A.1 Optimal majority requirement

Proposition 9. The optimal majority requirement in the conservative equilibrium, denoted by \( \tau_c \), satisfies

\[
-q_c(\tau_c) = \min\{\beta_c, -H^{-1}(\phi)\},
\]

and the optimal majority requirement in the activist equilibrium, denoted by \( \tau_a \), satisfies

\[
-q_a(\tau_a) = \max\{\beta_a, -H^{-1}(\phi)\}
\]

where \( q_c(\tau) \) and \( q_a(\tau) \) are given by (10) and (9).\(^{31}\)

Proof. Consider first the conservative equilibrium, which exists if and only if \( H(q_c) < \phi \). Recall \( W_c = v(\beta_c, q_c) \), where \( b_c = G^{-1}(1 - \lambda) \), \( \beta_c = \mathbb{E}[b|b < b_c] \), and \( q_c = -G^{-1}((1 - \lambda)(1 - \tau)) \). Using (6),

\[
\frac{\partial W_c}{\partial \tau} = -(\beta_c + q_c) \frac{\partial q_c}{\partial \tau} f(q_c)
\]

Using (10), we get \( \frac{\partial q_c}{\partial \tau} = \frac{1 - \lambda}{g(-q_c)} > 0 \). Plugging into \( \frac{\partial W_c}{\partial \tau} \), we get

\[
\frac{\partial W_c}{\partial \tau} = -(1 - \lambda)(\beta_c + q_c) \frac{f(q_c)}{g(-q_c)}
\]

and hence, \( \frac{\partial W_c}{\partial \tau} > 0 \iff -q_c > \beta_c \). Recall that the conservative equilibrium exists if and only if \( H(q_c) < \phi \iff -q_c < -H^{-1}(\phi) \). Also notice that \( -q_c(\tau) \) spans \([-\bar{b}, b_c]\) as a decreasing function of \( \tau \), and \( \beta_c \in (-\bar{b}, b_c) \). Therefore, there is a unique \( \hat{\tau}_c \in (0, 1) \) such that \( -q_c(\hat{\tau}_c) = \beta_c \). Thus, if \( \beta_c < -H^{-1}(\phi) \) then \( \tau_c = \hat{\tau}_c \), and if \( \beta_c \geq -H^{-1}(\phi) \) then the closet marginal voter to \( \beta_c \) that

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\(^{31}\) We analyze the optimal threshold in a given equilibrium rather than across all equilibria, because when multiple equilibria exist, unless a selection is imposed, the optimal threshold is not well defined.
implies a conservative equilibrium is \(-q^* = -H^{-1}(\phi)\). Thus, \(-q_e(\tau_e) = \min\{\beta_e, -H^{-1}(\phi)\}\) as required.\(^{32}\)

Next, consider the activist equilibrium, which exists if and only if \(H(q_a) > \phi\). Recall \(W_a = v(\beta_a, q_a)\), where \(b_a = G^{-1}(\lambda)\), \(\beta_a = \mathbb{E}[b | b > b_a]\), and \(q_a = -G^{-1}(1 - \tau(1 - \lambda))\). Using (6),

\[
\frac{\partial W_a}{\partial \tau} = - (\beta_a + q_a) \frac{\partial q_a}{\partial \tau} f(q_a) .
\]

Using (10), we get \(\frac{\partial q_a}{\partial \tau} = \frac{1 - \lambda}{g(-q_a)} > 0\). Plugging into \(\frac{\partial W_a}{\partial \tau}\), we get

\[
\frac{\partial W_a}{\partial \tau} = - (1 - \lambda) (\beta_a + q_a) \frac{f(q_a)}{g(-q_a)} ,
\]

and hence, \(\frac{\partial W_a}{\partial \tau} > 0 \iff -q_a > \beta_a\). Recall that the activist equilibrium exists if and only if \(H(q_a) > \phi \iff -q_a > -H^{-1}(\phi)\). Also notice that \(-q_a(\tau)\) spans \([b_a, \overline{b}]\) as a decreasing function of \(\tau\), and \(\beta_a \in (b_a, \overline{b})\). Therefore, there is a unique \(\hat{\tau}_a \in (0, 1)\) such that \(-q_a(\hat{\tau}_a) = \beta_a\). Thus, if \(\beta_a > -H^{-1}(\phi)\) then \(\tau_a = \hat{\tau}_a\), and if \(\beta_a \leq -H^{-1}(\phi)\) then the closet marginal voter to \(\beta_a\) implies an activist equilibrium is \(-q^* = -H^{-1}(\phi)\). Thus, \(-q_a(\tau_a) = \max\{\beta_a, -H^{-1}(\phi)\}\) as required. ■

**A.2 Effect of \(\lambda\) on the benefits of delegation to the optimal board**

In this section, we examine the effect of trading frictions on the benefit from delegation. Specifically, we ask how the comparison between delegation to an optimal board and decision-making via shareholder voting depends on liquidity \(\lambda\). For this purpose, we assume that whenever multiple equilibria exist in the voting game, shareholders will coordinate on the equilibrium with the highest expected welfare. Hence, we are interested in the benefit from delegation

\[ D(\lambda) \equiv W^* - \max\{W_a, W_c\} , \]

where \(W_a\) and \(W_c\) are given by (15). From part (iii) of Proposition 7, \(D(\lambda) \geq 0\) for all \(\lambda \in (0, 1)\). A direct implication of part (ii) of Proposition 7 is that if expected shareholder welfare in the voting equilibrium decreases with \(\lambda\), which happens under the conditions identified in Proposition 6 part (ii), then the benefit from delegation to the

