Internet Appendix for "Proxy Advisory Firms: The Economics of Selling Information to Voters"

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The first part of the Internet Appendix presents the supplementary analysis for the proofs in the main Appendix. The second part presents a detailed analysis of litigation pressure and policy proposals.

A. Supplementary analysis for the proofs

1. Supplementary analysis for the proof of Proposition 1: Proof that for any q, the equilibrium $w_s(0) = 0$, $w_s(1) = 1$, and $w_0 = \frac{1}{2}$ exists.

Consider the decision of shareholder i with signal s_i when other informed shareholders (i.e., shareholders that acquired private signals) vote according to strategy $w_s(s_j)$, and uninformed shareholders (i.e., shareholders that did not acquire private signals) vote according to strategy $w_0 = \frac{1}{2}$. Given q, the probability that each shareholder votes "for" in state $\theta \in \{0, 1\}$ equals

$$\Pr[v_j = 1|\theta = 1] = q(w_s(1)p + w_s(0)(1-p)) + (1-q)\frac{1}{2} = qp + (1-q)\frac{1}{2},$$
$$\Pr[v_j = 1|\theta = 0] = q(w_s(1)(1-p) + w_s(0)p) + (1-q)\frac{1}{2} = q(1-p) + (1-q)\frac{1}{2}.$$

Shareholder i's vote affects the decision if $\frac{N-1}{2}$ other shareholders vote "for" and $\frac{N-1}{2}$ vote "against." The expected value of the proposal to shareholder i in this case is

$$\tilde{u}(s_i) = \mathbb{E}[u(1,\theta)|s_i, PIV_i] = \Pr[\theta = 1|s_i, PIV_i] - \Pr[\theta = 0|s_i, PIV_i],$$

where PIV_i denotes the event in which shareholder i's vote determines the outcome (i.e., if $\sum_{i \neq j} v_j = \frac{N-1}{2}$). Applying the Bayes' rule,

$$\tilde{u}(s_i) = \frac{\Pr[s_i|\theta=1]\Pr\left[\sum_{j\neq i} v_j = \frac{N-1}{2}|\theta=1\right] - \Pr[s_i|\theta=0]\Pr\left[\sum_{j\neq i} v_j = \frac{N-1}{2}|\theta=0\right]}{\Pr[s_i|\theta=1]\Pr\left[\sum_{j\neq i} v_j = \frac{N-1}{2}|\theta=1\right] + \Pr[s_i|\theta=0]\Pr\left[\sum_{j\neq i} v_j = \frac{N-1}{2}|\theta=0\right]}$$
$$= D(s_i) \times (\Pr[s_i|\theta = 1] - \Pr[s_i|\theta = 0]) \left(\frac{1}{2} + q\left(p - \frac{1}{2}\right)\right)^{\frac{N-1}{2}} \left(\frac{1}{2} - q\left(p - \frac{1}{2}\right)\right)^{\frac{N-1}{2}},$$

where $D(s_i) > 0$. The best response of shareholder i is to vote "for" ($v_i = 1$) if $\tilde{u}(s_i) \geq 0$ and vote "against" ($v_i = 0$) if $\tilde{u}(s_i) \leq 0$. When $s_i = 1$, $\Pr[s_i|\theta = 1] - \Pr[s_i|\theta = 0] = 2p - 1 > 0$. When $s_i = 0$, $\Pr[s_i|\theta = 1] - \Pr[s_i|\theta = 0] = 1 - 2p < 0$. Therefore, the optimal strategy of shareholder i is indeed $v_i = s_i$. Hence, $w_s(s) = s$ is an equilibrium.

Similarly, for an uninformed shareholder, the expected value of the proposal conditional on

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being pivotal is

$$\tilde{u}_0 = D_0 \times \left(\begin{array}{c} \left(qp + (1-q)\frac{1}{2}\right)^{\frac{N-1}{2}} \left(1 - qp - (1-q)\frac{1}{2}\right)^{\frac{N-1}{2}} \\ -\left(q(1-p) + (1-q)\frac{1}{2}\right)^{\frac{N-1}{2}} \left(1 - q(1-p) - (1-q)\frac{1}{2}\right)^{\frac{N-1}{2}} \end{array} \right) = 0,$$

for some D_0, and hence it is indeed optimal to mix between voting "for" and "against."

2. Proof that $\underline{c}$ and $\bar{c}$ decrease in N and approach zero as $N \to \infty$. Denote both as functions of N via $\underline{c}_N$ and $\bar{c}_N$, respectively. Using their expressions in (7) and the fact that $C_{N+1}^{\frac{N+1}{2}} = \frac{4N}{N+1} C_{N-1}^{\frac{N-1}{2}}$, we get $\underline{c}_{N+2} = \underline{c}_N \frac{N}{N+1}\left(1 - (2p-1)^2\right) < \underline{c}_N$ and $\bar{c}_{N+2} = \bar{c}_N \frac{N}{N+1} < \bar{c}_N$. Furthermore, $\lim_{N\to\infty} \underline{c}_N = 0$ and $\lim_{N\to\infty} \bar{c}_N = 0$, because $\lim_{N\to\infty} P\left(q, N-1, \frac{N-1}{2}\right) = 0$ for any $q \in (0,1)$.

3. Value of signals. We derive the value of the private signal $V_s(q_r, q_s)$ and the value of the advisor's recommendation $V_r(q_r, q_s)$ to shareholder i for given q_r, q_s.

3.1. Value of a private signal. Shareholder i's vote only makes a difference only if $\sum_{j\neq i} v_j = \frac{N-1}{2}$. Conditional on $s_i = 1$ and on being pivotal, his utility from being informed is $\frac{1}{2}\mathbb{E}\left[u(1,\theta)|s_i = 1, PIV_i\right]$. Similarly, conditional on being pivotal and his private signal being $s_i = 0$, the shareholder's utility from being informed is $-\frac{1}{2}\mathbb{E}\left[u(1,\theta)|s_i = 0, PIV_i\right]$. Overall, the shareholder's value of acquiring a private signal is

$$\begin{aligned} V_s(q_r, q_s) = {} & \Pr(s_i = 1)\Pr(PIV_i|s_i = 1)\tfrac{1}{2}\mathbb{E}\left[u(1,\theta)|s_i = 1, PIV_i\right] \\ & - \Pr(s_i = 0)\Pr(PIV_i|s_i = 0)\tfrac{1}{2}\mathbb{E}\left[u(1,\theta)|s_i = 0, PIV_i\right]. \end{aligned}$$

By the symmetry of the model, $\mathbb{E}\left[u(1,\theta)|s_i = 1, PIV_i\right] = -\mathbb{E}\left[u(1,\theta)|s_i = 0, PIV_i\right]$ and $\Pr(PIV_i|s_i = 1) = \Pr(PIV_i|s_i = 0)$, so we get

$$\begin{aligned} & V_s(q_r, q_s) = \tfrac{1}{2}\Pr(PIV_i|s_i = 1)\,\mathbb{E}\left[u(1,\theta)|s_i = 1, PIV_i\right] \\ & = \tfrac{1}{2}\Pr(PIV_i|s_i = 1)\left(\Pr(\theta = 1|s_i = 1, PIV_i) - \Pr(\theta = 0|s_i = 1, PIV_i)\right) = \left(p - \tfrac{1}{2}\right)\Pr(PIV_i), \end{aligned}$$

where

$$\begin{aligned} \Pr(PIV_i) & = \Pr(PIV_i|\theta = 1) = \pi\Pr(PIV_i|r = 1, \theta = 1) + (1-\pi)\Pr(PIV_i|r = 0, \theta = 1) \\ & = \pi P\left(\tfrac{1}{2}q_n + q_r + q_s p, N-1, \tfrac{N-1}{2}\right) + (1-\pi) P\left(\tfrac{1}{2}q_n - q_r + q_s p, N-1, \tfrac{N-1}{2}\right). \end{aligned}$$

Hence, $V_s(q_r, q_s)$ is given by (9).

3.2. Value of the advisor's signal. As before, shareholder i's vote makes a difference only if $\sum_{j\neq i} v_j = \frac{N-1}{2}$. Conditional on $r = 1$ and on being pivotal, his utility from being informed is $\frac{1}{2}\mathbb{E}\left[u(1,\theta)|r = 1, PIV_i\right]$. Similarly, conditional on $r = 0$ and on being pivotal, shareholder i's utility from being informed is $-\frac{1}{2}\mathbb{E}\left[u(1,\theta)|r = 0, PIV_i\right]$. Overall, the shareholder's value of acquiring the advisor's signal is

$$\begin{aligned} V_r(q_r, q_s) = {} & \Pr(r = 1)\Pr(PIV_i|r = 1)\tfrac{1}{2}\mathbb{E}\left[u(1,\theta)|r = 1, PIV_i\right] \\ & - \Pr(r = 0)\Pr(PIV_i|r = 0)\tfrac{1}{2}\mathbb{E}\left[u(1,\theta)|r = 0, PIV_i\right]. \end{aligned}$$

By the symmetry of the model, $\mathbb{E}[u(1,\theta)|r=1,PIV_i]=-\mathbb{E}[u(1,\theta)|r=0,PIV_i]$ and $\Pr(PIV_i|r=1)=\Pr(PIV_i|r=0)$, so we get

$$\begin{gathered}V_r(q_r,q_s)=\tfrac{1}{2}\Pr(PIV_i|r=1)\,\mathbb{E}[u(1,\theta)|r=1,PIV_i]\\=\tfrac{1}{2}\Pr(PIV_i|r=1)\left(\Pr(\theta=1|r=1,PIV_i)-\Pr(\theta=0|r=1,PIV_i)\right)\\=\tfrac{1}{2}\Pr(PIV_i|r=1,\theta=1)\Pr(r=1|\theta=1)-\tfrac{1}{2}\Pr(PIV_i|r=1,\theta=0)\Pr(r=1|\theta=0)\\=\tfrac{1}{2}\Pr(PIV_i|r=1,\theta=1)\,\pi-\tfrac{1}{2}\Pr(PIV_i|r=1,\theta=0)(1-\pi).\end{gathered}$$

Note that $\Pr(PIV_i|r=1,\theta=1)=P\left(q_r+q_sp+\frac{1}{2}q_n,N-1,\frac{N-1}{2}\right)$ and $\Pr(PIV_i|r=1,\theta=0)=P\left(q_r-q_sp+\frac{1}{2}q_n,N-1,\frac{N-1}{2}\right)$. Hence, $V_r(q_r,q_s)$ is given by (10).

4. Supplementary analysis for the proof of Proposition 2: Derivation of the condition under which equilibrium $w_s(s_i)=s_i$, $w_r(r)=r$, and $w_0=\frac{1}{2}$ exists.

According to Proposition 2, we can restrict attention to subgames that follow the information acquisition stage at which each shareholder i acquires r with probability q_r, acquires s_i with probability q_s, and stays uninformed with probability $q_n=1-q_r-q_s$. Such an equilibrium only exists if given q_r, q_s, it is optimal for a shareholder who acquired a signal to follow it. It will be useful to compute the probabilities that a random shareholder j votes for the proposal, conditional on the advisor's recommendation r and the true state θ:

$$\Pr[v_j=1|r=1,\theta=1] = q_r+q_sp+q_n\frac{1}{2}, \tag{IA1}$$

$$\Pr[v_j=1|r=0,\theta=1] = q_sp+q_n\frac{1}{2}, \tag{IA2}$$

$$\Pr[v_j=1|r=1,\theta=0] = q_r+q_s(1-p)+q_n\frac{1}{2}, \tag{IA3}$$

$$\Pr[v_j=1|r=0,\theta=0] = q_s(1-p)+q_n\frac{1}{2}. \tag{IA4}$$

First, consider a shareholder with private signal s_i. Since his vote affects the decision only when he is pivotal, he compares $\mathbb{E}[u(1,\theta)|s_i,PIV_i]$ with zero or, equivalently, $\Pr(\theta=1|s_i,PIV_i)$ with $\frac{1}{2}$, and votes "for" if and only if the former is higher. By Bayes' rule,

$$\Pr(\theta=s_i|s_i,PIV_i)=\frac{\Pr(PIV_i|\theta=s_i)\,p}{\Pr(PIV_i|\theta=s_i)\,p+\Pr(PIV_i|\theta\neq s_i)(1-p)}=p>\frac{1}{2},$$

where we used the independence of s_j and r from s_i conditional on θ: because of independence, v_j is independent from $\theta=s_i$ or $\theta\neq s_i$ (i.e., from whether shareholder i's private signal is correct or not). Therefore, it is always optimal for a shareholder who acquired a private signal to follow it.

Second, consider a shareholder that acquired r. A shareholder compares $\mathbb{E}[u(1,\theta)|r,PIV_i]$ with zero and votes "for" if and only if the former is higher. Using Bayes' rule and $\Pr(\theta)=\frac{1}{2}=\Pr(r)$, we get

$$\begin{gathered}\mathbb{E}[u(1,\theta)|r,PIV_i]\Pr(PIV_i|r)\\=\Pr(\theta=1|r,PIV_i)\Pr(PIV_i|r)-\Pr(\theta=0|r,PIV_i)\Pr(PIV_i|r)\\=\Pr(PIV_i|r,\theta=1)\Pr(r|\theta=1)-\Pr(PIV_i|r,\theta=0)\Pr(r|\theta=0).\end{gathered} \tag{IA5}$$

It is sufficient to consider $r=1$: since the model is symmetric, voting "against" is optimal for $r=0$ whenever voting "for" is optimal for $r=1$. When $r=1$, the shareholder finds it optimal to vote "for" if and only if

$$\frac{\Pr\left(PIV_i|r=\theta=1\right)}{\Pr\left(PIV_i|r=1,\theta=0\right)}\frac{\pi}{1-\pi}\geq 1. \tag{IA6}$$

By independence of s_i, s_j, $j\neq i$, and r conditional on θ,

$$\Pr\left(PIV_i|r,\theta\right)=\Pr\left(\sum_{j\neq i}v_j=\frac{N-1}{2}|r,\theta\right)=P\left(\Pr\left[v_j=1|r,\theta\right],N-1,\frac{N-1}{2}\right).$$

Plugging this into (IA6) gives

$$\frac{\pi}{1-\pi}\frac{P\left(\frac{1}{2}+\frac{q_r}{2}+q_s(p-\frac{1}{2}),N-1,\frac{N-1}{2}\right)}{P\left(\frac{1}{2}+\frac{q_r}{2}-q_s(p-\frac{1}{2}),N-1,\frac{N-1}{2}\right)}\geq 1. \tag{IA7}$$

The intuition for (IA7) is as follows. Consider a shareholder with the advisor's recommendation deciding whether to follow it. If $q_s>0$, a split vote is a signal that the advisor's recommendation is more likely to be incorrect ($r\neq\theta$), since a split vote is more likely when private signals of shareholders disagree with the advisor's recommendation than when they agree with it. Therefore, as long as $q_r>0$ and $q_s>0$, the information content from being pivotal lowers the shareholder's assessment of the precision of the advisor's recommendation. This logic is reflected in the left-hand side of (IA7), which gives the ratio of probabilities that the advisor is correct and incorrect: the first term ($\frac{\pi}{1-\pi}$) is the prior, while the second term reflects additional information from the fact that the vote is split.

