The Kondo Problem and Asymptotic Freedom

Final presentation for Solid State Physics II

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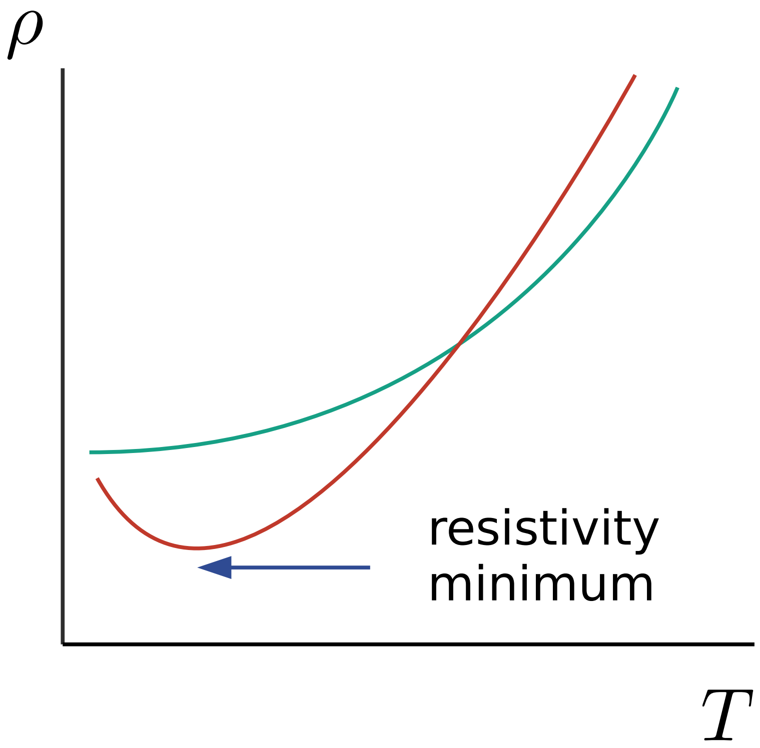
Outline

- History
- Kondo effect
- Kondo problem
- Asymptotic freedom: poor man’s scaling

History: resistance minimum

- Electron-impurity scattering: resistivity is T independent
- Electron-phonon scattering: resistivity increases with T
- Resistivity minimum at low T observed
  - *What causes this?*



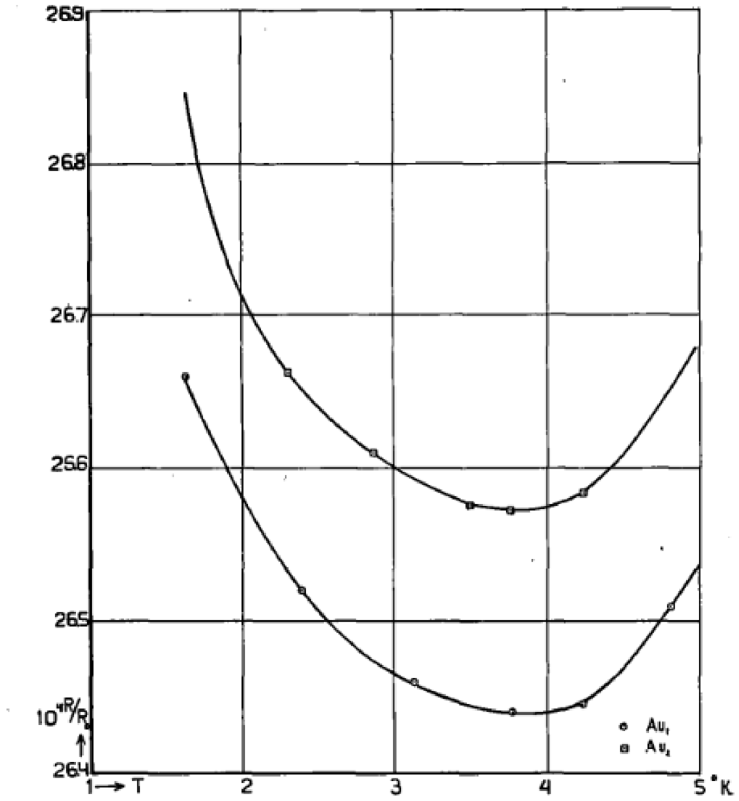


Fig. 1. Resistance of *Au* between 1°K. and 5°K.