\(^{32}\)As in the proof of Proposition 7, if \(q^* = H^{-1}(\phi)\), the tie-breaking rule we adopt implies that no shareholder trades. While this tie-breaking rule implies that the trading strategies of shareholders in the delegation equilibrium are not continuous in \(q^*\) when \(q^* = H^{-1}(\phi)\), the expected welfare of shareholders is nevertheless continuous in \(q^*\) when \(q^* = H^{-1}(\phi)\), which is the only relevant consideration in the derivation of the optimal majority requirement.
optimal board increases in $\lambda$:

**Corollary 4.** If the expected welfare in the voting equilibrium decreases in $\lambda$, then the benefit from delegation to the optimal board, $D(\lambda)$, increases in $\lambda$.

Next, we show that generally, the effect of trading frictions on the benefit from delegation is ambiguous, and $D(\lambda)$ may be increasing or decreasing. To see this, compare, for example, the conservative equilibrium in the voting game and delegation to an optimal conservative board. The welfare benefit of delegation is

$$D_c(\lambda) \equiv v(\beta_c, -\beta_c) - W_c = \beta_c[F(q_c) - F(-\beta_c)] + \int_{-\beta_c}^{q_c} qdF(q) - \int_{q_c}^{\infty} qdF(q),$$

and hence

$$\frac{\partial D_c}{\partial \lambda} = \frac{\partial q_c}{\partial \lambda} (q_c + \beta_c) f(q_c) + \frac{\partial \beta_c}{\partial \lambda} (F(q_c) - F(-\beta_c)).$$

Note that

$$q_c > -\beta_c \Leftrightarrow F(q_c) > F(-\beta_c).$$

Hence, the first and the second expression both change signs at $q_c = -\beta_c$. Since $\frac{\partial q_c}{\partial \lambda} > 0$ and $\frac{\partial \beta_c}{\partial \lambda} < 0$, the first expression is negative (positive) and the second expression is positive (negative) if $q_c + \beta_c < 0$ ($q_c + \beta_c > 0$). Therefore, $D_c(\lambda)$ may be increasing or decreasing depending on the relative size of these expressions. Intuitively, if the marginal voter is more (less) conservative then the average post-trade shareholder (i.e., $-q_c < (>\beta_c)$), then an increase in liquidity, which makes the marginal voter in the voting game even more conservative ($\frac{\partial q_c}{\partial \lambda} > 0$), increases (decreases) the benefit from delegation. On the other hand, an increase in liquidity also makes the average post-trade shareholder more conservative ($\frac{\partial \beta_c}{\partial \lambda} < 0$), which increases the efficiency of the voting equilibrium and thus reduces (increases) the benefit from delegation.

The next result shows which of these effects dominates when liquidity is relatively high or low.

**Proposition 10.** There exist $0 < \Lambda \leq \bar{\lambda} < 1$ such that:

(i) If $G(\mathbb{E}[b]) \neq 1 - \tau$, then $\Delta(\lambda') > \Delta(\lambda'')$ for all $\lambda' < \Lambda$ and $\lambda'' > \bar{\lambda}$.

(ii) If $G(\mathbb{E}[b]) = 1 - \tau$, then $\Delta(\lambda') > \Delta(\lambda'')$ for all $\lambda' \in (\Lambda, \bar{\lambda})$ and $\lambda'' \notin (\Lambda, \bar{\lambda})$. 

3
Intuitively, consider the generic case (i) in which the bias of the marginal voter in the no-trade equilibrium, $-q_{\text{NoTrade}}$, does not happen to be equal to the average bias of the pre-trade shareholder base. If liquidity is low and converges to zero ($\lambda \to 0$), then both voting equilibria converge to the no-trade equilibrium, which is then strictly inferior to the case with optimal delegation in which marginal voter and the average shareholder are aligned. Conversely, as liquidity increases, extreme shareholders can build larger positions in the firm and tilt the voting outcome in their favor more often, which reduces the benefit of delegation to an optimal board. Indeed, in the limit, as liquidity becomes large ($\lambda \to 1$), both the marginal trader and the marginal voter converge to the most extreme shareholder, so their preferences are fully aligned and delegation adds no value. Hence, the benefits of delegation are large if liquidity is low and small if liquidity is large. The case in which the no-trade equilibrium is efficient is different, because then the voting equilibria converge to an efficient no-trade equilibrium, delegation adds no value and the benefits from delegation arise only for intermediate values of liquidity (part (ii) of the proposition).

**Proof of Proposition 10.** Recall that $\lim_{\lambda \to 1} b_c = -\overline{b}$ and $\lim_{\lambda \to 1} b_a = \overline{b}$. Then we have $\lim_{\lambda \to 1} \beta_c = -\overline{b}$ and $\lim_{\lambda \to 1} \beta_a = \overline{b}$. Also recall that $\lim_{\lambda \to 1} (-q_c) = -\overline{b}$ and $\lim_{\lambda \to 1} (-q_a) = \overline{b}$. Therefore, in both the voting game and the delegation game, the marginal trader and the decision-maker (marginal voter or board, respectively) converge to the most extreme shareholder. Therefore, the expected welfare of shareholders in both cases is the same and equals the valuation of the most extreme shareholder. This means that $\lim_{\lambda \to 1} D(\lambda) = 0$.