Finally, consider an uninformed shareholder. Since the event of being pivotal is uninformative about state θ, such a shareholder is indifferent between voting "for" and "against" the proposal, so it is optimal for him to mix between the two options.

Therefore, if q_r and q_s satisfy (IA7), then voting in the direction of the signal that a shareholder has (private or advisor's) is an equilibrium. If (IA7) is violated, there is no equilibrium with a positive value of the advisor's recommendation. However, since all these sub-games imply zero value of recommendation of the advisor, they are not reached on equilibrium path if $q_r>0$. In particular, whenever $V_r\left(q_r,q_s\right)-f\geq 0$, which is implied by any equilibrium with $q_r>0$ (where $V_r\left(q_r,q_s\right)$ is the value of the advisor's recommendation to a shareholder), this condition is satisfied. Therefore, we do not verify (IA7) in subsequent derivations.

5. Supplementary analysis for the proof of Lemma 1.

The solutions to (A7), if they exist, are given by

$$\begin{aligned} q_r^1(\psi) &= \sqrt{\frac{1}{4}-\left(\frac{f+\frac{c}{2p-1}+\frac{2p}{2p-1}\psi}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} - \sqrt{\frac{1}{4}-\left(\frac{\frac{c}{2p-1}-f+\frac{2(1-p)}{2p-1}\psi}{(1-\pi)C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}, \\ q_s^1(\psi) &= \frac{1}{2p-1}\left(\sqrt{\frac{1}{4}-\left(\frac{f+\frac{c}{2p-1}+\frac{2p}{2p-1}\psi}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} + \sqrt{\frac{1}{4}-\left(\frac{\frac{c}{2p-1}-f+\frac{2(1-p)}{2p-1}\psi}{(1-\pi)C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}\right), \end{aligned} \tag{IA8}$$

$$\begin{aligned} q_r^2(\psi) &= \sqrt{\frac{1}{4}-\left(\frac{f+\frac{c}{2p-1}+\frac{2p}{2p-1}\psi}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} + \sqrt{\frac{1}{4}-\left(\frac{\frac{c}{2p-1}-f+\frac{2(1-p)}{2p-1}\psi}{(1-\pi)C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}, \\ q_s^2(\psi) &= \frac{1}{2p-1}\left(\sqrt{\frac{1}{4}-\left(\frac{f+\frac{c}{2p-1}+\frac{2p}{2p-1}\psi}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} - \sqrt{\frac{1}{4}-\left(\frac{\frac{c}{2p-1}-f+\frac{2(1-p)}{2p-1}\psi}{(1-\pi)C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}\right). \end{aligned} \tag{IA9}$$

Note that $q_j^1(0) = q_j^a$ and $q_j^2(0) = q_j^b$ for $j \in \{r, s\}$. Since $p \in \left(\frac{1}{2}, 1\right)$, it is easy to see that $q_r^2(\psi) + q_s^2(\psi) \leq q_r^1(\psi) + q_s^1(\psi)$ and that $q_r^1(\psi) + q_s^1(\psi)$ is strictly decreasing in ψ. Each solution satisfies $(q_r, q_s) > 0$ if and only if $f + \psi < \frac{2\pi-1}{2p-1}(c+\psi) \Leftrightarrow f < \bar{f} + \frac{2(\pi-p)\psi}{2p-1}$.

Proof of Claim 1: If $f \geq \bar{f}$, then there is no equilibrium $(q_r, q_s) > 0$.

First, since strictly positive solutions (A5)-(A6) do not exist for $f \geq \bar{f}$, there is no equilibrium $(q_r, q_s) > 0$ satisfying $q_r + q_s < 1$. Second, by contradiction, suppose there is an equilibrium $(q_r, q_s) > 0$ with $q_r + q_s = 1$. Then, $\frac{f+\frac{c}{2p-1}+\frac{2p}{2p-1}\psi}{\pi} \leq \frac{\frac{c}{2p-1}-f+\frac{2(1-p)}{2p-1}\psi}{1-\pi}$ and $q_r^i(\psi) + q_s^i(\psi) = 1$ for some $\psi \geq 0$ and some $i \in \{1, 2\}$. Since $q_r^2(\psi) + q_s^2(\psi) \leq q_r^1(\psi) + q_s^1(\psi)$, we have $q_r^1(\psi) + q_s^1(\psi) \geq 1$. This, together with the inequality above, implies

$$\begin{aligned} 1 &\leq \frac{2p}{2p-1}\sqrt{\frac{1}{4}-\left(\frac{f+\frac{c}{2p-1}+\frac{2p}{2p-1}\psi}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} + \frac{2(1-p)}{2p-1}\sqrt{\frac{1}{4}-\left(\frac{\frac{c}{2p-1}-f+\frac{2(1-p)}{2p-1}\psi}{(1-\pi) C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} \\ &\leq \frac{2}{2p-1}\sqrt{\frac{1}{4}-\left(\frac{f+\frac{c}{2p-1}+\frac{2p}{2p-1}\psi}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} \leq \frac{2}{2p-1}\sqrt{\frac{1}{4}-\left(\frac{\bar{f}+\frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} \\ &= \frac{2}{2p-1}\Lambda = q_0^*, \end{aligned}$$

which contradicts Assumption 1 that $q_0^* \in (0, 1)$.

Proof of Claim 2: If $\frac{2p}{2p-1}\sqrt{\frac{1}{4}-\left(\frac{\underline{f}_1+\frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} \leq 1$, there is an equilibrium $(q_r, q_s) > 0$ if and only if $f \in \left[\underline{f}_1, \bar{f}\right)$, where $\underline{f}_1$ is given by (A4).

Note that

$$q_r^b + q_s^b = \frac{2p}{2p-1}\sqrt{\frac{1}{4} - \left(\frac{f + \frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} - \frac{2(1-p)}{2p-1}\sqrt{\frac{1}{4} - \left(\frac{\frac{c}{2p-1} - f}{(1-\pi) C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} \tag{IA10}$$

is strictly decreasing in f. Also, when $f = \underline{f}_1$, the second term is zero and hence, given the inequality assumed by the claim, $q_r^b + q_s^b \leq 1$ for $f = \underline{f}_1$. Hence, $q_r^b + q_s^b \leq 1$ for any $f \in \left[\underline{f}_1, \bar{f}\right)$ with strict inequality for $f \neq \underline{f}_1$. As shown above, $\left(q_r^b, q_s^b\right) > 0$ for $f < \bar{f}$. Hence, there is an equilibrium $\left(q_r^b, q_s^b\right) > 0$ if $f \in \left[\underline{f}_1, \bar{f}\right)$. By Claim 1, there is no equilibrium $(q_r, q_s) > 0$ if $f \geq \bar{f}$. If $f < \underline{f}_1$, system (A3) has no solution, so there is no equilibrium $(q_r, q_s) > 0$ with $q_r + q_s < 1$. Finally, (A3) with $c + \psi$ and $f + \psi$ instead of c and f does not have a solution if $f < \frac{c}{2p-1} + \frac{2(1-p)}{2p-1}\psi - C_{N-1}^{\frac{N-1}{2}} 2^{1-N}(1-\pi) = \underline{f}_1 + \frac{2(1-p)}{2p-1}\psi$. Since $\psi \geq 0$, it does not have a solution if $f < \underline{f}_1$, so there is no equilibrium $(q_r, q_s) > 0$ with $q_r + q_s = 1$ in this case either.

Proof of Claim 3: If $\frac{2p}{2p-1}\sqrt{\frac{1}{4} - \left(\frac{\underline{f}_1 + \frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} > 1$, there exists $\underline{f}_2 \geq \underline{f}_1$ such that there is an equilibrium $(q_r, q_s) > 0$ if and only if $f \in \left[\underline{f}_2, \bar{f}\right)$.

By Claim 1, there is no equilibrium $(q_r, q_s) > 0$ if $f \geq \bar{f}$. Note that when $f = \bar{f}$, the two roots in (IA10) are equal, and hence $q_r^b + q_s^b$, given by (IA10), is below one. Also, when $f = \underline{f}_1$, the second term in (IA10) is zero and hence, given the inequality assumed by the claim, $q_r^b + q_s^b > 1$ for $f = \underline{f}_1$. Since $q_r^b + q_s^b$ is strictly decreasing in f, there is a unique $\hat{f}_1 \in \left(\underline{f}_1, \bar{f}\right)$ at which (IA10) equals one (and since $f < \bar{f}$, both q_r^b and q_s^b are strictly positive). Hence, if $f \in \left(\hat{f}_1, \bar{f}\right)$, there is an equilibrium $(q_r, q_s) > 0$ with $q_r + q_s < 1$. If $f = \hat{f}_1$, there is an equilibrium $(q_r, q_s) > 0$ with $q_r + q_s = 1$. Finally, if $f < \hat{f}_1$, then $q_r^a + q_s^a \geq q_r^b + q_s^b > 1$, so there is no equilibrium of type (q_r^a, q_s^a) or $\left(q_r^b, q_s^b\right)$.

Next, consider $f \leq \hat{f}_1$. Consider equilibria with $q_r + q_s = 1$. Define

$$\hat{f}_2 \equiv c + C_{N-1}^{\frac{N-1}{2}} 2^{1-N}\left(\pi(1-p)\left(\frac{(1-p)(3p-1)}{p^2}\right)^{\frac{N-1}{2}} - p(1-\pi)\right). \tag{IA11}$$

We next show that if $\hat{f}_2 \leq \hat{f}_1$, then the necessary and sufficient conditions for equilibrium of the type $\left(q_r^1(\psi), q_s^1(\psi)\right) > 0$ with $q_r^1(\psi) + q_s^1(\psi) = 1$ to exist (for some $\psi \geq 0$) is $f \in [\hat{f}_2, \hat{f}_1]$. To prove this, note that such an equilibrium exists if and only if f is such that equation $q_r^1(\psi) + q_s^1(\psi) = 1$ has a solution $\psi \geq 0$ with $q_r^1(\psi) > 0$ (condition $q_s^1(\psi) > 0$ is implied by it from (IA8)). Hence, ψ

must satisfy

$$\frac{2p}{2p-1}\sqrt{\frac{1}{4}-\left(\frac{f+\frac{c}{2p-1}+\frac{2p}{2p-1}\psi}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} \leq 1 \Leftrightarrow \psi \geq \psi_l, \tag{IA12}$$

$$\frac{1}{4}-\left(\frac{\frac{c}{2p-1}-f+\frac{2(1-p)}{2p-1}\psi}{(1-\pi)\, C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}} \geq 0 \Leftrightarrow \psi \leq \psi_h, \tag{IA13}$$

where the first inequality follows from $q_r^1(\psi)+q_s^1(\psi)=1$ and

$$\begin{aligned} \psi_l &= \tfrac{2p-1}{2p}\left(\left(\tfrac{(1-p)(3p-1)}{4p^2}\right)^{\frac{N-1}{2}}\pi C_{N-1}^{\frac{N-1}{2}}-f-\tfrac{c}{2p-1}\right), \\ \psi_h &= \tfrac{2p-1}{2(1-p)}\left(2^{1-N}(1-\pi)\, C_{N-1}^{\frac{N-1}{2}}+f-\tfrac{c}{2p-1}\right). \end{aligned} \tag{IA14}$$

Hence, this system is equivalent to

$$\psi_l \leq \psi \leq \psi_h. \tag{IA15}$$

Note that $\psi_h \geq \psi_l \Leftrightarrow f \geq \hat{f}_2$, given by (IA11). Therefore, if $f < \hat{f}_2$, there is no equilibrium $\left(q_r^1(\psi), q_s^1(\psi)\right) > 0$. We next show that if $f \in [\,\hat{f}_2, \hat{f}_1]$, so that (IA15) is non-empty, such an equilibrium exists. When $\psi = \psi_h$, (IA13) binds and since $\psi_l \leq \psi_h$, then (IA12) is satisfied and hence $q_r^1(\psi_h)+q_s^1(\psi_h) \leq 1$ (since it equals the left-hand side of (IA12) when (IA13) binds). When $\psi = \psi_l$, (IA12) binds and hence $q_r^1(\psi_l)+q_s^1(\psi_l) \geq 1$ (since it equals the left-hand side of (IA12) plus a non-negative number). As shown above, $q_r^1(0)+q_s^1(0) = q_r^a+q_s^a \geq q_r^b+q_s^b \geq 1$ for $f \leq \hat{f}_1$. Hence, when $\psi = \max\{0, \psi_l\}$, we have $q_r^1(\psi)+q_s^1(\psi) \geq 1$ for $f \leq \hat{f}_1$. Since $q_r^1(\psi)+q_s^1(\psi)$ is strictly decreasing in ψ, it must be that $\psi_h \geq 0$ (otherwise, $q_r^1(\psi_h)+q_s^1(\psi_h) > q_r^1(0)+q_s^1(0) \geq 1$). Thus, the interval $[\max\{0,\psi_l\}, \psi_h]$ is non-empty and by the intermediate value theorem there exists a unique $\psi^* \in [\max\{0,\psi_l\}, \psi_h]$ at which $q_r^1(\psi^*)+q_s^1(\psi^*) = 1$. Note also that for this ψ^*, $q_r^1(\psi^*) > 0$ (and $q_s^1(\psi^*) > 0$ follows from (IA8)). Indeed, suppose by contradiction that $q_r^1(\psi^*) \leq 0$. Since $q_r^1(0) = q_r^a > 0$ for $f < \bar{f}$, then by the intermediate value theorem, there exists $\psi^{**} \in (0, \psi^*]$ such that $q_r^1(\psi^{**}) = 0$. Since $\psi^{**} \leq \psi^*$ and $q_r^1(\psi)+q_s^1(\psi)$ is strictly decreasing in ψ, $q_r^1(\psi^{**})+q_s^1(\psi^{**}) \geq q_r^1(\psi^*)+q_s^1(\psi^*) = 1$, and hence $q_s^1(\psi^{**}) \geq 1$. Since $q_s^1(\psi^{**})$ and $q_r^1(\psi^{**})$ satisfy $V_s(q_r,q_s)-c = V_r(q_r,q_s)-f = \psi^{**}$ and $q_r^1(\psi^{**}) = 0$, we have $V_s\left(0, q_s^1(\psi^{**})\right) = c+\psi^{**}$, and hence $q_s^1(\psi^{**})$ is the equilibrium of the benchmark case with no advisor but with a higher cost, $\tilde{c} = c+\psi^{**}$. Since the cost if higher, it must be that $q_s^1(\psi^{**}) \leq q_s^1(0) = q_0^*$, but then $q_0^* \geq 1$, which contradicts Assumption 1. Hence, indeed, $q_r^1(\psi^*) > 0$. Therefore, there exists an equilibrium $\left(q_r^1(\psi), q_s^1(\psi)\right) > 0$ with $q_r^1(\psi)+q_s^1(\psi) = 1$ (for some $\psi \geq 0$) if and only if $f \in [\,\hat{f}_2, \hat{f}_1]$.