de Haas et al. Physica, Volume 1, Issue 7, p. 1115-1124 (1934)

History: magnetic impurity

- Resistivity minimum happens only when impurity is magnetic
- Indication that electron-magnetic impurity scattering increases with decreasing T
  - *How can we understand this?*

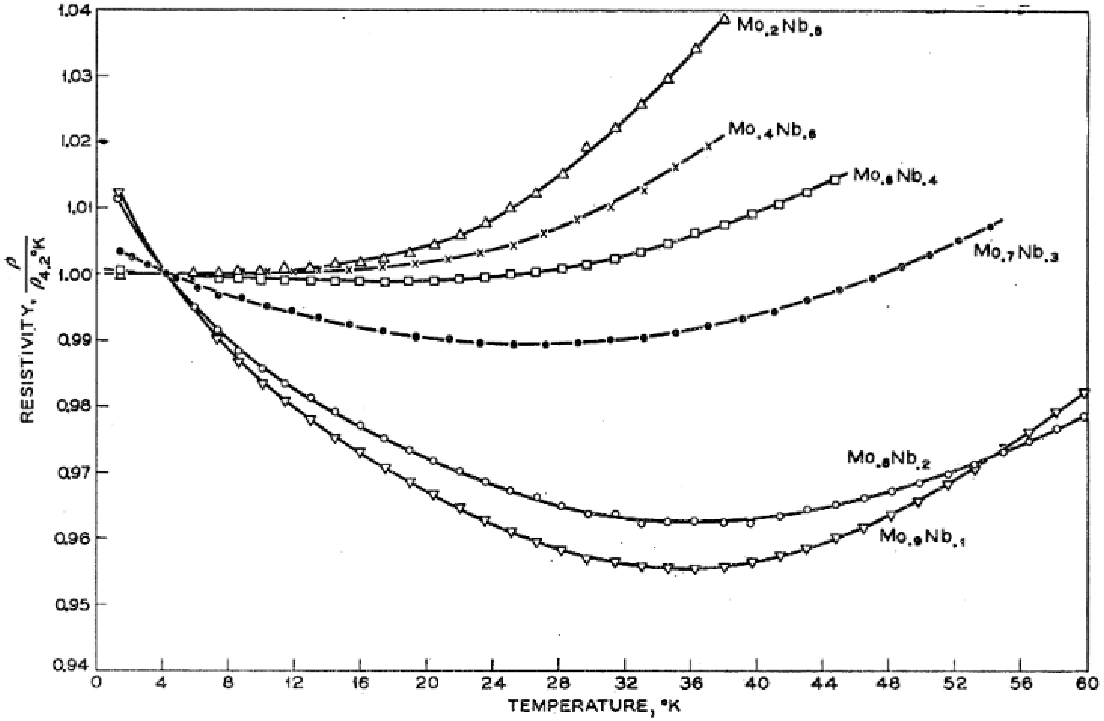


FIG. 3. Resistivity vs temperature for various Mo-Nb alloys containing 1% Fe. Resistivities are normalized at 4.2°K.

M Sarachik et al. Phys. Rev. 135, A1041 (1964)

Kondo Hamiltonian

$$H_{\text{ex}} = J \sum_{\mathbf{k}\mathbf{k}'n} \left\{ \left[c^\dagger_{\mathbf{k}'\uparrow} c_{\mathbf{k}\uparrow} - c^\dagger_{\mathbf{k}'\downarrow} c_{\mathbf{k}\downarrow} \right] S^z_n + c^\dagger_{\mathbf{k}'\downarrow} c_{\mathbf{k}\uparrow} S^+_n + c^\dagger_{\mathbf{k}'\uparrow} c_{\mathbf{k}\downarrow} S^-_n \right\}$$