Next, consider the limit $\lambda \to 0$. Recall that $\lim_{\lambda \to 0} q_c = \lim_{\lambda \to 0} q_a = q_{\text{NoTrade}}$ and $\lim_{\lambda \to 0} b_c = \overline{b}$ and $\lim_{\lambda \to 0} b_a = -\overline{b}$, and hence, $\lim_{\lambda \to 0} \beta_c = \lim_{\lambda \to 0} \beta_a = \mathbb{E}[b]$.

First, consider the case where $-q_{\text{NoTrade}} \neq \mathbb{E}[b]$. Since $v(\mathbb{E}[b], q)$ achieves its maximum at $q = -\mathbb{E}[b]$, we have

$$
\lim_{\lambda \to 0} W^*_m = e \cdot v(\mathbb{E}[b], -\mathbb{E}[b]) > e \cdot v(\mathbb{E}[b], q_{\text{NoTrade}}) = W^*_{\text{NoTrade}} = \lim_{\lambda \to 0} W^*_v.
$$

Thus, in this case, $\lim_{\lambda \to 0} D(\lambda) > 0$. Combining it with $\lim_{\lambda \to 1} D(\lambda) = 0$ and using the continuity of $W^*_m$ and $W^*_v$ in $\lambda$, implies that $D(\lambda') > D(\lambda'')$ for all $\lambda'$ sufficiently close to 0 and all $\lambda''$ sufficiently close to 1, which proves the statement in part (i).
Second, consider the case where \( q_{\text{NoTrade}} = E_b \). Then \( \lim_{\lambda \to 0} W^*_m = \lim_{\lambda \to 0} W^*_v \), so \( \lim_{\lambda \to 0} D(\lambda) = 0 \). Since \( \lim_{\lambda \to 1} D(\lambda) = 0 \), to prove the statement in part (ii), it is sufficient to show that there exists \( \lambda \in (0, 1) \) such that the benefit of delegation to an optimal board is strictly positive, i.e., the bias of the marginal voter in the voting game is different from the post-trade average shareholder. This follows immediately from the fact that \( q_c \) and \( q_a \) are equal to \( \beta_c \) or \( \beta_a \) only under knife-edge conditions on parameters.

### A.3 Trading without selling all shares

Our basic model assumes that shareholders can sell their entire endowment. In this Online Appendix, we relax this assumption to incorporate scenarios where trading frictions are particularly high and do not allow initial shareholders to exit the firm completely. Specifically, we assume that when trading, shareholders can buy up to \( x \) shares or sell up to \( y \in (0, e) \) shares, while retaining the remaining \( e - y \) shares. All other assumptions in the baseline model remain unchanged. We provide a general discussion of the model in which we allow for \( y < e \) in Section 8, offer a more rigorous analysis in Section 8, and gather the formal proofs in Section 8.

#### A.3.1 Discussion of the model with \( y < e \)

Note that this extension allows us to separate the effect of market depth (captured by \( x \) and \( y \), the amounts that shareholders can trade) from the effects of \( \frac{x}{y} \), which captures the asymmetry between trading frictions on the buy-side and those on the sell-side. For simplicity, in what follows, we set \( e = 1 \). The formal analysis of this extension is presented in Section 8, and we only summarize the key steps and conclusions here. As the following discussion demonstrates, our main results continue to hold in this extension.

When shareholders cannot exit their entire position in the firm, the post-trade shareholder base is composed of the buying shareholders, who hold \( 1 + x \) shares each, and the selling shareholders, who hold \( 1 - y \) shares each. This change does not materially affect the characterization of the equilibrium as given in Proposition 3. In particular, any equilibrium is either conservative or activist. For example, if the equilibrium is conservative, the marginal voter is given by

\[
-q_c = \begin{cases} 
G^{-1}\left(\frac{1-x}{1-x+y}\right) & \text{if } \frac{x(1-y)}{x+y} \leq \tau \\
G^{-1}(1 - \frac{\tau}{1-y}) & \text{if } \frac{x(1-y)}{x+y} > \tau
\end{cases}
\]  

(34)

5
and the marginal trader is given by

$$b_c = G^{-1}(1 - \frac{x}{x + y}).$$ \hspace{1cm} (35)$$

In this equilibrium, which exists if and only if $q_c > F^{-1}(1 - \phi)$, the shareholder buys $x$ shares if $b < b_c$ and sells $y$ shares otherwise. The proposal is accepted if and only if $q > q_c$, and the share price is given by $p_c = v(b_c, q_c)$.

As $y \to 1$, this setting converges to our baseline model and the activist and conservative equilibria co-exist. As $y \to 0$, the equilibrium becomes unique and converges to the no-trade benchmark.