Since $\psi_l = \psi_h$ for $f = \hat{f}_2$, then (IA12) and (IA13) bind for $\psi = \psi_h$. Thus, $q_r^2(\psi_h)+q_s^2(\psi_h) = 1$. By (IA8), $q_r^2(\psi_h) \in (0,1)$, and hence $q_s^2(\psi_h) = 1-q_r^2(\psi_h) \in (0,1)$. Hence, equilibrium of the type $\left(q_r^2(\psi), q_s^2(\psi)\right) > 0$ with $q_r^2(\psi)+q_s^2(\psi) = 1$ (for some $\psi \geq 0$) exists for $f = \hat{f}_2$. We next prove that there exists a cutoff level $\hat{f}_3 \leq \hat{f}_2$ such that equilibrium of the type $\left(q_r^2(\psi), q_s^2(\psi)\right) > 0$ with $q_r^2(\psi)+q_s^2(\psi) = 1$ (for some $\psi \geq 0$) exists for $f \in \left[\hat{f}_3, \hat{f}_2\right]$ and does not exist for $f < \hat{f}_3$. To see this, define

$$V(f) \equiv \min_{\psi \in [0,\psi_h(f)]} \{q_r^2(\psi, f) + q_s^2(\psi, f)\} = -\max_{\psi \in [0,\psi_h(f)]} \{-q_r^2(\psi, f) - q_s^2(\psi, f)\},$$

where $\psi_h(f)$ is given by (IA14). Define $\Phi = \{f \in [0, \hat{f}_2] : V(f) \leq 1\}$ and note that equilibrium of the type $(q_r^2(\psi), q_s^2(\psi)) > 0$ with $q_r^2(\psi)+q_s^2(\psi) = 1$ exists if and only if $f \in \Phi$. Indeed, if $V(f) > 1$, then $q_r^2(\psi, f) + q_s^2(\psi, f) > 1$ for any $\psi \geq 0$ (since for $\psi > \psi_h(f)$, this function is not well defined) and hence no such equilibrium exists. On the other hand, suppose that $V(f) \leq 1$ and is achieved at $\psi^*(f)$. Then $q_r^2(\psi^*(f), f) + q_s^2(\psi^*(f), f) \leq 1$. In addition, since $\psi_h(f) < \psi_l(f)$ for $f < \hat{f}_2$, then (IA12) is violated and (IA13) binds for $\psi = \psi_h(f)$, and hence $q_r^2(\psi_h(f), f) + q_s^2(\psi_h(f), f) > 1$ for $f < \hat{f}_2$. By the intermediate value theorem, there then exists $\psi \in [\psi^*(f), \psi_h(f)]$ such that $q_r^2(\psi, f) + q_s^2(\psi, f) = 1$. Since (IA9) implies that $q_r^2(\psi) \in (0, 1)$, and hence $q_s^2(\psi) = 1 - q_r^2(\psi) \in (0, 1)$ as well, this constitues an equilibrium.

Next, note that $V(f)$ is decreasing in f. Indeed, define the Lagrangian $L(f, \psi, \lambda, \mu) \equiv -q_r^2(\psi, f) - q_s^2(\psi, f) + \lambda\psi + \mu(\psi_h(f) - \psi)$ and note that $V(f) = -\max_{\psi,\lambda,\mu} L(f, \psi, \lambda, \mu)$. By Envelope theorem, $V'(f) = -L'_f(f, \psi^*, \lambda^*, \mu^*) = -\left(-q_r^2(\psi^*, f)'_f - q_s^2(\psi^*, f)'_f + \mu^*\psi'_h(f)\right)$. Note that $\mu^* \geq 0$ according to the Kuhn-Tucker conditions. Because $q_r^2(\psi, f) + q_s^2(\psi, f)$ decreases in f for a given ψ and $\psi'_h(f) \geq 0$, it follows that $V'(f) \leq 0$. The fact that $V'(f) \leq 0$ implies that $\Phi = \left[\hat{f}_3, \hat{f}_2\right]$ for some $\hat{f}_3 \leq \hat{f}_2$. Hence, equilibrium $(q_r^2(\psi), q_s^2(\psi)) > 0$ with $q_r^2(\psi)+q_s^2(\psi) = 1$ exists for $f \in \left[\hat{f}_3, \hat{f}_2\right]$ and does not exist for $f < \hat{f}_3$, as required. Moreover, note that $\hat{f}_3 \geq \underline{f}_1$. This is because, (IA9) does not have a solution if $\psi > \psi_h \Leftrightarrow f < \underline{f}_1 + \frac{2(1-p)}{2p-1}\psi$ and hence does not have a solution if $f < \underline{f}_1$.

Consider two cases. First, if $\hat{f}_2 \leq \hat{f}_1$, then combining the results above, equilibrium $(q_r, q_s) > 0$ with $q_r + q_s < 1$ exists if and only if $f \in \left(\hat{f}_1, \bar{f}\right)$, equilibrium $(q_r^1(\psi), q_s^1(\psi)) > 0$ with $q_r^1(\psi) + q_s^1(\psi) = 1$ exists if and only if $f \in [\hat{f}_2, \hat{f}_1]$, and equilibrium $(q_r^2(\psi), q_s^2(\psi)) > 0$ with $q_r^2(\psi) + q_s^2(\psi) = 1$ exists for $f \in \left[\hat{f}_3, \hat{f}_2\right]$ and does not exist for $f < \hat{f}_3$. Combined with Claim 1, this implies that equilibrium $(q_r, q_s) > 0$ exists if and only if $f \in [\hat{f}_3, \bar{f})$. Second, if $\hat{f}_2 > \hat{f}_1$, then equilibrium $(q_r, q_s) > 0$ with $q_r + q_s < 1$ exists if and only if $f \in \left(\hat{f}_1, \bar{f}\right)$, there exists an equilibrium $(q_r, q_s) > 0$ with $q_r + q_s = 1$ for $f = \hat{f}_1$, and equilibrium $(q_r^2(\psi), q_s^2(\psi)) > 0$ with $q_r^2(\psi)+q_s^2(\psi) = 1$ exists for $f \in \left[\hat{f}_3, \hat{f}_2\right]$ and does not exist for $f < \hat{f}_3$. Combined with Claim 1, this implies that equilibrium $(q_r, q_s) > 0$ exists if and only if $f \in [\min(\hat{f}_3, \hat{f}_1), \bar{f})$. Combining the two cases, we conclude that equilibrium $(q_r, q_s) > 0$ exists if and only if $f \in [\underline{f}_2, \bar{f})$, where $\underline{f}_2 \equiv \min(\hat{f}_3, \hat{f}_1)$. Since, as shown above, $\hat{f}_1 > \underline{f}_1$ and $\hat{f}_3 \geq \underline{f}_1$, then $\underline{f}_2 \geq \underline{f}_1$, which proves Claim 3.

6. Supplementary analysis for the proof of Proposition 3: Properties of (A16).

Let us fix fee f and vary π. Recall from Lemma 1 that equilibrium with complete crowding out exists if and only if $f < \bar{f} = \frac{2\pi-1}{2p-1}c$, i.e., $\pi > \frac{1}{2} + \frac{f}{c}\left(p - \frac{1}{2}\right)$. The derivative of the left-hand side of (A16) in π is:

$$2\sum_{k=\frac{N+1}{2}}^{N} P(p_a, N, k) - 1 + (2\pi - 1)\frac{dp_a}{d\pi}\sum_{k=\frac{N+1}{2}}^{N} P_q(p_a, N, k) > 0,$$

since $\sum_{k=\frac{N+1}{2}}^{N} P(p_a, N, k) > \frac{1}{2}$, $\sum_{k=\frac{N+1}{2}}^{N} P_q(p_a, N, k) > 0$, and $\frac{dp_a}{d\pi} > 0$. Indeed, $\sum_{k=\frac{N+1}{2}}^{N} P(p_a, N, k) > \frac{1}{2}$ because $\sum_{k=\frac{N+1}{2}}^{N} P(p_a, N, k) + \sum_{k=\frac{N+1}{2}}^{N} P(1-p_a, N, k) = 1$ and $P(p_a, N, k) > P(1-p_a, N, k)$ for $p_a > \frac{1}{2}$ and $k > N$. Second, $\sum_{k=\frac{N+1}{2}}^{N} P_q(p_a, N, k) > 0$ for $p_a > \frac{1}{2}$, as shown in the proof of Part 1. Finally, $\frac{dp_a}{d\pi} > 0$ follows directly from (A14). Therefore, the left-hand side of (A16) is strictly increasing in π.

Note also that the advisor's presence strictly decreases firm value for $\pi \to \frac{1}{2} + \frac{f}{c}\left(p - \frac{1}{2}\right)$. Indeed, in this case, $p_a \to p_0^*$, so we obtain

$$(2\pi - 1) \sum_{k=\frac{N+1}{2}}^{N} P(p_0^*, N, k) - \pi < \sum_{k=\frac{N+1}{2}}^{N} P(p_0^*, N, k) - 1 \Leftrightarrow 1 < 2 \sum_{k=\frac{N+1}{2}}^{N} P(p_0^*, N, k),$$

which is true, as just shown above, since $p_0^* > \frac{1}{2}$. Finally, when $\pi \to 1$, the advisor's presence strictly increases firm value. Indeed,

$$\lim_{\pi \to 1} p_a = \frac{1}{2} + \sqrt{\frac{1}{4} - \left(\frac{2f}{C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} > \frac{1}{2} + \sqrt{\frac{1}{4} - \left(C_{N-1}^{\frac{N-1}{2}} \frac{p - \frac{1}{2}}{c}\right)^{-\frac{2}{N-1}}} = p_0^*,$$

so the left-hand side of (A16) converges to

$$\sum_{k=\frac{N+1}{2}}^{N} P\left(\lim_{\pi \to 1} p_a, N, k\right) - 1 > \sum_{k=\frac{N+1}{2}}^{N} P(p_0^*, N, k) - 1$$

because $\sum_{k=\frac{N+1}{2}}^{N} P_q(q, N, k) > 0$ for $q > \frac{1}{2}$, as shown in the proof of Part 1.

7. Supplementary analysis for the proof of Proposition 5.

7.1. Proof that when $f = \underline{f}_1$, firm value is strictly higher in equilibrium with incomplete crowding out than in equilibrium with complete crowding out.

To see this, consider any equilibrium with $p_a > \frac{1}{2}$ and $p_d < \frac{1}{2}$. Since $\Omega_1(q_r, q_s) = P(p_a, N-1, \frac{N-1}{2})$, $\Omega_2(q_r, q_s) = P(p_d, N-1, \frac{N-1}{2})$ and since $p_a > \frac{1}{2}$ and $p_d < \frac{1}{2}$, we have $p_a = \varphi(\Omega_1)$ and $p_d = 1 - \varphi(\Omega_2)$, where φ is given by (A12). According to (A10), firm value is

$$\begin{aligned}\hat{U}(\Omega_1, \Omega_2) &= \textstyle\sum_{k=\frac{N+1}{2}}^{N} (\pi P(\varphi(\Omega_1), N, k) + (1-\pi) P(1-\varphi(\Omega_2), N, k)) - \frac{1}{2} \\ &= \textstyle\sum_{k=\frac{N+1}{2}}^{N} (\pi P(\varphi(\Omega_1), N, k) - (1-\pi) P(\varphi(\Omega_2), N, k)) + \frac{1}{2} - \pi \\ &= \pi f(\Omega_1) - (1-\pi) f(\Omega_2) + \frac{1}{2} - \pi,\end{aligned}$$

where $f(x) \equiv \sum_{k=\frac{N+1}{2}}^{N} P(\varphi(x), N, k)$. In equilibrium with complete crowding out and $f = \underline{f}_1$, we have $p_a = \frac{1}{2} + \frac{1}{2}q_r > \frac{1}{2}$, $p_d = \frac{1}{2} - \frac{1}{2}q_r < \frac{1}{2}$, and (according to (A1)) $\Omega_1(q_r, 0) = \Omega_2(q_r, 0) = \frac{2\underline{f}_1}{2\pi-1} \equiv \Omega_r$. Consider $\Omega_1 \equiv \Omega_r + \frac{1-\pi}{2\pi-1}\varepsilon$ and $\Omega_2 \equiv \Omega_r + \frac{\pi}{2\pi-1}\varepsilon$ with $\varepsilon \equiv (c\frac{2\pi-1}{2p-1} - \underline{f}_1)\frac{1}{\pi(1-\pi)} > 0$. Note that $\pi\Omega_1 - (1-\pi)\Omega_2 = 2\underline{f}_1$ and $\pi\Omega_1 + (1-\pi)\Omega_2 = \frac{c}{p-0.5}$, i.e., Ω_1 and Ω_2 satisfy (A2). Hence, for $f = \underline{f}_1$, equilibrium with incomplete crowding out is characterized by probabilities of being pivotal

Ω_1 and Ω_2. We next prove that $\hat{U}(\Omega_1,\Omega_2)=\hat{U}\left(\Omega_r+\frac{1-\pi}{2\pi-1}\varepsilon,\Omega_r+\frac{\pi}{2\pi-1}\varepsilon\right)>\hat{U}(\Omega_r,\Omega_r)$. Indeed, function $\tilde{U}(x)\equiv\hat{U}\left(\Omega_r+\frac{1-\pi}{2\pi-1}x,\Omega_r+\frac{\pi}{2\pi-1}x\right)$ for $x\geq 0$ is increasing because

$$\tilde{U}'(x)=\frac{\pi(1-\pi)}{2\pi-1}\left(f'\left(\Omega_r+\frac{1-\pi}{2\pi-1}x\right)-f'\left(\Omega_r+\frac{\pi}{2\pi-1}x\right)\right)=-\frac{\pi(1-\pi)}{2\pi-1}\int_{\Omega_r+\frac{1-\pi}{2\pi-1}x}^{\Omega_r+\frac{\pi}{2\pi-1}x}f''(y)\,dy>0$$

by Auxiliary Lemma A1. Hence, indeed, when $f=\underline{f}_1$, firm value is strictly higher in equilibrium with incomplete crowding out than in equilibrium with complete crowding out.