- Spin-exchange interaction between itinerant electron (k) and static impurity spin (n)
- Spin-flipping is allowed
- J (<)> 0 for (anti)ferromagnetism, assumed to be small [*Is this valid?*]
- Kondo's solution - up to second order Born series: $T = H_{\text{ex}} + H_{\text{ex}} G_0 H_{\text{ex}} + \ldots$
- Looks pretty hard
  - *How can we do perturbation theory with this?*
  - *Can it explain the Kondo minimum?*

J Kondo, Progress of Theoretical Physics, Vol. 32, No. 1 (1964)

O Madelung, Introduction to Solid State Theory, Springer-Verlag (1981)

1^{st} order processes

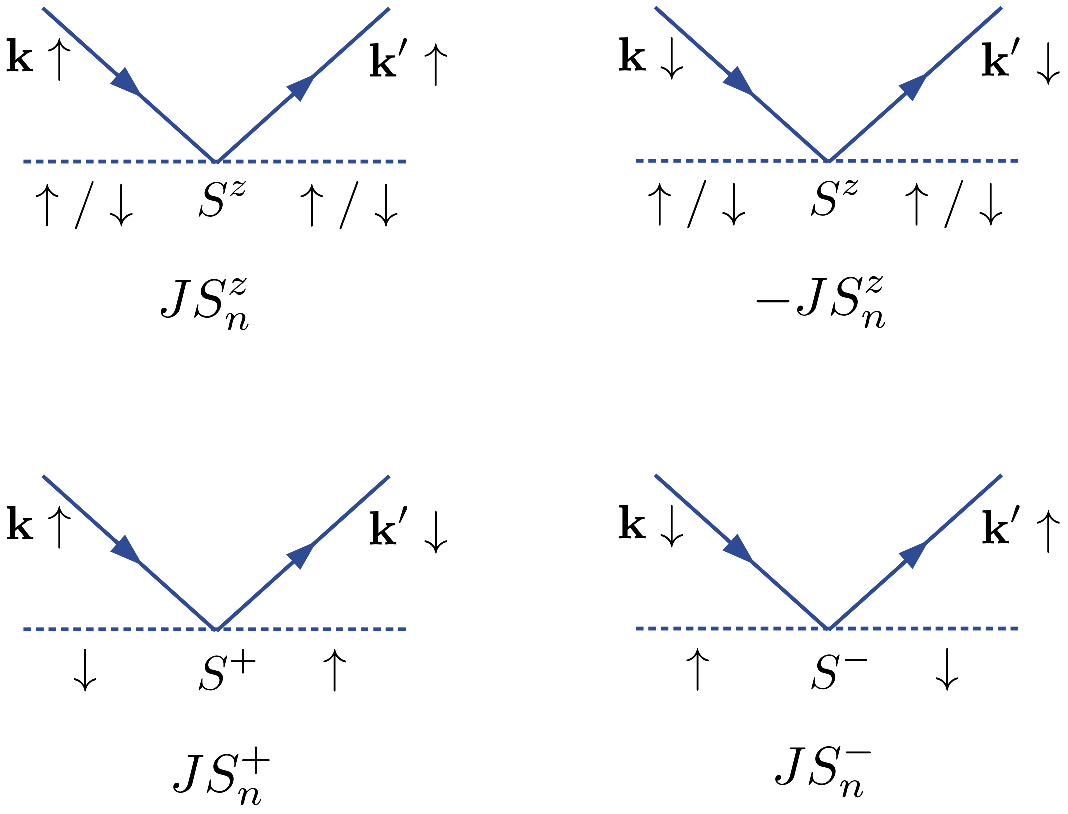
$$H_{\text{ex}} = J\sum_{\mathbf{k}\mathbf{k}'n}\left\{\left[c^\dagger_{\mathbf{k}'\uparrow}c_{\mathbf{k}\uparrow} - c^\dagger_{\mathbf{k}'\downarrow}c_{\mathbf{k}\downarrow}\right]S^z_n + c^\dagger_{\mathbf{k}'\downarrow}c_{\mathbf{k}\uparrow}S^+_n + c^\dagger_{\mathbf{k}'\uparrow}c_{\mathbf{k}\downarrow}S^-_n\right\}$$