The key difference that distinguishes the analysis with $y < 1$ from the baseline model is that the marginal voter can now be less extreme than the marginal trader if $y$ is sufficiently close to zero. Intuitively, when $y$ is very small, the supply of shares is very low, and only the most extreme shareholders, those with the highest willingness to pay, buy shares in equilibrium. In other words, the marginal trader is very extreme and, as $y \to 0$, converges to $-\bar{b}$ in the conservative equilibrium and to $\bar{b}$ in the activist equilibrium. By contrast, the post-trade shareholder base is very similar to the initial shareholder base because the volume of trade is low due to small $y$. As such, the marginal voter is relatively moderate and, as $y \to 0$, the marginal voter converges to $q_{\text{NoTrade}} \in (-\bar{b}, \bar{b})$.

As in the baseline model, the expected welfare of the pre-trade shareholder base is equal to the expected welfare of the post-trade shareholder base, because prices are just transfers from buying to selling shareholders. However, different from the baseline model, since selling shareholders cannot exit their entire position in the firm, the expected welfare of the post-trade shareholder base is now a weighted average of the buying shareholders’ expected welfare and the selling shareholders’ expected welfare, where the weight on the former is always larger than the weight on the latter. To see this explicitly, consider, for example, a conservative equilibrium. Then, the expected shareholder welfare is

$$W_c = (1 - y) \Pr \{b > b_c\} \mathbb{E} [v(b, q_c) | b > b_c] + (1 + x) \Pr \{b < b_c\} \mathbb{E} [v(b, q_c) | b < b_c].$$ \hspace{1cm} (36)$$

Market clearing implies that $(1 - y) \Pr \{b > b_c\} + (1 + x) \Pr \{b < b_c\} = 1$, and hence, indeed, $W_c$ is a weighted average of $\mathbb{E} [v(b, q_c) | b > b_c]$ and $\mathbb{E} [v(b, q_c) | b < b_c]$, the welfare of selling and buying shareholders, respectively. Since $(1 - y) \Pr \{b > b_c\}$ decreases in $y$, the weight that is put
on the selling shareholders is decreasing in $y$, as they hold a smaller and smaller fraction of the firm post-trade. As $y \to 0$, $W_c \to \mathbb{E}[v(b, q_{NoTrade})]$, and as $y \to 1$, $W_c \to \mathbb{E}[v(b, q_c) | b < b_c]$, just as in the baseline model.

As before, the expected shareholder welfare obtains its maximum exactly when the bias of the marginal voter is equal to the average bias of the post-trade shareholder. Thus, our results on the optimal majority rule and the optimal board naturally extend to this setup. For example, in the conservative equilibrium, the average bias of the post-trade shareholder base is $(1 - y) \Pr [b > b_c] \mathbb{E}[b | b > b_c] + (1 + x) \Pr [b < b_c] \mathbb{E}[b | b < b_c]$, which includes the biases of both buying and selling shareholders. Note that this bias is always strictly smaller than $\mathbb{E}[b]$, and similarly, the average bias of the post-trade shareholder base in the activist equilibrium is strictly larger than $\mathbb{E}[b]$. These observations imply that the bias of the optimal board is different from $\mathbb{E}[b]$ in this setup as well.

Finally, the extension to $y < 1$ allows us to revisit our results on the effect of liquidity on welfare when trading frictions have a symmetric effect on buy and sell orders. For this purpose we impose $x = y$ and consider the effect of increasing $x$ and $y$ by the same amount, which can be interpreted as an increase in market depth. We show that the expected shareholder welfare decreases in market depth under similar conditions to those specified in Proposition 6 part (ii). For example, the expected welfare in the conservative equilibrium $W_c$ decreases in market depth whenever $1 - F(q_c) - \phi$ is relatively small and the marginal voter in this equilibrium is more conservative than the average post-trade shareholder. The intuition is the same as in the baseline model.

A.3.2 Analysis

Define

$$\lambda(y) \equiv \frac{x}{y + x}. \quad (37)$$

We prove the following results. The first result is the analog of Proposition 3.

**Proposition 11.** Consider the setup of the baseline model where shareholders can only sell $y < 1$ of their shares. An equilibrium of the game with trading and voting always exists.
(i) A **conservative** equilibrium exists if and only if $q_c > F^{-1}(1 - \phi)$, where

$$q_c = \begin{cases} 
-G^{-1}((1 - \lambda(1)) (1 - \tau)) & \text{if } \lambda(y) (1 - y) \leq \tau \\
-G^{-1}(\frac{1-\tau-y}{1-y}) & \text{if } \lambda(y) (1 - y) > \tau 
\end{cases} . \tag{38}$$

In this equilibrium, the shareholder buys $x$ shares if $b < b_c$ and sells $y$ shares if $b > b_c$, where

$$b_c = G^{-1}(1 - \lambda(y)). \tag{39}$$

The proposal is accepted if and only if $q > q_c$, and the share price is given by $p_c = v(b_c, q_c)$.

(ii) An **activist** equilibrium exists if and only if $q_a < F^{-1}(1 - \phi)$, where

$$q_a = \begin{cases} 
-G^{-1}((1 - (1 - \lambda(1)) \tau)) & \text{if } \lambda(y) (1 - y) \leq 1 - \tau \\
-G^{-1}(\frac{1-\tau-y}{1-y}) & \text{if } \lambda(y) (1 - y) > 1 - \tau 
\end{cases} . \tag{40}$$

In this equilibrium, the shareholder buys $x$ shares if $b > b_a$ and sells $y$ shares if $b < b_a$, where

$$b_a = G^{-1}(\lambda(y)). \tag{41}$$

The proposal is accepted if and only if $q > q_a$, and the share price is given by $p_a = v(b_a, q_a)$.