7.2. Comparison of $\hat{\pi}$ and $\tilde{\pi}$. Simplifying,

$$P\left(\tfrac{1}{2}+\tfrac{1}{2\sqrt{N}},N-1,\tfrac{N-1}{2}\right)=C_{N-1}^{\frac{N-1}{2}}\left(\left(\tfrac{1}{2}+\tfrac{1}{2\sqrt{N}}\right)\left(\tfrac{1}{2}-\tfrac{1}{2\sqrt{N}}\right)\right)^{\frac{N-1}{2}}=C_{N-1}^{\frac{N-1}{2}}2^{1-N}\left(\tfrac{N-1}{N}\right)^{\frac{N-1}{2}},$$
$$P\left(\tfrac{1}{2},N-1,\tfrac{N-1}{2}\right)=C_{N-1}^{\frac{N-1}{2}}2^{1-N},$$

and hence

$$\hat{\pi}\equiv\frac{1}{2}+\frac{1}{2}\frac{C_{N-1}^{\frac{N-1}{2}}2^{1-N}-\frac{2c}{2p-1}}{P\left(\frac{1}{2},N-1,\frac{N-1}{2}\right)-P\left(\frac{1}{2}+\frac{1}{2\sqrt{N}},N-1,\frac{N-1}{2}\right)}.$$

Plugging in $p_0^*=\left(p-\frac{1}{2}\right)q_0^*+\frac{1}{2}=\Lambda+\frac{1}{2}$ into (A20), we get $\hat{\pi}\leq\tilde{\pi}$ if and only if

$$\frac{P\left(\frac{1}{2},N-1,\frac{N-1}{2}\right)-\frac{2c}{2p-1}}{P\left(\frac{1}{2},N-1,\frac{N-1}{2}\right)-P\left(\frac{1}{2}+\frac{1}{2\sqrt{N}},N-1,\frac{N-1}{2}\right)}\leq\frac{\sum_{k=\frac{N+1}{2}}^{N}P\left(\frac{1}{2}+\Lambda,N,k\right)-\frac{1}{2}}{\sum_{k=\frac{N+1}{2}}^{N}P(\frac{1}{2}+\frac{1}{2\sqrt{N}},N,k)-\frac{1}{2}}.$$

Furthermore, from the indifference condition in the benchmark case, $P\left(\frac{1}{2}+\Lambda,N-1,\frac{N-1}{2}\right)=\frac{2c}{2p-1}$, and hence, $\hat{\pi}\leq\tilde{\pi}$ if and only if

$$\frac{\sum_{k=\frac{N+1}{2}}^{N}P(\frac{1}{2}+\frac{1}{2\sqrt{N}},N,k)-\frac{1}{2}}{P\left(\frac{1}{2},N-1,\frac{N-1}{2}\right)-P\left(\frac{1}{2}+\frac{1}{2\sqrt{N}},N-1,\frac{N-1}{2}\right)}\leq\frac{\sum_{k=\frac{N+1}{2}}^{N}P\left(\frac{1}{2}+\Lambda,N,k\right)-\frac{1}{2}}{P\left(\frac{1}{2},N-1,\frac{N-1}{2}\right)-P\left(\frac{1}{2}+\Lambda,N-1,\frac{N-1}{2}\right)}.$$

Consider function

$$g(x)=\frac{L(x)}{P\left(\frac{1}{2},N-1,\frac{N-1}{2}\right)-P\left(x,N-1,\frac{N-1}{2}\right)},$$

where $L(x)=\sum_{k=\frac{N+1}{2}}^{N}P(x,N,k)-\frac{1}{2}$ is the same as defined in the proof of Lemma A3. Then, the above inequality is equivalent to $g\left(\frac{1}{2}+\frac{1}{2\sqrt{N}}\right)\leq g\left(\frac{1}{2}+\Lambda\right)$. Differentiating,

$$g'(x)=\frac{L'(x)\left(P\left(\frac{1}{2},N-1,\frac{N-1}{2}\right)-P\left(x,N-1,\frac{N-1}{2}\right)\right)+P_x\left(x,N-1,\frac{N-1}{2}\right)L(x)}{\left(P\left(\frac{1}{2},N-1,\frac{N-1}{2}\right)-P\left(x,N-1,\frac{N-1}{2}\right)\right)^2}$$

Using the expressions for $L'(x)$ and $P_x\left(x,N-1,\frac{N-1}{2}\right)$ in (IA31) and (IA32) in the proof of Lemma

A3, it follows that the sign of $g'(x)$ coincides with the sign of

$$\tilde{g}(x) = N\left(P\left(\frac{1}{2}, N-1, \frac{N-1}{2}\right) - P\left(x, N-1, \frac{N-1}{2}\right)\right) - \frac{(N-1)\left(x-\frac{1}{2}\right)}{x(1-x)} L(x).$$

Note that

$$\begin{aligned}\tilde{g}'(x) &= -NP_x\left(x, N-1, \tfrac{N-1}{2}\right) - (N-1)\left[\tfrac{x-\frac{1}{2}}{x(1-x)}\right]' L(x) - \tfrac{(N-1)\left(x-\frac{1}{2}\right)}{x(1-x)} L'(x) \\ &= -(N-1)\tfrac{x(1-x)+2\left(x-\frac{1}{2}\right)^2}{x^2(1-x)^2} L(x) < 0\end{aligned}$$

Since $\tilde{g}\left(\frac{1}{2}\right) = 0$, $\tilde{g}(x) < 0$ for $x \in \left(\frac{1}{2}, 1\right)$. Therefore, $g(x)$ is strictly decreasing in $x \in \left(\frac{1}{2}, 1\right)$. Hence, $\hat{\pi} \leq \tilde{\pi} \Leftrightarrow g\left(\frac{1}{2} + \frac{1}{2\sqrt{N}}\right) \leq g\left(\frac{1}{2} + \Lambda\right)$ is satisfied if and only if $\frac{1}{2} + \frac{1}{2\sqrt{N}} \geq \frac{1}{2} + \Lambda \Leftrightarrow \Lambda \leq \frac{1}{2\sqrt{N}}$. Note also that $\Lambda \leq \frac{1}{2\sqrt{N}} \Leftrightarrow \tilde{\pi} \leq 1$, as follows from (A20). Hence, if $\Lambda \leq \frac{1}{2\sqrt{N}}$, then $\hat{\pi} \leq \tilde{\pi}$ and $\tilde{\pi} \leq 1$, so the advisor improves the quality of decision-making compared to the benchmark case if and only if $\pi > \tilde{\pi}$. If $\Lambda > \frac{1}{2\sqrt{N}}$, then $\hat{\pi} > \tilde{\pi}$ and $\tilde{\pi} \geq 1$, so the advisor never improves the quality of decision-making. Hence, in both cases, the advisor improves the quality of decision-making compared to the benchmark case if and only if $\pi > \tilde{\pi}$.

8. Proof that the advisor does not benefit from adding i.i.d. noise to its recommendations. Consider the following extension of the model. Given the advisor's signal r, the signal that shareholder i observes is:

$$r_i = \begin{cases} r, & \text{with prob. } 1-\varepsilon, \\ 1-r, & \text{with prob. } \varepsilon, \end{cases}$$

for $\varepsilon \in [0, \frac{1}{2}]$. Signals r_i are independent across shareholders, conditional on r. If $\varepsilon = 0$, the model reduces to the basic model in which all shareholders observe the same recommendation.

By the same logic as in the basic model, at the voting stage, shareholders that acquired a certain signal must vote according to the signal. Consider the information acquisition stage. The values of signals are:

$$\begin{aligned} V_s(q_r, q_s, \varepsilon) &= \left(p - \tfrac{1}{2}\right)\left(\pi\Omega_1(q_r, q_s, \varepsilon) + (1-\pi)\Omega_2(q_r, q_s, \varepsilon)\right) \\ V_r(q_r, q_s, \varepsilon) &= (1-2\varepsilon)\tfrac{1}{2}\left(\pi\Omega_1(q_r, q_s, \varepsilon) - (1-\pi)\Omega_2(q_r, q_s, \varepsilon)\right) \end{aligned} \tag{IA16}$$

where the probabilities of a split vote conditional on the advisor being correct and incorrect are, respectively,

$$\begin{aligned} \Omega_1(q_r, q_s, \varepsilon) &= P_{N-1}\left(q_r(1-\varepsilon) + q_s p + q_n \tfrac{1}{2}\right) = P_{N-1}\left(\tfrac{1}{2} + \tfrac{q_r}{2}(1-2\varepsilon) + q_s\left(p - \tfrac{1}{2}\right)\right) \\ &= \Omega_1(q_r(1-2\varepsilon), q_s, 0) \\ \Omega_2(q_r, q_s, \varepsilon) &= P_{N-1}\left(q_r\varepsilon + q_s p + q_n \tfrac{1}{2}\right) = P_{N-1}\left(\tfrac{1}{2} - \tfrac{q_r}{2}(1-2\varepsilon) + q_s\left(p - \tfrac{1}{2}\right)\right) \\ &= \Omega_2(q_r(1-2\varepsilon), q_s, 0) \end{aligned} \tag{IA17}$$

where $P_{N-1}(x) \equiv P\left(x, N-1, \frac{N-1}{2}\right)$ and $q_n = 1 - q_r - q_s$. Hence,

$$\begin{aligned} V_s(q_r, q_s, \varepsilon) &= V_s(q_r(1-2\varepsilon), q_s, 0), \\ V_r(q_r, q_s, \varepsilon) &= (1-2\varepsilon) V_r(q_r(1-2\varepsilon), q_s, 0), \end{aligned} \tag{IA18}$$

where $V_i(q_r, q_s, 0) = V_i(q_r, q_s)$ are the values of the signals in the basic model with $\varepsilon = 0$. Denote $q_r^*(\varepsilon, f)$ and $q_s^*(\varepsilon, f)$ the equilibrium fractions of shareholders acquiring the advisor's and private signal, respectively, given ε and fee f.

For simplicity, we assume that c is sufficiently large, so that at least some shareholders will remain uninformed, i.e., $q_r^*(\varepsilon, f) + q_s^*(\varepsilon, f) < 1$ (this restriction is analogous to restriction $c > \hat{c}$ in Assumption 2 of the basic model, which guarantees, according to the proof of Lemma A3, that there is no equilibrium with $q_r + q_s = 1$). Then, the following three cases are possible:
(1) $q_r^*(\varepsilon, f) > 0$, $q_s^*(\varepsilon, f) > 0$, requiring $V_s(q_r^*(\varepsilon, f), q_s^*(\varepsilon, f), \varepsilon) = c$, $V_r(q_r^*(\varepsilon, f), q_s^*(\varepsilon, f), \varepsilon) = f$;
(2) $q_r^*(\varepsilon, f) = 0$, $q_s^*(\varepsilon, f) > 0$, requiring $V_s(q_r^*(\varepsilon, f), q_s^*(\varepsilon, f), \varepsilon) = c$, $V_r(q_r^*(\varepsilon, f), q_s^*(\varepsilon, f), \varepsilon) \leq f$;
(3) $q_r^*(\varepsilon, f) > 0$, $q_s^*(\varepsilon, f) = 0$, requiring $V_s(q_r^*(\varepsilon, f), q_s^*(\varepsilon, f), \varepsilon) \leq c$, $V_r(q_r^*(\varepsilon, f), q_s^*(\varepsilon, f), \varepsilon) = f$.

Hence, (IA18) implies that, respectively, either
(1) $V_s(q_r^*(\varepsilon, f)(1-2\varepsilon), q_s^*(\varepsilon, f)) = c$, $V_r(q_r^*(\varepsilon, f)(1-2\varepsilon), q_s^*(\varepsilon, f)) = \frac{f}{1-2\varepsilon}$ or
(2) $V_s(q_r^*(\varepsilon, f)(1-2\varepsilon), q_s^*(\varepsilon, f)) = c$, $q_r^*(\varepsilon, f)(q_r(1-2\varepsilon), q_s^*(\varepsilon, f)) \leq \frac{f}{1-2\varepsilon}$, or
(3) $V_s(q_r^*(\varepsilon, f)(1-2\varepsilon), q_s^*(\varepsilon, f)) \leq c$, $V_r(q_r^*(\varepsilon, f)(1-2\varepsilon), q_s^*(\varepsilon, f)) = \frac{f}{1-2\varepsilon}$.
This, in turn, implies that $q_r^{**} \equiv (1-2\varepsilon) q_r^*(\varepsilon, f)$ and $q_s^{**} = q_s^*(\varepsilon, f)$ will form an equilibrium (of the same type) in the model with $\varepsilon = 0$ and fee $\frac{f}{1-2\varepsilon}$ ($q_r^{**} \leq 1$ directly follows from $q_r^*(\varepsilon, f) \leq 1$ since $1 - 2\varepsilon \leq 1$). Hence, the advisor can achieve exactly the same profits by setting $\varepsilon = 0$ and fee $\frac{f}{1-2\varepsilon}$ (and perhaps even higher profits by setting a different fee).Therefore, the advisor can never strictly benefit from adding i.i.d. noise to the signals distributed to shareholders.

Intuitively, when the advisor adds noise to its signal, two effects act in opposite directions. The first, negative, effect is that given the same probabilities of being pivotal, noise $\varepsilon > 0$ decreases the shareholder's value from buying the advisor's recommendation because the signal now has a lower precision. This is captured in the term $(1-2\varepsilon)$ of the second equation of (IA16) and requires the advisor to decrease the fee by a factor $(1-2\varepsilon)$. The second, positive, effect is that given the same fraction q_r of shareholders subscribing to the advisor, the probability of each shareholder being pivotal is higher when $\varepsilon > 0$, because shareholders subscribing to the advisor will not always vote with each other. This allows the advisor to capture a higher fraction of shareholders while keeping their probability of being pivotal the same. This effect is captured by terms $(1-2\varepsilon)$ in each equation of (IA17). The two effects fully offset each other.