- Represent T matrix elements as diagrams
- Sum over spin states
- Scattering probability

$$W = \frac{2\pi}{\hbar}|T|^2$$

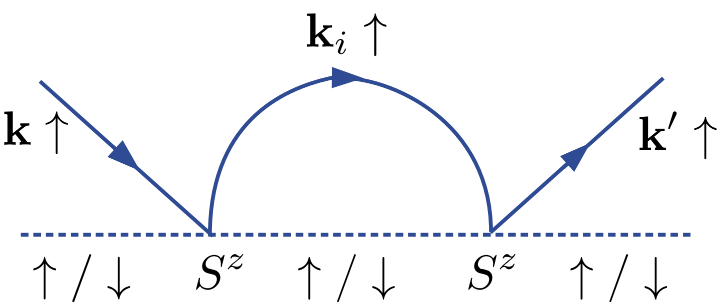
$$W \sim J^2\left(2(S^z_n)^2 + S^-_n S^+_n + S^+_n S^-_n\right)$$

- Scattering probability is T independent!
  - Try a higher order



2^{nd} order processes (no spin-flip)

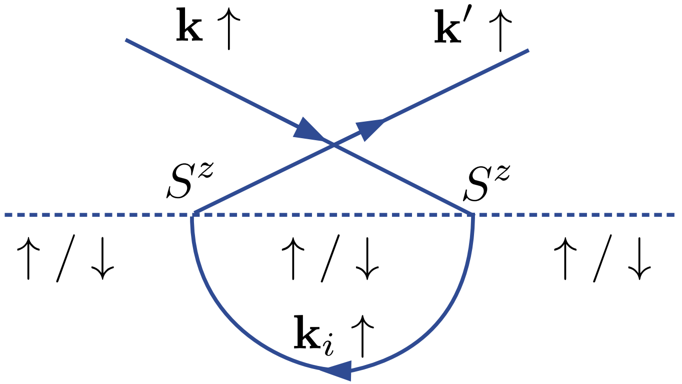
- 2 processes with no internal spin-flipping
- Calculate $\langle \mathbf{k}' \uparrow | T^{(2)} | \mathbf{k} \uparrow \rangle$



$$J^2 \langle \mathbf{k}' \uparrow | \sum_{\mathbf{k}_i} c^\dagger_{\mathbf{k}'\uparrow} c_{\mathbf{k}_i\uparrow} S^z \frac{1}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}_i}} c^\dagger_{\mathbf{k}_i\uparrow} c_{\mathbf{k}\uparrow} S^z | \mathbf{k} \uparrow \rangle$$

$$= -J^2 \sum_{\mathbf{k}_i} \frac{1}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}_i}} \left\langle c^\dagger_{\mathbf{k}_i\uparrow} c_{\mathbf{k}_i\uparrow} \right\rangle \langle \mathbf{k}' \uparrow | c^\dagger_{\mathbf{k}'\uparrow} c_{\mathbf{k}\uparrow} S^z S^z | \mathbf{k} \uparrow \rangle$$

$$= -J^2 (S^z)^2 \sum_{\mathbf{k}_i} \frac{1 - f(\epsilon_{\mathbf{k}_i})}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}_i}}$$



$$J^2 \langle \mathbf{k}' \uparrow | \sum_{\mathbf{k}_i} c^\dagger_{\mathbf{k}_i\uparrow} c_{\mathbf{k}\uparrow} S^z \frac{1}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}_i}} c^\dagger_{\mathbf{k}'\uparrow} c_{\mathbf{k}_i\uparrow} S^z | \mathbf{k} \uparrow \rangle$$

$$= -J^2 \sum_{\mathbf{k}_i} \frac{1}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}_i}} \left\langle c^\dagger_{\mathbf{k}_i\uparrow} c_{\mathbf{k}_i\uparrow} \right\rangle \langle \mathbf{k}' \uparrow | c^\dagger_{\mathbf{k}'\uparrow} c_{\mathbf{k}\uparrow} S^z S^z | \mathbf{k} \uparrow \rangle$$

$$= -J^2 (S^z)^2 \sum_{\mathbf{k}_i} \frac{f(\epsilon_{\mathbf{k}})}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}_i}}$$