(iii) Other equilibria do not exist.

The second result is the analog of Lemma 2.

**Lemma 5.** In any equilibrium, the expected welfare of the shareholder base pre-trade is equal to the expected welfare of the shareholder base post-trade. In particular,

$$W_c = (1 - y) \Pr[b > b_c] \mathbb{E}[v(b, q_c) | b > b_c] + (1 + x) \Pr[b < b_c] \mathbb{E}[v(b, q_c) | b < b_c] \tag{42}$$

and

$$W_a = (1 - y) \Pr[b < b_a] \mathbb{E}[v(b, q_a) | b < b_a] + (1 + x) \Pr[b > b_a] \mathbb{E}[v(b, q_a) | b > b_a]. \tag{43}$$
The third result is the analog of Lemma 3.

**Lemma 6.** The expected welfare obtains its maximum exactly when the bias of the marginal voter is equal to the average bias of the post-trade shareholder, which is given by
\[(1 - y) \lambda (y) \mathbb{E} [b|b > b_c] + (1 + x) (1 - \lambda (y)) \mathbb{E} [b|b < b_c] \text{ in the conservative equilibrium and by } (1 - y) \lambda (y) \mathbb{E} [b|b < b_a] + (1 + x) (1 - \lambda (y)) \mathbb{E} [b|b > b_a] \text{ in the activist equilibrium. Moreover, the average bias of the post-trade shareholder in the conservative (activist) equilibrium is strictly smaller (larger) than } \mathbb{E} [b].\]

Finally, the last result is the analog of Proposition 6 part (ii).

**Proposition 12.** Suppose \(x = y\). Then:

(i) If a conservative equilibrium exists (i.e., \(q_c > F^{-1} (1 - \phi)\)) then here exists \(\phi_c > 1 - F (q_c)\) such that the expected shareholder welfare in the conservative equilibrium \(W_c\) decreases in market depth (i.e., a change in \(y\) and in \(x\) by the same amount) if and only if \(\phi \in (1 - F (q_c), \phi_c)\) and the marginal voter in this equilibrium is more conservative than the average post-trade shareholder (i.e., \(-q_c < (1 - y) 0.5 \mathbb{E} [b|b > b_c] + (1 + y) 0.5 \mathbb{E} [b|b < b_c]\)).

(ii) If an activist equilibrium exists (i.e., \(q_a < F^{-1} (1 - \phi)\)) then there exists \(\phi_a < 1 - F (q_a)\) such that the expected shareholder welfare in the activist equilibrium \(W_a\) decreases in market depth (i.e., a change in \(y\) and in \(x\) by the same amount) if and only if \(\phi \in (\phi_a, 1 - F (q_a))\) and the marginal voter in this equilibrium is less conservative than the average post-trade shareholder (i.e., \(-q_a > (1 - y) 0.5 \mathbb{E} [b|b < b_a] + (1 + y) 0.5 \mathbb{E} [b|b > b_a]\)).

**A.3.3 Proofs**

**Proof of Proposition 11.** Notice that Lemma 1 continues to hold in this setup and the expected value of shareholder \(b\) is given by (6). We consider three cases. First, suppose that \(q^* > F^{-1} (1 - \phi)\) (conservative equilibrium). The proof of Proposition 2 can be repeated in a setup with \(y < 1\) to show that if \(q^* > F^{-1} (1 - \phi)\) then \(v(b, q^*)\) decreases in \(b\) and therefore there exists \(b_c\) such that \(v(b, q^*) > p \iff b < b_c\). The key difference is that the market clears if and only if
\[xG(b_c) = y (1 - G(b_c)) \iff G(b_c) = 1 - \lambda (y) .\]
After the trading stage, shareholders with \( b < b_c \), of which there \( 1 - \lambda(y) \), hold \( 1 + x \) shares. Shareholders with \( b > b_c \), of which there are \( \lambda(y) \), hold \( 1 - y \) shares. Recall shareholder \( b \) votes his shares for the proposal if and only if \( q > -b \). Therefore, if \( \lambda(y)(1 - y) \leq \tau \) then the marginal voter is among the buying shareholders, those with \( b < b_c \), and if \( \lambda(y)(1 - y) > \tau \) then the marginal voter is among the selling shareholders, those with \( b > b_c \). Let us write the identity of the marginal voter explicitly. If \( \lambda(y)(1 - y) \leq \tau \) then the marginal voter is determined by

\[
\int_{-b}^{-q_c} (1 + x) \, dG(b) = 1 - \tau \iff \\
G(-q_c) = \frac{1 - \tau}{1 + x} \iff \\
q_c = -G^{-1}\left((1 - \lambda(1))(1 - \tau)\right),
\]

just as in the baseline model (recall \( \lambda = \lambda(1) \) in the baseline model). If \( \lambda(y)(1 - y) > \tau \) then the marginal voter is determined by

\[
\int_{-b}^{b_c} (1 + x) \, dG(b) + \int_{-b}^{-q_c} (1 - y) \, dG(b) = 1 - \tau \iff \\
G(b_c)(1 + x) + (1 - y)(G(-q_c) - G(b_c)) = 1 - \tau \iff \\
G(b_c)(y + x) + (1 - y)G(-q_c) = 1 - \tau \iff \\
\lambda(y)(y + x) + (1 - y)G(-q_c) = 1 - \tau \iff \\
y + (1 - y)G(-q_c) = 1 - \tau \iff \\
q_c = -G^{-1}\left(\frac{1 - \tau - y}{1 - y}\right),
\]

as required. Hence, the cutoff in this “conservative” equilibrium is \( q_c \) as given by (38). Similarly to the proof of Proposition 2, the share price is \( p_c = v(b_c, q_c) \).

