end{aligned} \tag{76}$$

The logic behind Eq. (76) is as follows. Over the next period, with probability φ, the bidder leaves the market. With probability $(1-\varphi)(1-\gamma)$, it stays but does not get matched with a target, corresponding to the first term of Eq. (76). With probability γp, it gets matched with a target and draws $q \in Q_M$, in which case the bidder's skill remains unknown. Because a bidder of unknown skill does not divert value in the HU-equilibrium, this bidder can get an extra payoff from the club if the other bidder is of type L and has $q \in Q_M$ (this means that the type-L bidder was of type L at the beginning of the period as well and the fraction of such types is μ_L). This corresponds to the second term of Eq. (76). The last two terms of Eq. (76) reflect situations in which the bidder's skill is revealed next period. The third term corresponds to the case in which the bidder's skill is high, and it draws $q \in Q_H$ (in this case, it cannot lend its reputation to a type-L bidder because q conditional on $\chi = L$ is always below $\frac{1}{2} + d$), whereas the fourth term corresponds to the case in which the bidder's skill is low and it draws $q \in Q_L$ (in this case, it cannot borrow its reputation from a type-U or type-H bidder because q conditional on $\theta = U$ and q conditional on $\chi = H$ are always above $\frac{1}{2} - d$). Rearranging the terms and using Eq. (73) and

$$W^{HU}(L) = V^{HU}(L) + \frac{\gamma(1+r)}{r+\varphi}(\mu_H + \mu_U)\, p^2 \mathbb{E}\left[\frac{1}{2} S_{club} | q_1 \in Q_M, q_2 \in Q_M\right], \tag{77}$$

we obtain that $W_R^{HU}(U)$ equals

$$\begin{gathered} \frac{V_R^{HU}(H) + V^{HU}(L)}{2} \\ + \frac{2\mu_L(r+\varphi) + \gamma(1-p)(1-\varphi)}{r+\varphi+\gamma(1-p)(1-\varphi)} \frac{1+r}{2(r+\varphi)} \gamma p^2 \mathbb{E}\left[\frac{1}{2} S_{club} | q_1 \in Q_M, q_2 \in Q_M\right]. \end{gathered} \tag{78}$$

Next, suppose that the bidder diverts value. An analogous to Eqs. (76)–(78) derivation yields

$$\begin{gathered} W_{NR}^{HU}(U) = \frac{V_{NR}^{HU}(H) + V^{HU}(L)}{2} + \frac{\gamma(1+r)}{r+\varphi}(\mu_H + \mu_U)\, p^2 \mathbb{E}\left[\tfrac{1}{2} S_{club} | q_1 \in Q_M, q_2 \in Q_M\right] \\ + \frac{\gamma(1+r)(1-p)}{r+\varphi} \left(\begin{array}{c} p(\mu_H + \mu_U)\, \mathbb{E}\left[\tfrac{1}{2} S_{club} | q_1 \in Q_H, q_2 \in Q_M\right] \\ + (1-p)\left(\mu_H + \frac{\mu_U}{2}\right) \mathbb{E}\left[\tfrac{1}{2} S_{club} | q_1 \in Q_H, q_2 \in Q_H\right] \end{array} \right). \end{gathered} \tag{79}$$

Taking the difference and simplifying,

$$\begin{gathered} W_R^{HU}(U) - W_{NR}^{HU}(U) = \frac{V_R^{HU}(H) - V_{NR}^{HU}(H)}{2} \\ + \left(\frac{\gamma(1-p)(1-\varphi)}{r+\varphi+\gamma(1-p)(1-\varphi)} \left(\tfrac{1}{2} - \mu_L\right) - \mu_U \right) \frac{1+r}{r+\varphi} \gamma p^2 \mathbb{E}\left[\tfrac{1}{2} S_{club} | q_1 \in Q_M, q_2 \in Q_M\right] \\ - \frac{\gamma(1+r)(1-p)}{2(r+\varphi)} \left(\begin{array}{c} p(\mu_H + \mu_U)\, \mathbb{E}\left[\tfrac{1}{2} S_{club} | q_1 \in Q_H, q_2 \in Q_M\right] \\ + (1-p)\left(\mu_H + \frac{\mu_U}{2}\right) \mathbb{E}\left[\tfrac{1}{2} S_{club} | q_1 \in Q_H, q_2 \in Q_H\right] \end{array} \right). \end{gathered} \tag{80}$$