2^{nd} order processes (no spin-flip)

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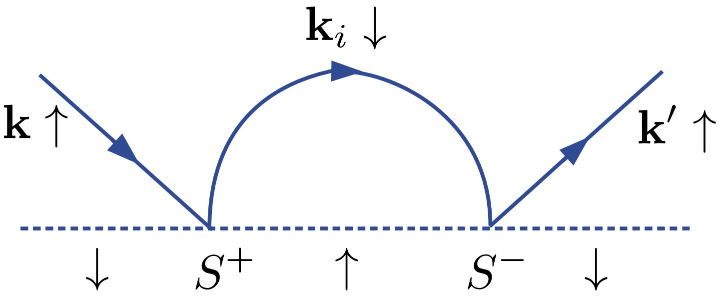
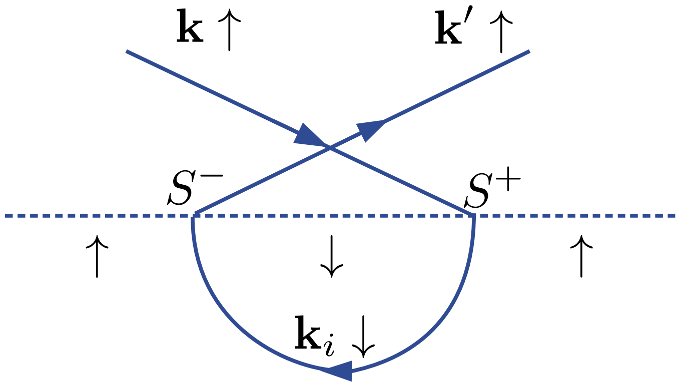
- Calculate $\langle \mathbf{k}' \uparrow | T^{(2)} | \mathbf{k} \uparrow \rangle$



$$J^2 \langle \mathbf{k}' \uparrow | \sum_{\mathbf{k}_i} c^\dagger_{\mathbf{k}'\uparrow} c_{\mathbf{k}_i\downarrow} S^- \frac{1}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}_i}} c^\dagger_{\mathbf{k}_i\downarrow} c_{\mathbf{k}\uparrow} S^+ | \mathbf{k} \uparrow \rangle$$

$$= -J^2 S^- S^+ \sum_{\mathbf{k}_i} \frac{1 - f(\epsilon_{\mathbf{k}_i})}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}_i}}$$



$$J^2 \langle \mathbf{k}' \uparrow | \sum_{\mathbf{k}_i} c^\dagger_{\mathbf{k}_i\downarrow} c_{\mathbf{k}\uparrow} S^+ \frac{1}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}_i}} c^\dagger_{\mathbf{k}'\uparrow} c_{\mathbf{k}_i\downarrow} S^- | \mathbf{k} \uparrow \rangle$$

$$= -J^2 S^+ S^- \sum_{\mathbf{k}_i} \frac{f(\epsilon_{\mathbf{k}})}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}_i}}$$

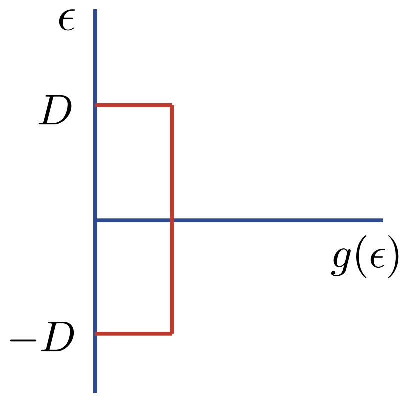
2^{nd} order processes (are we there yet?)