Second, suppose that \( q^* < F^{-1}(1 - \phi) \) (activist equilibrium). The proof of Proposition 2 can be repeated in a setup with \( y < 1 \) to show that if \( q^* < F^{-1}(1 - \phi) \) then \( v(b, q^*) \) increases in \( b \) and therefore there exists \( b_a \) such that \( v(b, q^*) > p \iff b > b_c \). The key difference is that the market clears if and only if

\[
x(1 - G(b_a)) = yG(b_a) \iff G(b_a) = \lambda(y).
\]
After the trading stage, shareholders with \( b > b_a \), of which there \( 1 - \lambda(y) \) hold \( 1 + x \) shares. Shareholders with \( b < b_a \), of which there are \( \lambda(y) \), hold \( 1 - y \) shares. Recall shareholder \( b \) votes his shares for the proposal if and only if \( q > -b \). Therefore, if \( \lambda(y) (1 - y) \leq 1 - \tau \) then the marginal voter is among the buying shareholders, those with \( b > b_a \), and if \( \lambda(y) (1 - y) > 1 - \tau \) then the marginal voter is among the selling shareholders, those with \( b < b_a \). Let us write the identity of the marginal voter explicitly. If \( \lambda(y) (1 - y) \leq 1 - \tau \) then the marginal voter is determined by

\[
\int_{q_a}^{b} (1 + x) \, dG(b) = \tau \iff \\
1 - G(q_a) = \frac{\tau}{1 + x} \iff \\
q_a = -G^{-1}(1 - (1 - \lambda(1)\tau)),
\]

just as in the baseline model. If \( \lambda(y) (1 - y) > 1 - \tau \) then the marginal voter is determined by

\[
\int_{q_a}^{b_a} (1 - y) \, dG(b) + \int_{b_a}^{b} (1 + x) \, dG(b) = \tau \iff \\
(1 - y)(G(b_a) - G(q_a)) + (1 + x)(1 - G(b_a)) = \tau \iff \\
G(q_a) = \frac{1 + x - \tau - (y + x)G(b_a)}{1 - y} \iff \\
G(q_a) = \frac{1 - \tau}{1 - y} \iff \\
q_a = -G^{-1} \left( \frac{1 - \tau}{1 - y} \right),
\]

as required. Hence, the cutoff in this “conservative” equilibrium is \( q_a \) as given by (40). Similarly to the proof of Proposition 2, the share price is \( p_a = v(b_a, q_a) \).

Third, the same arguments that are outlined in the proof of Proposition 2 to show that an equilibrium with \( F(q^*) = 1 - \phi \) does not exist, hold in this case as well.

Finally, notice that if \( \lambda(y) (1 - y) \leq \min \{ \tau, 1 - \tau \} \) or \( \lambda(y) (1 - y) \geq \max \{ \tau, 1 - \tau \} \) then \( q_a < q_c \). Suppose \( \tau < \lambda(y) (1 - y) < 1 - \tau \) then \( q_c = -G^{-1}(\frac{1 - \tau - y}{1 - y}) \) and \( q_a = -G^{-1}(1 - (1 - \lambda(1))\tau) \), and \( q_a < q_c \) if and only if \( -y < x \) which always holds. If \( 1 - \tau < \lambda(y) (1 - y) < \tau \) then \( q_c = -G^{-1} ((1 - \lambda(1)) (1 - \tau)) \) and \( q_a = -G^{-1}(\frac{1 - \tau}{1 - y}) \), and \( q_a < q_c \) if and only if \( -y < x \) which always holds. Since \( q_a < q_c \), in any event either \( F(q_c) > 1 - \phi \), \( F(q_a) < 1 - \phi \), or both. Therefore, an equilibrium always exists. ■
**Proof of Lemma 5.** The expected shareholder welfare in a conservative equilibrium is

\[
W_c = \Pr[b > b_c] \mathbb{E}[(1 - y) v(b, q_c) + y p_c | b > b_c] + \Pr[b < b_c] \mathbb{E}[(1 + x) v(b, q_c) - x p_c | b < b_c] \\
= \Pr[b > b_c] y p_c - \Pr[b < b_c] x p_c \\
+ (1 - y) \Pr[b > b_c] \mathbb{E}[v(b, q_c) | b > b_c] + (1 + x) \Pr[b < b_c] \mathbb{E}[v(b, q_c) | b < b_c]
\]