9. Auxiliary Lemma A1. *Function $f(x) \equiv \sum_{k=\frac{N+1}{2}}^{N} P(\varphi(x), N, k)$, where $\varphi(x)$ is defined by (A12), is strictly decreasing and strictly concave.*

Proof of Auxiliary Lemma A1. It will be useful to compute the derivative:

$$\varphi'(x) = -\frac{1}{C_{N-1}^{\frac{N-1}{2}}(N-1)\psi(x)}, \tag{IA19}$$

where

$$\psi(x) \equiv \left(\frac{x}{C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{N-3}{N-1}} \sqrt{\frac{1}{4} - \left(\frac{x}{C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}. \tag{IA20}$$

The first derivative of $f(x)$ is

$$f'(x) = \left(\sum_{k=\frac{N+1}{2}}^{N} P_q(\varphi(x), N, k) \right) \varphi'(x) < 0,$$

since $\varphi'(x) < 0$ and $\sum_{k=\frac{N+1}{2}}^{N} P_q(q, N, k) > 0$ for any $q > \frac{1}{2}$, including $q = \varphi(x)$. The former follows from (IA19). The latter follows from $\sum_{k=\frac{N+1}{2}}^{N} P_q(q, N, k) = -\sum_{k=0}^{\frac{N-1}{2}} P_q(q, N, k)$ and $P_q(q, N, k) = P(q, N, k) \frac{k-Nq}{q(1-q)} < 0$ for any $k < \frac{N}{2}$ because $q > \frac{1}{2}$. Therefore, $f(x)$ is strictly decreasing. The second derivative of $f(x)$ is

$$f''(x) = \left(\frac{d\varphi}{dx}\right)^2 \left(\sum_{k=\frac{N+1}{2}}^{N} P_{qq}(\varphi(x), N, k) \right) + \frac{d^2\varphi}{dx^2} \left(\sum_{k=\frac{N+1}{2}}^{N} P_q(\varphi(x), N, k) \right)$$

Since $\sum_{k=0}^{N} P_q(q, N, k) = 0$ and $\sum_{k=0}^{N} P_{qq}(q, N, k) = 0$,

$$\begin{aligned} f''(x) &= -\left(\frac{d\varphi}{dx}\right)^2 \left(\sum_{k=0}^{\frac{N-1}{2}} P_{qq}(\varphi(x), N, k) \right) - \frac{d^2\varphi}{dx^2} \left(\sum_{k=0}^{\frac{N-1}{2}} P_q(\varphi(x), N, k) \right) \\ &= -\frac{1}{\left(C_{N-1}^{\frac{N-1}{2}}\right)^2 (N-1)^2 \psi(x)^2} \left(\sum_{k=0}^{\frac{N-1}{2}} P_{qq}(\varphi(x), N, k) \right) - \frac{\psi'(x)}{C_{N-1}^{\frac{N-1}{2}} (N-1) \psi(x)^2} \left(\sum_{k=0}^{\frac{N-1}{2}} P_q(\varphi(x), N, k) \right) \end{aligned}$$

Plugging in P_q, P_{qq} and simplifying,

$$\begin{aligned} &\left(C_{N-1}^{\frac{N-1}{2}}\right)^2 (N-1)^2 \psi(x)^2 f''(x) \\ &= -\sum_{k=0}^{\frac{N-1}{2}} P(\varphi(x), N, k) \left[\left(\frac{k-N\varphi(x)}{\varphi(x)(1-\varphi(x))} \right)^2 - \frac{k}{\varphi(x)^2} - \frac{N-k}{(1-\varphi(x))^2} + C_{N-1}^{\frac{N-1}{2}} (N-1) \psi'(x) \left(\frac{k-N\varphi(x)}{\varphi(x)(1-\varphi(x))} \right) \right]. \end{aligned}$$

Next, we can calculate $\psi'(x)$:

$$\begin{aligned} C_{N-1}^{\frac{N-1}{2}} (N-1) \psi'(x) &= \left(\frac{N-3}{4} \left(\frac{x}{C_{N-1}^{\frac{N-1}{2}}} \right)^{\frac{-2}{N-1}} - N + 2 \right) \left(\frac{1}{4} - \left(\frac{x}{C_{N-1}^{\frac{N-1}{2}}} \right)^{\frac{2}{N-1}} \right)^{-\frac{1}{2}} \\ &= \frac{1}{\varphi(x) - \frac{1}{2}} \left(\frac{N-3}{4} \frac{1}{\varphi(x)(1-\varphi(x))} - N + 2 \right). \end{aligned} \tag{IA21}$$

Thus,

$$\begin{aligned} &\left(C_{N-1}^{\frac{N-1}{2}}\right)^2 (N-1)^2 \psi(x)^2 f''(x) = \\ &-\sum_{k=0}^{\frac{N-1}{2}} P(\varphi(x), N, k) \left[\left(\frac{k-N\varphi(x)}{\varphi(x)(1-\varphi(x))} \right)^2 - \frac{k}{\varphi(x)^2} - \frac{N-k}{(1-\varphi(x))^2} + \frac{1}{\varphi(x)-\frac{1}{2}} \left(\frac{N-3}{4} \frac{1}{\varphi(x)(1-\varphi(x))} - N + 2 \right) \left(\frac{k-N\varphi(x)}{\varphi(x)(1-\varphi(x))} \right) \right]. \end{aligned}$$

Multiplying by $(\varphi(x)(1-\varphi(x)))^2$:

$$\begin{array}{c} -\left(C_{N-1}^{\frac{N-1}{2}}\right)^2 (N-1)^2 \psi(x)^2 (\varphi(x)(1-\varphi(x)))^2 f''(x) \\ = \sum_{k=0}^{\frac{N-1}{2}} P(q,N,k) \left(\begin{array}{c} (k-Nq)^2 - k(1-q)^2 - (N-k)q^2 \\ + \frac{2(k-Nq)}{2q-1}\left(\frac{N-3}{4} - (N-2)q(1-q)\right) \end{array} \right) \equiv L(q), \end{array} \tag{IA22}$$

where we denote $\varphi(x)$ by $q \in \left(\frac{1}{2},1\right)$. It follows that $f''(x) < 0$ if $L(q) > 0$ for any $q \in \left(\frac{1}{2},1\right)$. To prove it, denote

$$\begin{array}{rcl} \zeta(q,k) & \equiv & (k-Nq)^2 - k(1-q)^2 - (N-k)q^2 + C(k-Nq) \\ & = & k(k-1) - (2(N-1)q - C)k + N(N-1)q^2 - CNq, \end{array}$$

where $C \equiv \frac{2}{2q-1}\left(\frac{N-3}{4} - (N-2)q(1-q)\right)$. Then,

$$L(q) = \sum_{k=0}^{\frac{N-1}{2}} P(q,N,k)\,k(k-1) - (2(N-1)q - C)\sum_{k=0}^{\frac{N-1}{2}} P(q,N,k)\,k + \left(N(N-1)q^2 - CNq\right)\sum_{k=0}^{\frac{N-1}{2}} P(q,N,k).$$

Consider the first two terms:

1. Term 1:

$$\begin{array}{c} \sum_{k=0}^{\frac{N-1}{2}} k(k-1) C_N^k q^k (1-q)^{N-k} = \sum_{k=2}^{\frac{N-1}{2}} k(k-1) \frac{N!}{k!(N-k)!} q^k (1-q)^{N-k} \\ = N(N-1)q^2 \sum_{m=0}^{\frac{N-1}{2}-2} P(q,N-2,m) = N(N-1)q^2 \Pr\left[k \leq \frac{N-1}{2} - 2 | k \sim B(N-2,q)\right]. \end{array}$$

2. Term 2:

$$\begin{array}{c} \sum_{k=0}^{\frac{N-1}{2}} k C_N^k q^k (1-q)^{N-k} = \sum_{k=1}^{\frac{N-1}{2}} k \frac{N!}{k!(N-k)!} q^k (1-q)^{N-k} \\ = qN\left(\sum_{k=0}^{\frac{N-1}{2}-1} P(q,N-1,k)\right) = qN \Pr\left[k \leq \frac{N-1}{2} - 1 | k \sim B(N-1,q)\right]. \end{array} \tag{IA23}$$

Hence,

$$\begin{array}{c} \frac{L(q)}{qN} = (N-1)q \Pr\left[k \leq \frac{N-1}{2} - 2 | k \sim B(N-2,q)\right] \\ -(2(N-1)q - C)\Pr\left[k \leq \frac{N-1}{2} - 1 | k \sim B(N-1,q)\right] \\ +((N-1)q - C)\Pr\left[k \leq \frac{N-1}{2} | k \sim B(N,q)\right]. \end{array}$$

Note that

$$\begin{array}{c} \Pr\left[k \leq \frac{N-1}{2} | k \sim B(N,q)\right] = I_{1-q}\left(\frac{N+1}{2}, \frac{N+1}{2}\right), \\ \Pr\left[k \leq \frac{N-1}{2} - 1 | k \sim B(N-1,q)\right] = I_{1-q}\left(\frac{N+1}{2}, \frac{N-1}{2}\right), \\ \Pr\left[k \leq \frac{N-1}{2} - 2 | k \sim B(N-2,q)\right] = I_{1-q}\left(\frac{N+1}{2}, \frac{N-3}{2}\right), \end{array} \tag{IA24}$$

where $I_{1-q}(\cdot)$ is the regularized incomplete beta function. According to the property of the regularized incomplete beta function, $I_x(a,b+1) = I_x(a,b) + \frac{x^a(1-x)^b}{bB(a,b)}$, where $B(a,b) = \frac{(a-1)!(b-1)!}{(a+b-1)!}$ is the beta function. Hence,

$$\begin{aligned}I_{1-q}\left(\tfrac{N+1}{2},\tfrac{N+1}{2}\right)&=I_{1-q}\left(\tfrac{N+1}{2},\tfrac{N-1}{2}\right)+\frac{(1-q)^{\frac{N+1}{2}}q^{\frac{N-1}{2}}}{\frac{N-1}{2}B\left(\frac{N+1}{2},\frac{N-1}{2}\right)}\\ I_{1-q}\left(\tfrac{N+1}{2},\tfrac{N-1}{2}\right)&=I_{1-q}\left(\tfrac{N+1}{2},\tfrac{N-3}{2}\right)+\frac{(1-q)^{\frac{N+1}{2}}q^{\frac{N-3}{2}}}{\frac{N-3}{2}B\left(\frac{N+1}{2},\frac{N-3}{2}\right)}.\end{aligned}\tag{IA25}$$

Plugging into the expression for $\frac{L(q)}{qN}$:

$$\begin{aligned}\frac{L(q)}{qN}&=(N-1)\,q\left(I_{1-q}\left(\tfrac{N+1}{2},\tfrac{N-1}{2}\right)-\frac{(1-q)^{\frac{N+1}{2}}q^{\frac{N-3}{2}}}{\frac{N-3}{2}B\left(\frac{N+1}{2},\frac{N-3}{2}\right)}\right)-(2\,(N-1)\,q-C)\,I_{1-q}\left(\tfrac{N+1}{2},\tfrac{N-1}{2}\right)\\&\quad+((N-1)\,q-C)\left(I_{1-q}\left(\tfrac{N+1}{2},\tfrac{N-1}{2}\right)+\frac{(1-q)^{\frac{N+1}{2}}q^{\frac{N-1}{2}}}{\frac{N-1}{2}B\left(\frac{N+1}{2},\frac{N-1}{2}\right)}\right)\\&=-\,(N-1)\,q\frac{(1-q)^{\frac{N+1}{2}}q^{\frac{N-3}{2}}}{\frac{N-3}{2}B\left(\frac{N+1}{2},\frac{N-3}{2}\right)}+((N-1)\,q-C)\,\frac{(1-q)^{\frac{N+1}{2}}q^{\frac{N-1}{2}}}{\frac{N-1}{2}B\left(\frac{N+1}{2},\frac{N-1}{2}\right)}.\end{aligned}$$

Dividing by $(1-q)^{\frac{N+1}{2}}\,q^{\frac{N-3}{2}}$ and simplifying,

$$\frac{L\,(q)}{(1-q)^{\frac{N+1}{2}}\,q^{\frac{N-1}{2}}N}=\frac{q\,(N-1)!}{\left(\frac{N-1}{2}\right)!\,\left(\frac{N-3}{2}\right)!}\,(2q-1)-C\frac{q\,(N-1)!}{\frac{N-1}{2}\,\left(\frac{N-1}{2}\right)!\,\left(\frac{N-3}{2}\right)!}.$$

Hence,

$$\begin{aligned}&\frac{L(q)\left(\frac{N-3}{2}\right)!\left(\frac{N-1}{2}\right)!(2q-1)}{(1-q)^{\frac{N+1}{2}}q^{\frac{N+1}{2}}N!}=(2q-1)^2-\frac{2}{N-1}\left(\frac{N-3}{2}-2\,(N-2)\,q\,(1-q)\right)\\&=\frac{4}{N-1}q^2-\frac{4}{N-1}q+\frac{2}{N-1}\Leftrightarrow\frac{L(q)\left(\frac{N-3}{2}\right)!\left(\frac{N-1}{2}\right)!(2q-1)(N-1)}{(1-q)^{\frac{N+1}{2}}q^{\frac{N+1}{2}}N!2}=2q^2-2q+1.\end{aligned}\tag{IA26}$$

Since $2q^2-2q+1>0$, we conclude that $L\,(q)>0$ for any $q\in(\frac{1}{2},1)$. Therefore, $f''\,(x)<0$, which completes the proof.

10. Auxiliary Lemma A2. *Function $\tilde{f}\,(x)$, defined by (IA34), is strictly concave.*
Proof of Auxiliary Lemma A2. Differentiating $\tilde{f}\,(x)$ and using the definition of $f\,(x)$,

$$\tilde{f}''\,(x)=f''\,(x)-2\varphi'\,(x)-x\varphi''\,(x)\,.$$

Using $f''\,(x)$ from the proof of Auxiliary Lemma A1 above, in particular, expressions (IA22), (IA26), (IA19), and the derivative of (IA19), we can write

$$\tilde{f}''\,(x)=-x\frac{(2\varphi\,(x)^2-2\varphi\,(x)+1)N}{(2\varphi\,(x)-1)\,\varphi\,(x)\,(1-\varphi\,(x))\,(C_{N-1}^{\frac{N-1}{2}}\,(N-1)\,\psi\,(x))^2}+\frac{2}{C_{N-1}^{\frac{N-1}{2}}\,(N-1)\,\psi\,(x)}-\frac{x\psi'\,(x)}{C_{N-1}^{\frac{N-1}{2}}\,(N-1)\,\psi\,(x)^2}.$$

Multiplying both sides by $(C_{N-1}^{\frac{N-1}{2}}\,(N-1)\,\psi\,(x))^2$, using (IA21), (IA20), and (A12) and simplifying gives

$$\left(C_{N-1}^{\frac{N-1}{2}}\,(N-1)\,\psi\,(x)\right)^2\tilde{f}''\,(x)=-\frac{N-1}{2}\frac{x}{\varphi\,(x)\,(1-\varphi\,(x))\,(2\varphi\,(x)-1)}<0$$

since $\varphi(x) \in \left(\frac{1}{2}, 1\right)$. Therefore, $\tilde{f}(x)$ is strictly concave.

11. Lemma A3 (comparison of shareholder welfare across equilibria). *Suppose that $c \in (\hat{c}, \bar{c})$, where $\hat{c}$ is defined in the proof. Then, in the range $f \in \left[\underline{f}, \bar{f}\right)$, all equilibria can be ranked in their shareholder welfare (expected value of the proposal minus information acquisition costs). Specifically, there exist three equilibria, with equilibrium (a) having the highest and (c) having the lowest shareholder welfare: (a) equilibrium with incomplete crowding out of private information acquisition and $q_r \leq (2p-1) q_s$, given by (A5) in the Appendix; (b) equilibrium with incomplete crowding out of private information acquisition and $q_r \geq (2p-1) q_s$, given by (A6) in the Appendix; (c) equilibrium with complete crowding out of private information acquisition: $q_s = 0$ and q_r given by (13). Equilibria (a) and (b) coincide when $f = \underline{f}$.*

When $f = \bar{f}$, there exist two equilibria: (a) equilibrium that is equivalent to the benchmark case, $q_s = q_0^*$, $q_r = 0$, and (b) equilibrium with complete crowding out: $q_s = 0$, $q_r \in (0,1)$, and equilibrium (a) has higher shareholder welfare than (b).