Using Eqs. (75) and (80), $W_R^{HU}(U) - W_{NR}^{HU}(U)$ can be rewritten as

$$\begin{aligned}&\frac{W_R^{HU}(H) - W_{NR}^{HU}(H)}{2} \\ &-\frac{\gamma(1+r)}{2(r+\varphi)}\frac{(r+\varphi)\mu_U p^2}{r+\varphi+\gamma(1-p)(1-\varphi)}\mathbb{E}\left[\frac{1}{2}S_{club}|q_1 \in Q_M, q_2 \in Q_M\right].\end{aligned} \tag{81}$$

Eq. (81) implies $W_R^{HU}(U) - W_{NR}^{HU}(U) < \frac{W_R^{HU}(H) - W_{NR}^{HU}(H)}{2} < W_R^{HU}(H) - W_{NR}^{HU}(H)$. Therefore, the HU-equilibrium exists if and only if condition

$$\lambda X_B - (X_B - D^*)^+ \leq W_R^{HU}(U) - W_{NR}^{HU}(U) \tag{82}$$

is satisfied, where $W_R^{HU}(U) - W_{NR}^{HU}(U)$ is given by Eq. (80). Using the fact that $\mu_L = \mu_H = \frac{1-\mu_U}{2}$, it is easy to see that $W_R^{HU}(U) - W_{NR}^{HU}(U) < \frac{V_R^{HU}(H) - V_{NR}^{HU}(H)}{2}$. Also, Eq. (59) implies that $V_R^{HU}(U) - V_{NR}^{HU}(U) > \frac{V_R^{HU}(H) - V_{NR}^{HU}(H)}{2}$. Hence, $W_R^{HU}(U) - W_{NR}^{HU}(U) < V_R^{HU}(U) - V_{NR}^{HU}(U)$, which completes the proof.

Proof of Corollary 2

Let $\rho \equiv \frac{\gamma}{r}$. As shown in the proof of Lemma A.1, $(1+r)\left[z_1(q_1) - (z_R(q_2))^+\right]_0^{\frac{q_1\Delta_g}{1+r}}$ does not depend on r. Hence, if $\varphi = 0$, Eqs. (56) and (58) imply that there exist ρ_1 and ρ_2 such that without club deals, the H-equilibrium (HL-equilibrium) exists if and only if $\rho \geq \rho_1$ ($\rho \geq \rho_2$). Moreover, because $z_{1\{\chi_2=H\}}(q_2) \leq z_1(q_2)$ and using FOSD,

$$\begin{aligned}&\mathbb{E}\left[\left[z_1(q_1) - \left(z_{1\{\chi_2=H\}}(q_2)\right)^+\right]_0^{\frac{q_1\Delta_g}{1+r}} \mid \chi_1 = H\right] \\ &\geq \mathbb{E}\left[\left[z_1(q_1) - (z_1(q_2))^+\right]_0^{\frac{q_1\Delta_g}{1+r}} \mid \chi_1 = L\right].\end{aligned} \tag{83}$$

Hence, $\rho_2 \geq \rho_1$.

Next, note that $(1+r)S_{club}$ does not depend on r either. Hence, if $\varphi = 0$, conditions (13) and (14) imply that there exist ρ_1^c and ρ_2^c such that, with club deals, the H-equilibrium (HL-equilibrium) exists if and only if $\rho \geq \rho_1^c$ ($\rho \geq \rho_2^c$). Moreover, $\rho_2^c \geq \rho_2^c$ because the right-hand side of condition (13) is smaller than the right-hand side of condition (14).

Indeed, using Eqs. (12), (56), and (58), this is equivalent to

$$\begin{aligned}\tfrac{1}{2}\mathbb{E}\left[S_{club}\middle|\ \chi_1 = L\right] &\le \mathbb{E}\left[\left[z_1\left(q_1\right) - \left(z_{1\{\theta_2=H\}}\left(q_2\right)\right)^+\right]_0^{\frac{q_1\Delta_g}{1+r}} \mid \chi_1 = H\right] \\ &+ \left(\tfrac{1}{2}\mathbb{E}\left[\tfrac{1}{2}S_{club}|\chi_1 = L, \chi_2 = H\right] - \tfrac{1}{2}\mathbb{E}\left[\tfrac{1}{2}S_{club}|\chi_1 = H, \chi_2 = H\right]\right).\end{aligned} \tag{84}$$