Notice that \( \Pr[b < b_c] = \frac{y}{y + x} \) and hence

\[
\Pr[b > b_c] y p_c - \Pr[b < b_c] x p_c = \left( \frac{x}{y + x} - \frac{y}{y + x} \right) p_c = 0
\]

Then

\[
W_c = (1 - y) \Pr[b > b_c] \mathbb{E}[v(b, q_c) | b > b_c] + (1 + x) \Pr[b < b_c] \mathbb{E}[v(b, q_c) | b < b_c],
\]

as required. Similarly, the expected shareholder welfare in an activist equilibrium is

\[
W_a = \Pr[b < b_a] \mathbb{E}[(1 - y) v(b, q_a) + y p_a | b < b_a] + \Pr[b > b_a] \mathbb{E}[(1 + x) v(b, q_a) - x p_a | b > b_a] \\
= \Pr[b < b_a] y p_a - \Pr[b > b_a] x p_a \\
+ (1 - y) \Pr[b < b_a] \mathbb{E}[v(b, q_a) | b < b_a] + (1 + x) \Pr[b > b_a] \mathbb{E}[v(b, q_a) | b > b_a]
\]

Notice that \( \Pr[b > b_a] = \frac{y}{y + x} \) and hence

\[
\Pr[b < b_a] y p_a - \Pr[b > b_a] x p_a = 0
\]

Then

\[
W_a = (1 - y) \Pr[b < b_a] \mathbb{E}[v(b, q_a) | b < b_a] + (1 + x) \Pr[b > b_a] \mathbb{E}[v(b, q_a) | b > b_a],
\]

as required. \(\blacksquare\)
Proof of Lemma 6. Notice that
\[
\frac{\partial W_c}{\partial q^*} = -f(q^*)[(1-y)\lambda(y)(\mathbb{E}[b|b>b_c] + q^*) + (1+x)(1-\lambda(y))(\mathbb{E}[b|b<b_c] + q^*)]
\]
\[
= -f(q^*)[(1-y)\lambda(y)\mathbb{E}[b|b>b_c] + (1+x)(1-\lambda(y))\mathbb{E}[b|b<b_c] + q^*]
\]
Thus, the optimal cutoff satisfies
\[
-q^* = (1-y)\lambda(y)\mathbb{E}[b|b>b_c] + (1+x)(1-\lambda(y))\mathbb{E}[b|b<b_c].
\]
Since \(\lambda(y)\mathbb{E}[b|b>b_c] + (1-\lambda(y))\mathbb{E}[b|b<b_c] = \mathbb{E}[b]\) and \(x(1-\lambda(y)) = y\lambda(y)\), we have
\[
(1-y)\lambda(y)\mathbb{E}[b|b>b_c] + (1+x)(1-\lambda(y))\mathbb{E}[b|b<b_c] < \mathbb{E}[b] ⇔
\]
\[
-y\lambda(y)\mathbb{E}[b|b>b_c] + x(1-\lambda(y))\mathbb{E}[b|b<b_c] < 0 ⇔
\]
\[
x(1-\lambda(y))\mathbb{E}[b|b<b_c] < y\lambda(y)\mathbb{E}[b|b>b_c] \Leftrightarrow
\]
\[
\mathbb{E}[b|b<b_c] < \mathbb{E}[b|b>b_c],
\]
which always holds.
Also notice that
\[
\frac{\partial W_a}{\partial q^*} = -f(q^*)[(1-y)\lambda(y)(\mathbb{E}[b|b>b_a] + q^*) + (1+x)(1-\lambda(y))(\mathbb{E}[b|b>b_a] + q^*)]
\]
\[
= -f(q^*)[(1-y)\lambda(y)\mathbb{E}[b|b<b_a] + (1+x)(1-\lambda(y))\mathbb{E}[b|b>b_a] + q^*]
\]
Thus, the optimal cutoff satisfies
\[
-q^* = (1-y)\lambda(y)\mathbb{E}[b|b<b_a] + (1+x)(1-\lambda(y))\mathbb{E}[b|b>b_a].
\]
Since \(\lambda(y)\mathbb{E}[b|b<b_a] + (1-\lambda(y))\mathbb{E}[b|b>b_a] = \mathbb{E}[b]\) and \(x(1-\lambda(y)) = y\lambda(y)\), we have
\[
(1-y)\lambda(y)\mathbb{E}[b|b<b_a] + (1+x)(1-\lambda(y))\mathbb{E}[b|b>b_a] > \mathbb{E}[b] ⇔
\]
\[
-y\lambda(y)\mathbb{E}[b|b<b_a] + x(1-\lambda(y))\mathbb{E}[b|b>b_a] > 0 ⇔
\]
\[
x(1-\lambda(y))\mathbb{E}[b|b>b_a] > y\lambda(y)\mathbb{E}[b|b<b_a] \Leftrightarrow
\]
\[
\mathbb{E}[b|b>b_a] > \mathbb{E}[b|b<b_a],
\]
13
as required. ■

Proof of Proposition 12. Notice that if \( x = y \) then \( \lambda(y) = 0.5 \), \( b_c = b_a = G^{-1}(0.5) \), and from the expressions in Proposition 11 it can be verified that \( \frac{\partial q_c(y)}{\partial y} > 0 > \frac{\partial q_a(y)}{\partial y} \).