- Total $$\langle \mathbf{k}' \uparrow | T^{(2)} | \mathbf{k} \uparrow \rangle = -J^2 \sum_{\mathbf{k}_i} \frac{1}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}_i}} \left\{ (S^z)^2 + S^- S^+ [1 - f(\epsilon_{\mathbf{k}_i})] + S^+ S^- f(\epsilon_{\mathbf{k}_i}) \right\}$$ $$= -J^2 \sum_{\mathbf{k}_i} \frac{1}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}_i}} \left\{ S^2 - S^z [1 - 2f(\epsilon_{\mathbf{k}_i})] \right\}$$

T-dependence!

- Let's evaluate this assuming
  - constant density of states, bandwidth [-D,D], 0 K Fermi

$$\langle \mathbf{k}' \uparrow | T^{(2)} | \mathbf{k} \uparrow \rangle \approx -J^2 \int_{-D}^{D} \mathrm{d}\epsilon_{\mathbf{k}_i} g_0 \frac{1}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}_i}} \left\{ S^2 - S^z [1 - 2f(\epsilon_{\mathbf{k}_i})] \right\}$$
$$= -J^2 g_0 \left\{ \int_{-D}^{D} \mathrm{d}\epsilon_{\mathbf{k}_i} \frac{S^2 - S^z}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}_i}} + \int_{-D}^{\epsilon_F} \mathrm{d}\epsilon_{\mathbf{k}_i} \frac{2S^z}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}_i}} \right\}$$
$$\approx 2J^2 g_0 S^z \ln \left| \frac{k_B T}{D} \right|, \epsilon_{\mathbf{k}} << D, \epsilon_{\mathbf{k}} - \epsilon_F \approx k_B T$$



- 2^{nd} order processes where initial and final spins are flipped give T independent total T matrix element.

2^{nd} order processes (Kondo's eureka moment!)

- Scattering probability including 1^{st} and 2^{nd} orders

$$W(\mathbf{k}\uparrow,\mathbf{k}'\uparrow) \propto \left(J + J^2 g_0 \ln\left|\frac{k_B T}{D}\right|\right)^2 \approx J^2 + 2J^3 g_0 \ln\left|\frac{k_B T}{D}\right|$$

- Check what happens at low T

$$T \to 0, \ln\left|\frac{k_B T}{D}\right| \to -\infty$$

- So for antiferromagnetic coupling (J < 0) we have infinite resistivity!
- Electron-magnetic impurity scattering increases as you go to low T - Kondo minimum explained!
- *Is this the end of the story?*

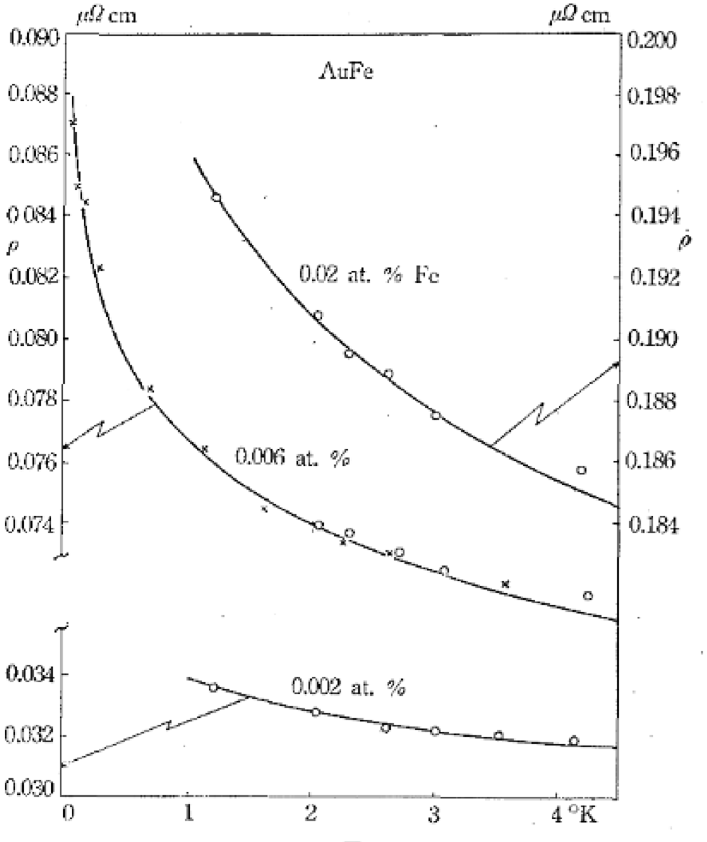


Fig. 1. Comparison of experimental and theoretical ρ–T curves for dilute AuFe alloys.