Proof of Lemma A3. We start by defining $\hat{c}$ in the statement of the lemma. Consider (q_r^a, q_s^a) given by (A5) and define

$$S(f,c) \equiv q_r^a + q_s^a = \frac{2p}{2p-1}\sqrt{\frac{1}{4} - \left(\frac{f + \frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} + \frac{2(1-p)}{2p-1}\sqrt{\frac{1}{4} - \left(\frac{\frac{c}{2p-1} - f}{(1-\pi) C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}.$$

Consider the following function of c:

$$\bar{S}(c) \equiv \max_{f \in \left[\underline{f}_1(c), \bar{f}(c)\right]} S(f,c), \tag{IA27}$$

where $\underline{f}_1(c) = \frac{c}{2p-1} - 2^{1-N}(1-\pi) C_{N-1}^{\frac{N-1}{2}}$ and $\bar{f}(c) = \frac{2\pi-1}{2p-1}c$, as defined before. We show that $\bar{S}(c)$ is strictly decreasing in c. Indeed,

$$\begin{aligned} S\left(\underline{f}_1(c), c\right) &= \frac{2p}{2p-1}\sqrt{\frac{1}{4} - \left(\frac{\frac{2c}{2p-1} - 2^{1-N}(1-\pi) C_{N-1}^{\frac{N-1}{2}}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}, \\ S\left(\bar{f}(c), c\right) &= \frac{2}{2p-1}\sqrt{\frac{1}{4} - \left(\frac{2c}{(2p-1) C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}. \end{aligned} \tag{IA28}$$

For any c, one of three cases must hold: (1) $\bar{S}(c) = S\left(\underline{f}_1(c), c\right)$; (2) $\bar{S}(c) = S(f,c)$ for some $f \in \left(\underline{f}_1(c), \bar{f}(c)\right)$; (3) $\bar{S}(c) = S\left(\bar{f}(c), c\right)$. As clear from (IA28), $S\left(\underline{f}_1(c), c\right)$ and $S\left(\bar{f}(c), c\right)$, corresponding to cases (1) and (3), are strictly decreasing in c. In case (2), i.e., when (IA27) reaches the maximum at an interior point $f^*(c)$, we can apply the envelope theorem: $\bar{S}'(c) = \frac{\partial S(f^*(c), c)}{\partial c} < 0$. Together, this implies that $\bar{S}(c)$ is strictly decreasing in c.

Note also that when $c = \underline{c}$, defined in (7), then $S\left(\bar{f}(\underline{c}), \underline{c}\right) = 1$. Hence, $\bar{S}(\underline{c}) \geq 1$. In addition, when $c = \bar{c}$, defined in (7), then $\bar{f}(\bar{c}) = \underline{f}_1(\bar{c})$, and hence $\bar{S}(\bar{c}) = S\left(\bar{f}(\bar{c}), \bar{c}\right) = 0$. Since $\bar{S}(c)$ is strictly decreasing in c, there exists a unique $\hat{c} \in [\underline{c}, \bar{c})$ at which $\bar{S}(\hat{c}) = 1$, and $\bar{S}(c) < 1$ for any

$c \in (\hat{c}, \bar{c})$. To sum up, we define

$$\hat{c} \equiv \bar{S}^{-1}(1),$$

where $\bar{S}(c)$ is given by (IA27).

Suppose that $c \in (\hat{c}, \bar{c})$. Then, $\frac{2p}{2p-1}\sqrt{\frac{1}{4} - \left(\frac{\underline{f}_1 + \frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} = S\left(\underline{f}_1, c\right) \leq \bar{S}(c) < 1$, and hence $\underline{f} = \underline{f}_1$ according to (A8). According to the proof of Lemma 1, there exists an equilibrium $(q_r, q_s) > 0$ if and only if $f \in \left[\underline{f}, \bar{f}\right)$. Let us find all such equilibria. Since $\bar{S}(c) < 1$, then $q_r^a + q_s^a < 1$ for any $f \in \left[\underline{f}, \bar{f}\right)$. Therefore, $q_r^b + q_s^b \leq q_r^a + q_s^a < 1$. In addition, $(q_r^a, q_s^a) > 0$ and $\left(q_r^b, q_s^b\right) > 0$ because $f < \bar{f}$. Thus, both equilibria (A5) and (A6) exist. Since $q_r^1(\psi) + q_s^1(\psi)$ is strictly decreasing in ψ and $q_r^1(0) + q_s^1(0) = q_r^a + q_s^a < 1$, we have $q_r^2(\psi) + q_s^2(\psi) \leq q_r^1(\psi) + q_s^1(\psi) < 1$ for any $\psi \geq 0$, where $\left(q_r^i(\psi), q_s^i(\psi)\right)$, $i = 1, 2$, represent potential solutions for $q_r + q_s = 1$ and are given by (IA8) and (IA9) in the Internet Appendix. Therefore, there is no equilibrium with $q_s + q_r = 1$ when $f \in \left[\underline{f}, \bar{f}\right)$. Thus, in addition to equilibrium with complete crowding out, there exist exactly two other equilibria when $f \in \left[\underline{f}, , \bar{f}\right)$, and these equilibria feature incomplete crowding out with $q_r + q_s < 1$: (A5) with $q_r^a \leq (2p-1)\, q_s^a$ and (A6) with $q_r^b \geq (2p-1)\, q_s^b$.

The expected welfare of a shareholder is the expected per-share value of the proposal, $U(q_r, q_s)$, given by (A10), minus the expected information acquisition cost:

$$W(q_r, q_s) = \sum_{k=\frac{N+1}{2}}^{N} \left(\pi P(p_a, N, k) + (1-\pi) P(p_d, N, k)\right) - \frac{1}{2} - q_r f - q_s c. \tag{IA29}$$

We next rank these three equilibria in shareholder welfare for $f \in \left[\underline{f}, \bar{f}\right)$ and show that the equilibrium with incomplete crowding out of private information and $q_r < (2p-1)\, q_s$ has the highest shareholder welfare, followed by the equilibrium with incomplete crowding out of private information and $q_r > (2p-1)\, q_s$, which is followed by the equilibrium with complete crowding out of private information.

First, we show that the equilibrium with incomplete crowding out of private information and $q_r > (2p-1)\, q_s$, denoted (q_s^b, q_r^b), has lower shareholder welfare than the equilibrium with incomplete crowding out and $q_r < (2p-1)\, q_s$, denoted (q_s^a, q_r^a). As shown above, $q_r + q_s < 1$. Using (A9), we get $q_r = p_a - p_d$ and $q_s = \frac{p_a + p_d - 1}{2p-1}$, and plugging these into (IA29), $W(q_r, q_s)$ can be rewritten as

$$\sum_{k=\frac{N+1}{2}}^{N} \left(\pi P(p_a, N, k) + (1-\pi) P(p_d, N, k)\right) - \left(f + \frac{c}{2p-1}\right) p_a - \left(\frac{c}{2p-1} - f\right) p_d - \frac{1}{2} + \frac{c}{2p-1}.$$

Using (A2),

$$W(q_r, q_s) = \pi \left(\sum_{k=\frac{N+1}{2}}^{N} P(p_a, N, k) - \Omega_1 p_a\right) + (1-\pi)\left(\sum_{k=\frac{N+1}{2}}^{N} P(p_d, N, k) - \Omega_2 p_d\right) - \frac{1}{2} + \frac{c}{2p-1}.$$

According to (A2), (A3), and (A9), p_a, Ω_1, and Ω_2 are identical in both equilibria and $p_d(q_s^b, q_r^b) = 1 - p_d(q_s^a, q_r^a) < \frac{1}{2}$. Therefore, to show that $W(q_r^a, q_s^a) > W\left(q_r^b, q_s^b\right)$, it is necessary and sufficient to show that for $p_d > \frac{1}{2}$

$$\sum_{k=\frac{N+1}{2}}^{N} P\left(p_d, N, k\right) - \Omega_2 p_d > \sum_{k=\frac{N+1}{2}}^{N} P\left(1-p_d, N, k\right) - \Omega_2\left(1-p_d\right).$$

Using $\sum_{k=\frac{N+1}{2}}^{N} P\left(1-q, N, k\right) = \sum_{k=\frac{N+1}{2}}^{N} P\left(q, N, N-k\right) = 1 - \sum_{k=\frac{N+1}{2}}^{N} P\left(q, N, k\right)$ and $\Omega_2 = P(p_d, N-1, \frac{N-1}{2})$, this is equivalent to

$$\sum_{k=\frac{N+1}{2}}^{N} P\left(p_d, N, k\right) - \frac{1}{2} > (p_d - \frac{1}{2})P(p_d, N-1, \frac{N-1}{2}). \tag{IA30}$$

Denote the left-hand side and the right-hand side by $L\left(p_d\right)$ and $R\left(p_d\right)$, respectively. Differentiating the left-hand side of (IA30),

$$\begin{aligned} L'\left(x\right) &= \textstyle\sum_{k=\frac{N+1}{2}}^{N} P_x\left(x, N, k\right) = -\sum_{k=0}^{\frac{N-1}{2}} P_x\left(x, N, k\right) = -\frac{1}{x(1-x)}\left(\sum_{k=0}^{\frac{N-1}{2}} P\left(x, N, k\right)\left(k - Nx\right)\right) \\ &= -\tfrac{1}{x(1-x)}\left(\textstyle\sum_{k=0}^{\frac{N-1}{2}} kP\left(x, N, k\right) - Nx\sum_{k=0}^{\frac{N-1}{2}} P\left(x, N, k\right)\right) \end{aligned}$$

Note that $\sum_{k=0}^{\frac{N-1}{2}} P\left(x, N, k\right) = I_{1-x}\left(\frac{N+1}{2}, \frac{N+1}{2}\right)$, where $I_x\left(a, b\right)$ is the regularized incomplete beta function. In addition, according to (IA23) and (IA24) and (IA25) in the proof of Auxiliary Lemma A1,

$$\sum_{k=0}^{\frac{N-1}{2}} kP\left(x, N, k\right) = NxI_{1-x}\left(\frac{N+1}{2}, \frac{N-1}{2}\right) = Nx\left[I_{1-x}\left(\frac{N+1}{2}, \frac{N+1}{2}\right) - \frac{\left(1-x\right)^{\frac{N+1}{2}} x^{\frac{N-1}{2}}}{\frac{N-1}{2}B\left(\frac{N+1}{2}, \frac{N-1}{2}\right)}\right],$$

where $B\left(a, b\right)$ is the beta function. Hence,

$$x\left(1-x\right)L'\left(x\right) = Nx\frac{\left(1-x\right)^{\frac{N+1}{2}} x^{\frac{N-1}{2}}}{\frac{N-1}{2}B\left(\frac{N+1}{2}, \frac{N-1}{2}\right)} = \frac{\left(\left(1-x\right)x\right)^{\frac{N+1}{2}} N!}{\left(\frac{N-1}{2}\right)!\left(\frac{N-1}{2}\right)!} = Nx\left(1-x\right)P\left(x, N-1, \frac{N-1}{2}\right). \tag{IA31}$$

Differentiating the right-hand side of (IA30),

$$\begin{aligned} R'\left(x\right) &= P_x\left(x, N-1, \tfrac{N-1}{2}\right)\left(x - \tfrac{1}{2}\right) + P\left(x, N-1, \tfrac{N-1}{2}\right) \\ &= P\left(x, N-1, \tfrac{N-1}{2}\right)\left(\tfrac{\frac{N-1}{2} - (N-1)x}{x(1-x)}\left(x - \tfrac{1}{2}\right) + 1\right) = P\left(x, N-1, \tfrac{N-1}{2}\right)\left(1 - \tfrac{(N-1)\left(x-\frac{1}{2}\right)^2}{x(1-x)}\right) \\ &< P\left(x, N-1, \tfrac{N-1}{2}\right)N = L'\left(x\right). \end{aligned} \tag{IA32}$$

Since $L\left(\frac{1}{2}\right) = R\left(\frac{1}{2}\right) = 0$, it follows that $L\left(x\right) > R\left(x\right)$ for any $x > \frac{1}{2}$. Hence, indeed, $W\left(q_r^a, q_s^a\right) > W\left(q_r^b, q_s^b\right)$.

Second, we show that the equilibrium with incomplete crowding out of private information and $q_r > \left(2p-1\right)q_s$, denoted $\left(q_r^b, q_s^b\right)$, has higher shareholder welfare than the equilibrium with complete crowding out of private information, denoted $\left(q_r^c, 0\right)$, whenever the two co-exist, i.e., $f \in [\underline{f}, \bar{f})$.

Consider function $\varphi(x) \in \left(\frac{1}{2}, 1\right)$ defined as the higher root of $x = P(\varphi(x), N-1, \frac{N-1}{2})$ and given by (A12). Since $\Omega_1 = P\left(p_a, N-1, \frac{N-1}{2}\right)$, $\Omega_2 = P\left(p_d, N-1, \frac{N-1}{2}\right)$, and since $p_a > \frac{1}{2}$ and $p_d < \frac{1}{2}$ in both of these equilibria, we have $p_a = \varphi(\Omega_1)$ and $p_d = 1 - \varphi(\Omega_2)$. Plugging these expressions for p_a and p_d, we can re-write (IA29) as

$$\begin{aligned} &\textstyle\sum_{k=\frac{N+1}{2}}^{N} \left(\pi P\left(\varphi(\Omega_1), N, k\right) + (1-\pi) P\left(1-\varphi(\Omega_2), N, k\right)\right) - \frac{1}{2} - q_r f - q_s c \\ =& \textstyle\sum_{k=\frac{N+1}{2}}^{N} \left(\pi P\left(\varphi(\Omega_1), N, k\right) - (1-\pi) P\left(\varphi(\Omega_2), N, k\right)\right) + \frac{1}{2} - \pi - q_r f - q_s c, \end{aligned} \tag{IA33}$$

where we used $\sum_{k=\frac{N+1}{2}}^{N} P(1-x, N, k) = \sum_{k=0}^{\frac{N-1}{2}} P(x, N, k) = 1 - \sum_{k=\frac{N+1}{2}}^{N} P(x, N, k)$ to get to the second line.

For equilibrium (q_r^b, q_s^b), let us plug $q_r = p_a - p_d$, $q_s = \frac{p_a + p_d - 1}{2p-1}$, $p_a = \varphi(\Omega_1)$, and $p_d = 1 - \varphi(\Omega_2)$ into (IA33). Then, using (A2) and simplifying, we can write shareholder welfare $W\left(q_r^b, q_s^b\right)$ as the following function of Ω_1 and Ω_2:

$$\hat{W}(\Omega_1, \Omega_2) = \pi \tilde{f}(\Omega_1) - (1-\pi)\tilde{f}(\Omega_2) + \frac{1}{2} - \pi,$$

where

$$\tilde{f}(x) \equiv \sum_{k=\frac{N+1}{2}}^{N} P\left(\varphi(x), N, k\right) - x\left(\varphi(x) - \frac{1}{2}\right) \tag{IA34}$$

and Ω_1 and Ω_2 are given by (A2).