Consider the conservative equilibrium first. In this case,

\[
W_c = (1 - y) 0.5 \mathbb{E} [v(b, q_c) | b > b_c] + (1 + y) 0.5 \mathbb{E} [v(b, q_c) | b < b_c]
\]

\[
= (1 - y) 0.5 \left( \mathbb{E} [b | b > b_c] (1 - F(q_c) - \phi) + v_0 + (1 - F(q_c)) \mathbb{E} [\theta | q > q_c] \right) + (1 + y) 0.5 \left( \mathbb{E} [b | b < b_c] (1 - F(q_c) - \phi) + v_0 + (1 - F(q_c)) \mathbb{E} [\theta | q > q_c] \right)
\]

and

\[
\frac{\partial W_c}{\partial y} = -0.5 \left( \mathbb{E} [b | b > b_c] (1 - F(q_c) - \phi) + v_0 + (1 - F(q_c)) \mathbb{E} [\theta | q > q_c] \right) + 0.5 \left( \mathbb{E} [b | b < b_c] (1 - F(q_c) - \phi) + v_0 + (1 - F(q_c)) \mathbb{E} [\theta | q > q_c] \right)
\]

\[-f(q_c) \frac{\partial q_c}{\partial y} [(1 - y) 0.5 \mathbb{E} [b | b > b_c] + (1 + y) 0.5 \mathbb{E} [b | b < b_c] + q_c]
\]

\[-f(q_c) \frac{\partial q_c}{\partial y} [(1 - y) 0.5 \mathbb{E} [b | b > b_c] + (1 + y) 0.5 \mathbb{E} [b | b < b_c] + q_c]
\]

Therefore, \( \frac{\partial W_c}{\partial y} < 0 \) if and only if

\[
\phi < \phi_c \equiv (1 - F(q_c)) + f(q_c) \frac{\partial q_c}{\partial y} \frac{(1 - y) 0.5 \mathbb{E} [b | b > b_c] + (1 + y) 0.5 \mathbb{E} [b | b < b_c] + q_c}{0.5 (\mathbb{E} [b | b > b_c] - \mathbb{E} [b | b < b_c])}
\]

Recall the conservative equilibrium exists if and only if \( \phi > 1 - F(q_c) \). Thus the interval in which \( \frac{\partial W_c}{\partial y} < 0 \) is non-empty if and only if \( 1 - F(q_c) < \phi_c \), which holds if and only if \( -q_c < (1 - y) 0.5 \mathbb{E} [b | b > b_c] + (1 + y) 0.5 \mathbb{E} [b | b < b_c] \). This completes part (i).

Consider the activist equilibrium. If \( x = y \) then

\[
W_a = (1 - y) 0.5 \mathbb{E} [v(b, q_a) | b < b_a] + (1 + y) 0.5 \mathbb{E} [v(b, q_a) | b > b_a]
\]

\[
= (1 - y) 0.5 \left( \mathbb{E} [b | b < b_a] (1 - F(q_a) - \phi) + v_0 + (1 - F(q_a)) \mathbb{E} [\theta | q > q_a] \right) + (1 + y) 0.5 \left( \mathbb{E} [b | b > b_a] (1 - F(q_a) - \phi) + v_0 + (1 - F(q_a)) \mathbb{E} [\theta | q > q_a] \right)
\]
and

\[
\frac{\partial W_a}{\partial y} = -0.5 \left( \mathbb{E}[b|b < b_a] (1 - F(q_a) - \phi) + v_0 + (1 - F(q_a)) \mathbb{E}[\theta|q > q_a] \right) + 0.5 \left( \mathbb{E}[b|b > b_a] (1 - F(q_a) - \phi) + v_0 + (1 - F(q_a)) \mathbb{E}[\theta|q > q_a] \right)
\]

\[
- f(q_a) \frac{\partial q_a}{\partial y} ((1 - y) 0.5\mathbb{E}[b|b < b_a] + (1 + y) 0.5\mathbb{E}[b|b > b_a] + q_a)
\]

\[
= 0.5 \left( \mathbb{E}[b|b > b_a] - \mathbb{E}[b|b < b_a] \right) (1 - F(q_a) - \phi)
\]

\[
- f(q_a) \frac{\partial q_a}{\partial y} ((1 - y) 0.5\mathbb{E}[b|b < b_a] + (1 + y) 0.5\mathbb{E}[b|b > b_a] + q_a)
\]

Therefore, \(\frac{\partial W_\ast}{\partial y} < 0\) if and only if

\[
\phi > \phi_a \equiv 1 - F(q_a) - f(q_a) \frac{\partial q_a}{\partial y} \frac{(1 - y) 0.5\mathbb{E}[b|b < b_a] + (1 + y) 0.5\mathbb{E}[b|b > b_a] + q_a}{0.5 (\mathbb{E}[b|b > b_a] - \mathbb{E}[b|b < b_a])}
\]

Recall the activist equilibrium exists if and only if \(\phi < 1 - F(q_a)\). Thus the interval in which \(\frac{\partial W_\ast}{\partial y} < 0\) is non-empty if and only if \(1 - F(q_a) > \phi_a\), which holds if and only if \(-q_a > (1 - y) 0.5\mathbb{E}[b|b < b_a] + (1 + y) 0.5\mathbb{E}[b|b > b_a]\). This completes part (ii).