J Kondo, Progress of Theoretical Physics, Vol. 32, No. 1 (1964)

Kondo problem

$$W(\mathbf{k}\uparrow,\mathbf{k}'\uparrow) \propto \left(J + J^2 g_0 \ln\left|\frac{k_B T}{D}\right|\right)^2 \approx J^2 + 2J^3 g_0 \ln\left|\frac{k_B T}{D}\right|$$

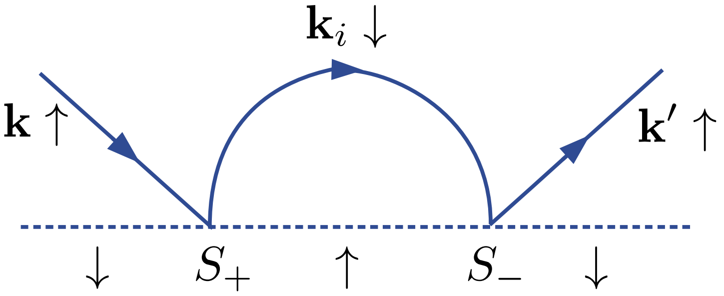
- But experimental 0 K resistivity is finite!
- 2nd order correction is not small for

$$k_B T_K = D \exp\left(-\frac{1}{2Jg_0}\right)$$

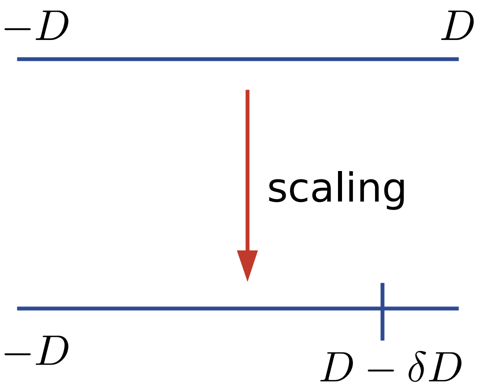
- Higher order perturbative calculation showed that resistivity is infinite at Kondo temperature, not 0 K.
- Failure of Kondo theory to produce finite results is the Kondo problem.
  - *Why should the coupling be always small?*

A Abrikosov, Physics 2, 5 (1965)

Anderson’s poor man’s scaling



- A renormalization group approach
  - Microscopic Hamiltonian → Effective low energy Hamiltonian
  - *What is the renormalized coupling at low energy?*
- Integrate out T-matrix element from [D-δD, D]



$$\int_{D-\delta D}^{D} \mathrm{d}\epsilon_{\mathbf{k}_i} \frac{g_0}{\epsilon_{\mathbf{k}} - D} = g_0 \frac{\delta D}{\epsilon_{\mathbf{k}} - D}, \epsilon_{\mathbf{k}_i} \approx D$$

$$\delta \langle \mathbf{k}' \uparrow | T^{(2)} | \mathbf{k} \uparrow \rangle = J^2 g_0 \frac{\delta D}{\epsilon_{\mathbf{k}} - D} \langle \mathbf{k}' \uparrow | c^\dagger_{\mathbf{k}'\uparrow} c_{\mathbf{k}_D \downarrow} S^- c^\dagger_{\mathbf{k}_D \downarrow} c_{\mathbf{k}\uparrow} S^+ | \mathbf{k} \uparrow \rangle$$

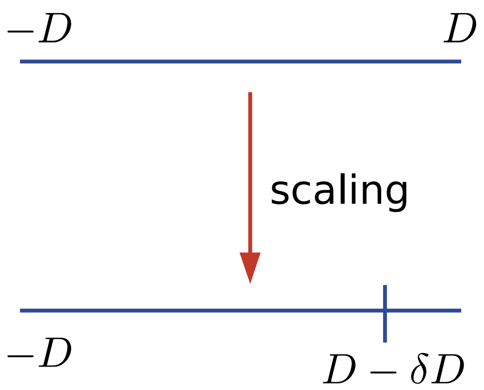
- Scaling produces a correction to coupling constant!