Similarly, for equilibrium $(q_r^c, 0)$, let us plug $q_r = p_a - p_d$, $q_s = 0$, $p_a = \varphi(\Omega_1)$, and $p_d = 1 - \varphi(\Omega_2)$ into (IA33). Using the fact that in this equilibrium, $\Omega_1 = \Omega_2 = \Omega_r = \frac{2f}{2\pi-1}$ and simplifying, we can write shareholder welfare $W(q_r^c, 0)$ as $\hat{W}(\Omega_r, \Omega_r)$, where $\Omega_r = \frac{2f}{2\pi-1}$.

Note next that $\Omega_1 = \Omega_r + \frac{1-\pi}{2\pi-1}\varepsilon$ and $\Omega_2 = \Omega_r + \frac{\pi}{2\pi-1}\varepsilon$, where $\varepsilon \equiv (c\frac{2\pi-1}{2p-1} - f)\frac{1}{\pi(1-\pi)} > 0$ since $f < \bar{f}$. Thus, to prove that $W(q_r^b, q_s^b) > W(q_r^c, 0)$, it is necessary and sufficient to prove that $\hat{W}\left(\Omega_r + \frac{1-\pi}{2\pi-1}\varepsilon, \Omega_r + \frac{\pi}{2\pi-1}\varepsilon\right) > \hat{W}(\Omega_r, \Omega_r)$. Define function $\tilde{W}(x) \equiv \hat{W}\left(\Omega_r + \frac{1-\pi}{2\pi-1}x, \Omega_r + \frac{\pi}{2\pi-1}x\right)$ for $x \geq 0$. Differentiating,

$$\tilde{W}'(x) = \frac{\pi(1-\pi)}{2\pi-1}\left(\tilde{f}'\left(\Omega_r + \frac{1-\pi}{2\pi-1}x\right) - \tilde{f}'\left(\Omega_r + \frac{\pi}{2\pi-1}x\right)\right) = -\frac{\pi(1-\pi)}{2\pi-1}\int_{\Omega_r + \frac{1-\pi}{2\pi-1}x}^{\Omega_r + \frac{\pi}{2\pi-1}x} \tilde{f}''(y)\, dy.$$

Auxiliary Lemma A2 shows that function $\tilde{f}(\cdot)$ is strictly concave, and hence $\tilde{W}'(x) > 0$ for any $x > 0$. Thus, $\hat{W}\left(\Omega_r + \frac{1-\pi}{2\pi-1}\varepsilon, \Omega_r + \frac{\pi}{2\pi-1}\varepsilon\right) = \tilde{W}(\varepsilon) > \tilde{W}(0) = \hat{W}(\Omega_r, \Omega_r)$, which proves the statement.

Combing the two results above, we can conclude that when $c \in (\hat{c}, \bar{c})$ and when $f \in [\underline{f}, \bar{f})$, multiple existing equilibria rank in shareholder welfare in the following way: The equilibrium with incomplete crowding out of private information and $q_r < (2p-1)\, q_s$ has the highest shareholder welfare, followed by the equilibrium with incomplete crowding out of private information and $q_r > (2p-1)\, q_s$, which is followed by the equilibrium with complete crowding out of private information.

Finally, when $f \to \bar{f}$, the equilibrium (q_r^a, q_s^a) converges to equilibrium $q_s = q_0^*$, $q_r = 0$, and the equilibrium $\left(q_r^b, q_s^b\right)$ converges to equilibrium with complete crowding out, $q_s = 0$, $q_r \in (0, 1)$. By monotonicity and the welfare comparison above, it follows that when $f = \bar{f}$, the first equilibrium has higher shareholder welfare than the second.

12. Proof that the model is equivalent to a more general setup with $u(1,1) - u(0,1) = u(0,0) - u(1,0)$

Suppose that $u(1,1) - u(0,1) = u(0,0) - u(1,0) = 1$. We show that shareholders make exactly the same information acquisition and voting decisions as in the current setup.

1. Same voting decisions. Whenever a shareholder decides how to vote, he votes "for" if and only if his utility from voting "for" is greater than his utility from voting "against" conditional on the event of being pivotal and whatever information he knows. Denote this information set I_i. The shareholder's utility from voting for minus his utility from voting against conditional on this information set is

$$\begin{aligned} & \Pr(\theta = 1 | I_{piv}) (u(1,1) - u(0,1)) + \Pr(\theta = 0 | I_{piv}) (u(1,0) - u(0,0)) \\ = \; & \Pr(\theta = 1 | I_{piv}) - \Pr(\theta = 0 | I_{piv}), \end{aligned}$$

which is the same as in the basic model. Hence, given the same information acquisition decisions, shareholders make the same voting decisions.

2. Same information acquisition decisions. Shareholder i's vote only makes a difference only if the votes of other shareholders are split. Denote this set of events by PIV_i. Let us find the value of any signal to the shareholder. Denote the signal acquired by the shareholder (private or advisor's) by S_i. If $S_i = 1$, then by acquiring the signal, the shareholder votes "for" for sure, instead of randomizing between voting "for" and "against". Hence, conditional on $S_i = 1$ and on being pivotal, his utility from being informed is

$$\mathbb{E}[u(1,\theta) | S_i = 1, PIV_i] - \frac{\mathbb{E}[u(1,\theta) + u(0,\theta) | S_i = 1, PIV_i]}{2} = \frac{\mathbb{E}[u(1,\theta) - u(0,\theta) | S_i = 1, PIV_i]}{2}$$

Similarly, conditional on being pivotal and the signal being $S_i = 0$, the shareholder's utility from being informed is

$$\mathbb{E}[u(0,\theta) | S_i = 1, PIV_i] - \frac{\mathbb{E}[u(1,\theta) + u(0,\theta) | S_i = 1, PIV_i]}{2} = \frac{\mathbb{E}[u(0,\theta) - u(1,\theta) | S_i = 1, PIV_i]}{2}$$

Overall, the shareholder's value of acquiring the signal is

$$\begin{gathered} \Pr(S_i = 1) \Pr(PIV_i | S_i = 1) \tfrac{\mathbb{E}[u(1,\theta) - u(0,\theta) | S_i = 1, PIV_i]}{2} \\ - \Pr(S_i = 0) \Pr(PIV_i | S_i = 0) \tfrac{\mathbb{E}[u(1,\theta) - u(0,\theta) | S_i = 1, PIV_i]}{2}, \end{gathered}$$

Since, by assumption, $u(1,1) - u(0,1) = 1$ and $u(1,0) - u(0,0) = -1$ and are the same as in the basic model, the value of any signal to the shareholder is the same as in the basic model, and hence the shareholders make the same information acquisition decisions.

13. Supplementary analysis for the proof of Proposition 8.

To prove that $\lim_{t \to 0} \pi^*(t) = 1$, we prove the auxiliary result that $R'(\pi)$ is bounded away from zero for π in the neighborhood of 1.

To prove this property, we first prove that $R(\pi)$ is strictly increasing in π. The envelope theorem implies that at any point at which $R(\pi)$ is differentiable,

$$R'(\pi) = Nf^*(\pi)\frac{\partial q_r}{\partial \pi}(f^*(\pi),\pi), \tag{IA35}$$

where $f^*(\pi)$ denotes the fee chosen by the advisor when the precision of the signal equals π. If, given π, the equilibrium features complete crowding out of private information acquisition, then $q_r(f^*(\pi),\pi)$ is given by (13), so

$$\frac{\partial q_r}{\partial \pi}(f^*(\pi),\pi) = \frac{4}{(N-1)\left(\pi-\frac{1}{2}\right)q_r(f^*(\pi),\pi)}\left(\frac{f^*(\pi)}{C_{N-1}^{\frac{N-1}{2}}(\pi-\frac{1}{2})}\right)^{\frac{2}{N-1}}. \tag{IA36}$$

If the equilibrium features incomplete crowding out of private information acquisition, then $q_r(f^*(\pi),\pi)$ is given by (A5), so

$$(N-1)\frac{\partial q_r}{\partial \pi}(f^*(\pi),\pi) = \frac{\left(\frac{f^*(\pi)+\frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}{\pi\sqrt{\frac{1}{4}-\left(\frac{f^*(\pi)+\frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}} + \frac{\left(\frac{\frac{c}{2p-1}-f^*(\pi)}{(1-\pi)C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}{(1-\pi)\sqrt{\frac{1}{4}-\left(\frac{\frac{c}{2p-1}-f^*(\pi)}{(1-\pi)C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}}. \tag{IA37}$$

Since $f^*(\pi) > 0$ for any $\pi \in \left(\frac{1}{2},1\right)$, (IA36)–(IA37) imply that $\frac{\partial q_r}{\partial \pi}(f^*(\pi),\pi) > 0$ at any $\pi \in \left(\frac{1}{2},1\right)$ at which $R(\pi)$ is differentiable. Thus, $R'(\pi) > 0$ at any $\pi \in \left(\frac{1}{2},1\right)$ at which $R(\pi)$ is differentiable. In addition, at any point π at which $R(\pi)$ is not differentiable, it cannot be decreasing, since for any such π the seller can choose the fee that is optimal for $\pi-\varepsilon$ for infinitesimal positive ε and achieve higher revenues. Therefore, $R(\pi)$ is strictly increasing in $\pi \in \left(\frac{1}{2},1\right)$.

Next, we prove that $R'(\pi)$ is bounded away from zero for π in the neighborhood of 1. When π is close enough to 1, (A3) has no solution, and hence equilibrium cannot feature incomplete crowding out. In equilibrium with complete crowding out, $R'(\pi)$ is given by (IA35)–(IA36), so to prove that it is bounded away from zero, it is sufficient to prove that $f^*(\pi)$ is bounded away from zero. Suppose this is not the case, that is, $\lim_{\pi\to 1} f^*(\pi) = 0$. Since $q_r < 1$, this implies $\lim_{\pi\to 1} R(\pi) = 0$. This, however, is not possible since $R(\pi)$ is strictly increasing, as shown above. This completes the proof of this statement.

B. Analysis of regulation

B.1. Litigation pressure

Suppose that a shareholder gets an additional payoff $\Delta > 0$ if it subscribes to and follows the advisor's recommendation. Given q_r and q_s, the gross value to a shareholder from acquiring a private signal and the recommendation of the advisor is $V_s(q_r,q_s)$ and $V_r(q_r,q_s)+\Delta$, respectively. As before, the value from staying uninformed is zero. Therefore, for a fixed fee f, the game is identical to the subgame of the basic model with fee $f-\Delta$. The equilibrium probability that a shareholder buys and follows the advisor is therefore given by $q_r(f-\Delta)$, where $q_r(\cdot)$ is given by (14). Specifically, if $f < \underline{f}+\Delta$, the equilibrium features complete crowding out of private information acquisition ($q_r = q_r^H(f-\Delta)$ and $q_s = 0$), while if $f \in [\underline{f}+\Delta, \overline{f}+\Delta)$, it features incomplete crowding out ($q_r = q_r^L(f-\Delta)$ and $q_s > 0$). Since $q_r(\cdot)$ is decreasing in fee f, for any fee f, the demand for the advisor's recommendation is higher than in the basic model. The advisor responds to the increased demand by increasing its fee.

The next proposition summarizes the effect of an increase in regulatory pressure Δ on the informativeness of decision-making:

Proposition B.1 (litigation pressure). *A marginal increase in Δ:*

1. *decreases firm value if the equilibrium features incomplete crowding out of private information acquisition (i.e., equilibrium fee exceeds $\underline{f}+\Delta$);*
2. *does not affect firm value if the equilibrium features complete crowding out of private information acquisition and limit pricing (i.e., equilibrium fee equals $\underline{f}+\Delta$);*
3. *increases firm value if the equilibrium features complete crowding out of private information acquisition and unconstrained maximization (i.e., equilibrium fee is below $\underline{f}+\Delta$).*

Proposition B.1 suggests that greater litigation pressure is a delicate issue. It increases the demand for the advisor's recommendation for any quality of the advisor's recommendation, which has two effects. On the one hand, it increases the incentives to vote informatively. On the other hand, it shifts the incentives from doing proprietary research to following the advisor's recommendations. As a consequence, the total effect on the quality of decision-making depends on the quality of the advisor's information. As the basic model shows, if the quality is low, there is overreliance on the advisor's recommendation and inefficient crowding out of private information production. In this case, higher litigation pressure leads to even more inefficient crowding out of private information production, which reduces the quality of decision-making. In contrast, if the quality of the advisor's recommendation is high, there is underreliance on the advisor's recommendation, because the profit-maximizing advisor prices information so as not to sell it to all shareholders. In this case, greater litigation pressure increases the quality of decision-making by increasing the fraction of shareholders who follow the advisor instead of voting uninformatively.

B.2. Reducing proxy advisory fees

Consider the effect of a marginal reduction in the fee charged by the advisor from the equilibrium f^* to a lower level. As the next proposition shows, whether such a reduction in market power is beneficial depends on the equilibrium information acquisition decisions by shareholders, and in particular, on how much private information they acquire. To see this, suppose, first, that given the equilibrium fee f^*, shareholders do not acquire any private information. In this case, it is optimal (for the quality of decision-making) that more shareholders rely on the advisor, since following the advisor dominates uninformed voting. Therefore, if complete crowding out of private information acquisition occurs in equilibrium, a marginal reduction of the advisor's fee increases the informativeness of voting. In contrast, if the equilibrium features incomplete crowding out of private information acquisition, a reduction in the advisor's fee has a negative effect of crowding out some of this private information acquisition. By the same logic as in Proposition 3, this is inefficient and lowers the quality of decision-making. The following result formalizes these arguments:

Proposition B.2 (restricting market power). *A marginal reduction in the advisor's fee increases firm value if equilibrium features complete crowding out of private information acquisition, but decreases firm value if equilibrium features incomplete crowding out of private information acquisition.*

Proposition B.2 implies that restricting the advisor's market power will lead to more informative voting only if the advisor's information is sufficiently precise. In contrast, if the advisor's information is imprecise, decreasing its market power will lower the quality of decision-making because it will lead to even greater overreliance on the advisor's recommendations.

B.3 Disclosing the quality of recommendations

In this section, we examine how disclosing the quality of the advisor's recommendations affects the informativeness of decision-making. Specifically, consider the following modification of our baseline setting. The actual precision of the advisor's signal can be high or low, $\pi \in \{\pi_l, \pi_h\}$, $\pi_l < \pi_h$, with probabilities μ_l and μ_h, $\mu_h + \mu_l = 1$. Let $\bar{\pi} \equiv \mu_l \pi_l + \mu_h \pi_h$ denote the expected precision of the signal.