Because $z_{1\{\theta_2=H\}}\left(q_2\right) \le z_1\left(q_2\right)$, it is sufficient to prove that

$$\begin{aligned}\tfrac{1}{2}\mathbb{E}\left[S_{club}\middle|\ \chi_1 = L\right] &\le \mathbb{E}\left[S_{club}\middle|\ \chi_1 = H\right] + \tfrac{1}{4}\mathbb{E}\left[S_{club}|\chi_1 = L, \chi_2 = H\right] \\ -\tfrac{1}{4}\mathbb{E}\left[S_{club}|\chi_1 = H, \chi_2 = H\right] &\Leftrightarrow \tfrac{1}{4}\mathbb{E}\left[S_{club}\middle|\ \chi_1 = L, \chi_2 = L\right] \\ &\le \tfrac{1}{4}\mathbb{E}\left[S_{club}\middle|\ \chi_1 = H, \chi_2 = H\right] + \tfrac{1}{2}\mathbb{E}\left[S_{club}\middle|\ \chi_1 = H, \chi_2 = L\right].\end{aligned} \tag{85}$$

By FOSD, $E[S_{club}|\ \chi_1 = L, \chi_2 = L] < \frac{1}{4}E[S_{club}|\ \chi_1 = H, \chi_2 = L]$, which proves the inequality.

Under the efficiency refinement, the equilibrium without (with) club deals is: (1) the N-equilibrium if $\rho < \rho_1$ ($\rho < \rho_1^c$), (2) the H-equilibrium if $\rho_1 \le \rho < \rho_2$ ($\rho_1^c \le \rho < \rho_2^c$), and (3) the HL-equilibrium if $\rho \ge \rho_2$ ($\rho \ge \rho_2^c$). Hence, the expected value from buyouts when club deals are not allowed is given by

$$\begin{aligned}&\mathbb{E}_{\chi_1,\chi_2}\mathbb{E}\left[\max\left(0, z_0\left(q_1\right), z_0\left(q_2\right)\right)\right], && \rho < \rho_1, \\ &\mathbb{E}_{\chi_1,\chi_2}\mathbb{E}\left[\max\left(0, z_{1\{\chi_1=H\}}\left(q_1\right), z_{1\{\chi_2=H\}}\left(q_2\right)\right)\right], && \rho_1 < \rho < \rho_2, \\ &\mathbb{E}_{\chi_1,\chi_2}\mathbb{E}\left[\max\left(0, z_1\left(q_1\right), z_2\left(q_2\right)\right)\right], && \rho > \rho_2,\end{aligned} \tag{86}$$

and the expected value from buyouts when club deals are allowed is given by

$$\begin{aligned}&\mathbb{E}_{\chi_1,\chi_2}\mathbb{E}\left[\max\left(0, z_0\left(q_1\right), z_0\left(q_2\right)\right)\right], && \rho < \rho_1^c, \\ &\mathbb{E}_{\chi_1,\chi_2}\mathbb{E}\left[\max\left(0, z_{1\{\chi_1=H \text{ or } \chi_2=H\}}\left(q_1\right), z_{1\{\chi_1=H \text{ or } \chi_2=H\}}\left(q_2\right)\right)\right], && \rho_1^c < \rho < \rho_2^c, \\ &\mathbb{E}_{\chi_1,\chi_2}\mathbb{E}\left[\max\left(0, z_1\left(q_1\right), z_2\left(q_2\right)\right)\right], && \rho > \rho_2^c.\end{aligned} \tag{87}$$

Comparing these expressions and using the fact that according to Part 2 of Proposition 5, $\rho_1^c > \rho_1$ and $\rho_2^c > \rho_2$, completes the proof.

Supplementary analysis for the proof of Proposition 6

Consider the environment in which PE firm i does not have a reputation for non-diversion and hence has the target raise debt with face value $(1-\lambda)\,X_B$, while PE firm j has a reputation for non-diversion and hence has the target raise debt with face value $D > (1-\lambda) X_B$ and does not divert value. We prove that no equilibrium exists in which PE firm i wins with positive probability; there exist infinitely many equilibria in which PE firm i wins with probability zero; and the resulting surplus of PE firm i from the auction is zero.

By contradiction, suppose there exists a realization of signals s_i and s_j of PE firms i and j, respectively, for which PE firm i wins with positive probability. Let p denote the price at which PE firm i acquires the target in this case. Conditional on the withdrawing strategy of the rival bidder, the payoff of each PE firm is strictly increasing in its signal. Therefore, the price at which each PE firm withdraws must be weakly increasing in its signal. Let $S_j(p)$ $(S_i(p))$ be the set of types of PE firm j (i), such that it withdraws at price p if and only if $s_j \in S_j(p)$ $(s_i \in S_i(p))$. Because type s_j of PE firm j is better off not continuing bidding at price p,