$$J(D - \delta D) = J(D) - 2J^2 g_0 \frac{\delta D}{D}, \epsilon_{\mathbf{k}} << D$$

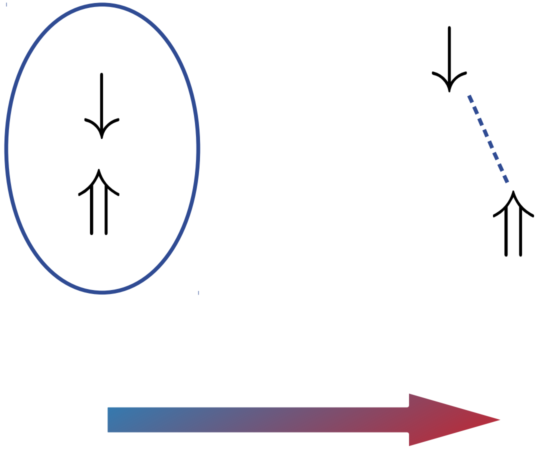
P W Anderson, Journal of Physics C: Solid State Physics, 3(12), 2436 (1970)

P Coleman, Introduction to Many-Body Physics, Cambridge (2015)

Anderson’s poor man’s scaling

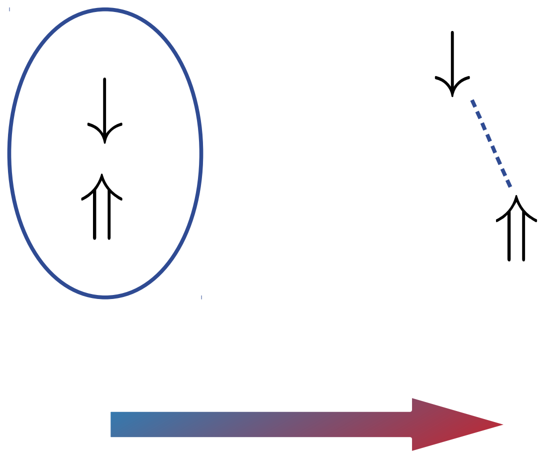


- A little algebra gives $$\frac{\partial \ln|\lambda|}{\partial(\frac{1}{D})} = -2\lambda, \lambda \equiv g_0 J$$
- For antiferromagnetic case, the right hand side is positive.
  - As D is made smaller (approach low energy), coupling gets stronger!
- Physical picture
  - At low energies (below Kondo T), antiferromagnetic coupling between an electron and a magnetic impurity increases: they form a singlet state. The impurity is screened out!
  - The rest of the conduction electrons can’t feel the impurity any more. The Kondo divergence is removed.
  - At high energies (above Kondo T), impurity sheds off screening electrons and acts as a free scattering center.
  - This is known as asymptotic freedom.



Summary

- Low T resistivity minimum observation explained by perturbative Kondo theory.
- This theory predicts infinite resistivity at low T.
- Renormalization of coupling constant shows that at low T coupling gets stronger.
- Thus, at low T, some conduction electrons form singlet states with impurities, screening them out from the view of other conduction electrons.
- At high T, coupling is weakened, screening electrons are shed off, and magnetic scattering is restored. That is, magnetic impurity moment is asymptotically free.



Other references

Book:

G Mahan, Condensed Matter in a Nutshell, Princeton (2011)

Lecture notes:

V Shenoy, http://www.physics.iisc.ernet.in/~shenoy/LectureNotes/kondo.pdf

N Andrei, https://www.cond-mat.de/events/correl15/talks/andrei.pdf

J Schalkowski, https://itp.uni-frankfurt.de/~valenti/TALKS_BACHELOR/Kondomodel.pdf

Online article:

A Hewson and J Kondo, http://www.scholarpedia.org/article/Kondo_effect