Let us compare the quality of decision-making in two regimes – when the precision of the advisor's signal is publicly disclosed and when it remains unknown to the shareholders. If the precision of the advisor's signal is disclosed, the timing of the game is as follows. First, precision $\pi \in \{\pi_l, \pi_h\}$ is realized and learned by all parties. Then, the advisor decides on the fee it charges for its recommendation. After that, shareholders non-cooperatively decide what signals to acquire and how to vote. If the precision of the advisor's signal is not disclosed, the timing of the game is identical to that in the previous sections: The advisor sets the fee it charges, shareholders decide what signal to acquire, not knowing whether $\pi = \pi_l$ or $\pi = \pi_h$, and then decide how to vote. The proof of the proposition below shows that the equilibrium in this game coincides with the equilibrium of the basic model for $\pi = \bar{\pi}$.

We make a simplifying assumption that uncertainty about the precision of the advisor's signal is rather high:

Assumption (high precision uncertainty). $\pi_l = \frac{1}{2}$ *and* π_h *is such that complete crowding out of private information acquisition occurs in equilibrium of the basic model with* $\pi = \pi_h$.

This assumption implies that if the quality of the advisor's information is low, its signal is completely uninformative. Clearly, if shareholders know that the advisor's signal is pure noise, no shareholder buys it, and the equilibrium is identical to the benchmark model without the advisor. In contrast, if the quality of the advisor's information is high and shareholders know about it, no shareholder acquires private information.

The next proposition gives sufficient conditions under which disclosure improves the quality of decision-making:

Proposition B.3 (disclosure of precision). *Firm value is strictly higher when the precision of the advisor's signal is disclosed if at least one of the following conditions is satisfied:*

1. $V^*(\pi_h) > V_0$, *i.e., firm value is higher with the advisor than without when* $\pi = \pi_h$; *or*
2. *Complete crowding out of private information acquisition occurs when* $\pi = \bar{\pi}$.

The intuition is as follows. Disclosing the precision of the advisor's recommendations allows shareholders to tailor their information acquisition decisions to the quality of the recommendations: shareholders do not acquire the advisor's recommendations if $\pi = \frac{1}{2}$ and do not acquire private information if $\pi = \pi_h$. Under the first condition in Proposition B.3, such tailored information acquisition decisions are rather efficient: they ensure that the advisor's recommendations do not affect the vote when they are uninformative, and that they have a relatively large effect on the vote when they are sufficiently informative ($V^*(\pi_h) > V_0$). Hence, disclosure leads to more informed voting decisions than if shareholders made their decisions based on the average precision $\bar{\pi}$ and sometimes relied on the advisor's recommendations when they are completely uninformative. A similar argument applies under the second condition in Proposition B.3: without disclosure, shareholders do not acquire private information and completely rely on the advisor's recommendations, even though they are sometimes uninformative. In contrast, with disclosure, shareholders perform independent research when the advisor's recommendations are uninformative, leading to more informed voting decisions.

Interestingly, however, disclosing the precision of the advisor's recommendations does not always improve the quality of decision-making: Disclosure may encourage even stronger crowding out of private information acquisition and decrease firm value. To see this, consider the numerical example of Figure 3 and suppose that $\pi_l = \frac{1}{2}$, $\pi_h = 0.7$, and $\mu_l = \mu_h = \frac{1}{2}$, so that $\bar{\pi} = 0.6$. Without disclosure, expected firm value is given by $V^*(0.6)$, which, as Figure 3c demonstrates, is very close to value V_0 in the benchmark case without the advisor. This is because the expected precision of the advisor's signal is sufficiently low, so that there is relatively little crowding out of private information acquisition. In contrast, with disclosure, expected firm value is the average of V_0 and $V^*(0.7)$, and this average is lower than $V^*(0.6)$. Thus, in this example, disclosure makes voting decisions less informed and decreases firm value. The reason is that when $\pi = \pi_h$, the advisor's recommendations are not precise enough to improve decision-making but are sufficiently precise to completely crowd out private information acquisition. This inefficient crowding out of private information when $\pi = \pi_h$ is detrimental for firm value, and even the more efficient decision-making when $\pi = \pi_l$ is not sufficient to counteract its negative effect.

Proofs for the section "Analysis of regulation"

Proof of Proposition B.1. Let $f^{**}(\Delta)$ denote the equilibrium fee that the advisor charges. Consider part 1 of the proposition. Since $q_r + q_s < 1$ by Assumption 2, then $f^{**}(\Delta) = \arg\max_f f q_r^L(f - \Delta)$, where q_r^L is given by (A5). Using a change of variable $\phi \equiv f - \Delta$, we have:

$$f^{**}(\Delta) = \Delta + \arg\max_{\phi} (\phi + \Delta)\, q_r^L(\phi).$$

We first prove that the maximizer ϕ, denoted $\phi^{**}(\Delta)$, is decreasing in Δ. Consider any $\Delta_2 > \Delta_1$. Denoting $\phi^{**}(\Delta_i) = \phi_i$, $i \in \{1, 2\}$, we have

$$\begin{aligned}
(\phi_2 + \Delta_2)\, q_r^L(\phi_2) &\geq (\phi_1 + \Delta_2)\, q_r^L(\phi_1), \\
(\phi_1 + \Delta_1)\, q_r^L(\phi_1) &\geq (\phi_2 + \Delta_1)\, q_r^L(\phi_2),
\end{aligned}$$

or equivalently,

$$\begin{aligned}\phi_2 q_r^L(\phi_2) - \phi_1 q_r^L(\phi_1) &\geq \Delta_2 \left[q_r^L(\phi_1) - q_r^L(\phi_2)\right], \\ \phi_2 q_r^L(\phi_2) - \phi_1 q_r^L(\phi_1) &\leq \Delta_1 \left[q_r^L(\phi_1) - q_r^L(\phi_2)\right],\end{aligned}$$

implying

$$\Delta_2 \left[q_r^L(\phi_1) - q_r^L(\phi_2)\right] \leq \Delta_1 \left[q_r^L(\phi_1) - q_r^L(\phi_2)\right]. \tag{IA38}$$

Suppose, by contradiction, that $\phi_2 > \phi_1$. Since $\frac{\partial q_r^L(\phi)}{\partial \phi} < 0$ by (A5), then $q_r^L(\phi_2) < q_r^L(\phi_1)$, and hence (IA38) implies $\Delta_2 \leq \Delta_1$, giving a contradiction. Hence, $\phi_2 \leq \phi_1$, i.e., $\phi^{**}(\Delta)$ is decreasing in Δ.

Since $\frac{\partial q_r^L(\phi)}{\partial \phi} < 0$ and $\phi^{**}(\Delta)$ is decreasing in Δ, then the equilibrium probability that a shareholder acquires information from the advisor, $q_r^L(\phi^{**}(\Delta))$, increases in Δ. Hence, according to (A9), $p_a - p_d = q_r$ increases in Δ. By the argument similar to that in the proof of Proposition 3, firm value decreases in Δ. Indeed, since $q_r + q_s < 1$, a marginal increase in Δ increases the distance between $x_a = P(p_a, N-1, \frac{N-1}{2})$ and $x_d = P(p_d, N-1, \frac{N-1}{2})$, while keeping the total probability of being pivotal, $\pi x_a + (1-\pi) x_d$, unchanged at $\frac{2c}{2p-1}$. According to (A10), firm value equals $\pi f(x_a) + (1-\pi) f(x_d) - \frac{1}{2}$, where $f(x) = \sum_{k=\frac{N+1}{2}}^{N} P(\varphi(x), N, k)$ and $\varphi(x)$ is defined by (A12). Since, according to Auxiliary Lemma A1, function $f(x)$ is concave, firm value decreases with the distance between x_a and x_d when $\pi x_a + (1-\pi) x_d$ remains unchanged, and hence decreases with Δ.

Consider part 2 of the proposition. In this case, $f^{**}(\Delta) = \underline{f} + \Delta$, and hence $q_r = q_r^H(f^{**}(\Delta) - \Delta) = q_r^H(\underline{f})$. Thus, both q_r and $q_s = 0$ are unaffected by a marginal change in Δ, and hence firm value is unaffected by Δ as well.

Finally, consider part 3 of the proposition. In this case, $f^{**}(\Delta) = \arg\max_f f q_r^H(f - \Delta)$. Using a change of variable $\phi \equiv f - \Delta$, we have:

$$f^{**}(\Delta) = \Delta + \arg\max_{\phi} (\phi + \Delta) q_r^H(\phi).$$

Since the cross-partial derivative of the maximized function ($\frac{\partial q_r^H(\phi)}{\partial \phi}$) is negative, the maximizer ϕ, denoted $\phi^*(\Delta)$, is decreasing in Δ. Therefore, the equilibrium probability that a shareholder acquires information from the advisor, $q_r^H(\phi^*(\Delta))$, increases in Δ. As shown in the proof of Proposition 3, $\sum_{k=\frac{N+1}{2}}^{N} P_q(q, N, k) > 0$ for $q > \frac{1}{2}$ and hence, according to (A18), firm value increases in Δ.

Proof of Proposition B.2. First, suppose that complete crowding out of private information acquisition occurs in equilibrium. Then $q_r = q_r^H(f)$ is given by (13), and a marginal decrease in f increases q_r. The expected value of the proposal is given by (A18). As shown in the proof of Proposition 3, $\sum_{k=\frac{N+1}{2}}^{N} P_q(q, N, k) > 0$ for $q > \frac{1}{2}$ and hence expected value increases when f decreases. Next, consider the case of incomplete crowding out of private information acquisition. Since $q_r + q_s < 1$ by Assumption 2, (q_r, q_s) are given by (A5). A marginal decrease in f increases q_r and hence, according to (A9), $p_a - p_d = q_r$ increases. By the argument similar to that in the proof of Proposition 3, firm value increases in f. Indeed, a marginal decrease in f increases the distance between $x_a = P(p_a, N-1, \frac{N-1}{2})$ and $x_d = P(p_d, N-1, \frac{N-1}{2})$. while keeping the total probability of being pivotal, $\pi x_a + (1-\pi)\, x_d$, unchanged at $\frac{2c}{2p-1}$. According to (A10), firm value equals $\pi f(x_a) + (1-\pi) f(x_d) - \frac{1}{2}$, where $f(x) = \sum_{k=\frac{N+1}{2}}^{N} P(\varphi(x), N, k)$ and $\varphi(x)$ is defined by (A12). Since, according to Auxiliary Lemma A1, function $f(x)$ is concave, firm value decreases with the distance between x_a and x_d when $\pi x_a + (1-\pi)\, x_d$ remains unchanged, and hence decreases when f decreases.

Proof of Proposition B.3. We first show that if the precision of the advisor's signal is not disclosed, the equilibrium of the game is the same as in the basic model but where the precision of the advisor's signal is the expected value of π, $\bar{\pi} \equiv \mu_l \pi_l + \mu_h \pi_h$. Indeed, fix the equilibrium probabilities q_r and q_s with which each shareholder acquires the advisor's signal and his private signal, and consider the information acquisition decision of any shareholder, taking the strategies of other shareholders as given. Denote $V_s(q_r, q_s, \pi)$ and $V_r(q_r, q_s, \pi)$ the shareholder's values from acquiring the private and public signal, respectively, if the precision of the advisor's signal is known to be π. These values are given by expressions (9) and (10). Then, the values from acquiring the private and public signal if the shareholder does not know the realization of π are $\bar{V}_s \equiv \mu_l V_s(q_r, q_s, \pi_l) + \mu_h V_s(q_r, q_s, \pi_h)$ and $\bar{V}_r \equiv \mu_l V_r(q_r, q_s, \pi_l) + \mu_h V_r(q_r, q_s, \pi_h)$. Because, $\Omega_1(q_r, q_s)$ and $\Omega_2(q_r, q_s)$ do not depend on π, (9) and (10) imply that $V_s(q_r, q_s, \pi)$ and $V_r(q_r, q_s, \pi)$ are linear in π. Hence, $\bar{V}_s = V_s(q_r, q_s, \bar{\pi})$ and $\bar{V}_r = V_r(q_r, q_s, \bar{\pi})$. This proves that the equilibrium of the game without disclosure coincides with the equilibrium of the basic model with precision $\bar{\pi}$.

Denote $V^*(\pi)$ the expected value of the proposal in the equilibrium of the basic model when the precision of the advisor's signal is π. The argument above implies that the expected value of the proposal in the game without disclosure is given by $V^*(\bar{\pi})$. Since the expected value of the proposal in the game with disclosure is $\mu_l V^*(\frac{1}{2}) + \mu_h V^*(\pi_h)$ and since $V^*(\frac{1}{2}) = V_0$, given by (8), we want to prove that under each of the conditions of the proposition, $\mu_l V_0 + \mu_h V^*(\pi_h) > V^*(\bar{\pi})$.

Consider the first condition, i.e., suppose that $V^*(\pi_h) > V_0$. First, if $\bar{\pi}$ is such that $V^*(\bar{\pi}) \leq V_0$, we have $\mu_l V_0 + \mu_h V^*(\pi_h) > V_0 \geq V^*(\bar{\pi})$, as required. Second, consider $\bar{\pi}$ such that $V^*(\bar{\pi}) > V_0$. The proof of Proposition 5 implies that this can only be true if $\bar{\pi} > \tilde{\pi}$ and $f^* = f_m$, and hence $V^*(\bar{\pi})$ is given by (A19). Since $V^*(\pi_h) > V_0$, $V^*(\pi_h)$ is also given by (A19). Hence,

$$
\begin{aligned}
V^*(\bar{\pi}) &= (2\bar{\pi} - 1) \left(\sum_{k=\frac{N+1}{2}}^{N} P\left(\tfrac{1}{2} + \tfrac{1}{2\sqrt{N}}, N, k\right) - \tfrac{1}{2}\right) \\
&= \mu_h (2\pi_h - 1) \left(\sum_{k=\frac{N+1}{2}}^{N} P\left(\tfrac{1}{2} + \tfrac{1}{2\sqrt{N}}, N, k\right) - \tfrac{1}{2}\right) = \mu_h V^*(\pi_h) < \mu_l V_0 + \mu_h V^*(\pi_h),
\end{aligned}
$$

as required.

Next, consider the second condition of the proposition. If $V^*(\pi_h) > V_0$, then the first condition of the proposition, which has been proved above to be sufficient, applies. Hence, consider $V^*(\pi_h) \leq V_0$. Since complete crowding out of private information acquisition occurs for $\bar{\pi}$, it also occurs for π_h since $\pi_h > \bar{\pi}$. In the range of complete crowding out of private information acquisition, the quality of decision-making $V^*(\pi)$ is strictly increasing in π, and hence $V^*(\pi_h) > V^*(\bar{\pi})$. Hence, $\mu_l V_0 + \mu_h V^*(\pi_h) \geq V^*(\pi_h) > V^*(\bar{\pi})$, as required.