$$V_0 + \frac{(\pi E_i(p) + \pi s_j + 2(1-\pi)\mathbb{E}[\kappa])(\Delta_X + g(D)) - q_T(\Delta_X + g_0)}{1+r} \leq p, \tag{88}$$

where $E_i(p)$ is the expected signal of PE firm i, conditional on it withdrawing at price p: $E_i(p) = E[s_i | s_i \in S_i(p)]$. If the set of types of PE firm i withdrawing at price p is empty, then PE firm j winning at price p is an off-equilibrium event. In this case, $E_i(p)$ denotes the belief of PE firm j about the expected signal of PE firm i conditional on this off-equilibrium event. By monotonicity, $E_i(p)$ is greater than or equal to the lowest type of s_i that has not withdrawn yet at price p.[25] Condition (88) must hold for any $s_j \in S_j(p)$. Integrating (88) over $s_j \in S_j(p)$ yields

$$V_0 + \frac{(\pi(E_i(p) + E_j(p)) + 2(1-\pi)\mathbb{E}[\kappa])(\Delta_X + g(D)) - q_T(\Delta_X + g_0)}{1+r} \leq p. \tag{89}$$

First, suppose that $S_i(p)$ is non-empty. Because type $s_i \in S_i(p)$ of PE firm i withdraws at price p, it weakly prefers continuation at price $p - \varepsilon$ for an infinitesimal $\varepsilon > 0$. Therefore,

$$V_0 + \frac{(\pi s_i + \pi E_j(p-\varepsilon) + 2(1-\pi)\mathbb{E}[\kappa])(\Delta_X + g_0) - q_T(\Delta_X + g_0)}{1+r} \geq p - \varepsilon. \tag{90}$$

By monotonicity of the withdrawal strategy, $E_j(p) \geq E_j(p-\varepsilon)$. Applying this inequality

[25] We impose a reasonable restriction on the off-the-equilibrium-path beliefs here. Specifically, if in equilibrium PE firm i with type s_i withdraws prior to price p with probability one, then the belief of PE firm j from observing withdrawal of PE firm i at price p cannot be lower than s_i.

to condition (90) and integrating over $s_i \in S_i(p)$ yields

$$V_0 + \frac{(\pi(E_i(p) + E_j(p)) + 2(1-\pi)\mathbb{E}[\kappa])(\Delta_X + g_0) - q_T(\Delta_X + g_0)}{1+r} \geq p - \varepsilon. \tag{91}$$

Second, suppose that $S_i(p)$ is empty. Then, the lowest type of s_i that has not withdrawn yet at price p must prefer continuation at price $p - \varepsilon$. Because $E_i(p)$ is greater than or equal to this type and because $E_j(p) \geq E_j(p-\varepsilon)$, we again obtain (91). Thus, (91) holds regardless of whether $S_i(p)$ is empty or not. Combining conditions (89) and (91), we obtain

$$(\pi(E_i(p) + E_j(p)) + 2(1-\pi)\mathbb{E}[\kappa])(g(D) - g_0) \leq \varepsilon(1+r). \tag{92}$$

This is a contradiction for any equilibrium $D > (1-\lambda)X_B$ because $g(D) > g_0$ and the right-hand side is infinitesimal.

Finally, we prove that there exist infinitely many equilibria in which PE firm i always loses. Let $b_i(s)$ denote the price at which PE firm i with signal s withdraws from the auction, and let $\underline{s}$ $(\bar{s})$ denote the lowest (highest) possible realization of s. Then, any strategy profile in which PE firm i with signal s bids up to $b_i(s)$ satisfying

$$V_0 \leq b_i(s) \leq V_0 + \frac{(\pi(s + \underline{s}) + 2(1-\pi)\mathbb{E}[\kappa])(\Delta_X + g_0) - q_T(\Delta_X + g_0)}{1+r}, \tag{93}$$

and PE firm j with signal s bids up to $b_j(s)$ satisfying

$$\begin{aligned} &V_0 + \tfrac{(\pi(s+\bar{s})+2(1-\pi)\mathbb{E}[\kappa])(\Delta_X+g_0)-q_T(\Delta_X+g_0)}{1+r} \leq b_j(s) \\ &\leq V_0 + \tfrac{(\pi(s+\bar{s})+2(1-\pi)\mathbb{E}[\kappa])(\Delta_X+g(D))-q_T(\Delta_X+g_0)}{1+r}, \end{aligned} \tag{94}$$

is an equilibrium. PE firm i does not benefit from a deviation because, given the strategy of firm j, it can never win at a price below its valuation. PE firm j does not benefit from a deviation because it always wins at a price less than or equal to its valuation.