CHAPTER 8

Economic Growth
I: Capital Accumulation and Population Growth

Modified for ECON 2204 by Bob Murphy
IN THIS CHAPTER, YOU WILL LEARN:

- the closed economy Solow model
- how a country’s standard of living depends on its saving and population growth rates
- how to use the “Golden Rule” to find the optimal saving rate and capital stock
Why growth matters

- Data on infant mortality rates:
  - 20% in the poorest 1/5 of all countries
  - 0.4% in the richest 1/5

- In Pakistan, 85% of people live on less than $2/day.

- One-fourth of the poorest countries have had famines during the past 3 decades.

- Poverty is associated with oppression of women and minorities.

*Economic growth raises living standards and reduces poverty*....
Income and poverty in the world selected countries, 2010

% of population living on $2/day or less

Income per capita in U.S. dollars

Indonesia
Senegal
Zambia
Nigeria
Kyrgyz Republic
Georgia
Peru
Panama
Mexico
Uruguay
Poland

$0 $2,000 $4,000 $6,000 $8,000 $10,000 $12,000 $14,000

Income per capita in U.S. dollars
Cross Country Growth Data

Visit gapminder.org for data and graphic on various indicators of well-being including:

- Life expectancy
- Infant mortality
- Malaria deaths per 100,000
- Cell phone users per 100 people
Why growth matters

Anything that effects the long-run rate of economic growth – even by a tiny amount – will have huge effects on living standards in the long run.

<table>
<thead>
<tr>
<th>annual growth rate of income per capita</th>
<th>increase in standard of living after...</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>...25 years</td>
</tr>
<tr>
<td>2.0%</td>
<td>64.0%</td>
</tr>
<tr>
<td>2.5%</td>
<td>85.4%</td>
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</table>
Why growth matters

- If the annual growth rate of U.S. real GDP per capita had been just one-tenth of one percent higher from 2000–2010, the average person would have earned $2,782 more during the decade.
The Solow model

- due to Robert Solow, won Nobel Prize for contributions to the study of economic growth
- a major paradigm:
  - widely used in policy making
  - benchmark against which most recent growth theories are compared
- looks at the determinants of economic growth and the standard of living in the long run
How Solow model is different from Chapter 3’s model

1. $K$ is no longer fixed: investment causes it to grow, depreciation causes it to shrink

2. $L$ is no longer fixed: population growth causes it to grow

3. the consumption function is simpler
How Solow model is different from Chapter 3’s model

4. no $G$ or $T$
   (only to simplify presentation; we can still do fiscal policy experiments)

5. cosmetic differences
The production function

- In aggregate terms: $Y = F(K, L)$
- Define: $y = \frac{Y}{L} =$ output per worker
  $k = \frac{K}{L} =$ capital per worker
- Assume constant returns to scale:
  $zY = F(zK, zL)$ for any $z > 0$
- Pick $z = \frac{1}{L}$. Then
  $\frac{Y}{L} = F\left(\frac{K}{L}, 1\right)$
  $y = F(k, 1)$
  $y = f(k)$ where $f(k) = F(k, 1)$
The production function

Output per worker, $y$

Capital per worker, $k$

$f(k)$

$MPK = f(k + 1) - f(k)$

Note: this production function exhibits diminishing MPK.
The national income identity

- \( Y = C + I \) (remember, no \( G \))
- In “per worker” terms:
  \[
  y = c + i
  \]
  where \( c = \frac{C}{L} \) and \( i = \frac{I}{L} \)
The consumption function

- \( s = \) the saving rate, the fraction of income that is saved
  \( (s \) is an exogenous parameter)\)

  **Note:** \( s \) is the *only* lowercase variable that is *not equal to*
  its uppercase version divided by \( L \)

- Consumption function: \( c = (1-s)y \)
  
  *(per worker)*
Saving and investment

- Saving (per worker) \( = y - c \)
  \( = y - (1-s)y \)
  \( = sy \)

- National income identity is \( y = c + i \)

Rearrange to get: \( i = y - c = sy \)

(investment = saving, like in chap. 3!)

- Using the results above,
  \( i = sy = sf(k) \)
Output, consumption, and investment

Output per worker, $y$

Capital per worker, $k$

$f(k)$

$sf(k)$

$y_1$

$c_1$

$i_1$

$k_1$
Depreciation

Depreciation per worker, $\delta k$

$\delta = \text{the rate of depreciation}$

$= \text{the fraction of the capital stock that wears out each period}$

Diagram: A line graph with the x-axis labeled "Capital per worker, $k$" and the y-axis labeled "Depreciation per worker, $\delta k$". The line has a slope of $\delta$ and passes through the point (1, $\delta$).
Capital accumulation

*The basic idea: Investment increases the capital stock, depreciation reduces it.*

Change in capital stock \[ \Delta k \] = investment – depreciation

\[ \Delta k = i - \delta k \]

Since \( i = sf(k) \), this becomes:

\[ \Delta k = sf(k) - \delta k \]
Population growth

- Assume the population and labor force grow at rate $n$ (exogenous):

$$\frac{\Delta L}{L} = n$$

- EX: Suppose $L = 1,000$ in year 1 and the population is growing at 2% per year ($n = 0.02$).

- Then $\Delta L = nL = 0.02 \times 1,000 = 20$, so $L = 1,020$ in year 2.
Break-even investment

- \((\delta + n)k = \text{break-even investment}\), the amount of investment necessary to keep \(k\) constant.

- Break-even investment includes:
  - \(\delta k\) to replace capital as it wears out
  - \(nk\) to equip new workers with capital
    (Otherwise, \(k\) would fall as the existing capital stock is spread more thinly over a larger population of workers.)
The equation of motion for \( k \)

- With population growth, the equation of motion for \( k \) is:

\[
\Delta k = sf(k) - (\delta + n)k
\]

- Actual investment
- Break-even investment
The Solow model diagram

\[ \Delta k = sf(k) - (\delta + n)k \]

Investment, break-even investment

\(sf(k)\)

\((\delta + n)k\)

Capital per worker, \(k\)

\(k^*\)
The impact of population growth

An increase in $n$ causes an increase in break-even investment, leading to a lower steady-state level of $k$. 

Investment, break-even investment

$\frac{(\delta + n_2)k}{(\delta + n_1)k} \quad sf(k)$

Capital per worker, $k$

$k_1^*$, $k_2^*$
Prediction:

- The Solow model predicts that countries with higher population growth rates will have lower levels of capital and income per worker in the long run.

- Are the data consistent with this prediction?
International evidence on population growth and income per person

Income per person in 2010 (log scale)

Population growth (percent per year, average 1961-2010)
Prediction:

- The Solow model predicts that countries with higher rates of saving and investment will have higher levels of capital and income per worker in the long run.
- Are the data consistent with this prediction?
International evidence on investment rates and income per person

Income per person in 2014 (logarithmic scale)

Correlation = 0.71

Effective investment rate (average 1995–2014)
The Golden Rule: Introduction

- Different values of \( s \) lead to different steady states. How do we know which is the “best” steady state?
- The “best” steady state has the highest possible consumption per person: \( c^* = (1-s) f(k^*) \).
- An increase in \( s \)
  - leads to higher \( k^* \) and \( y^* \), which raises \( c^* \)
  - reduces consumption’s share of income \( (1-s) \), which lowers \( c^* \).
- So, how do we find the \( s \) and \( k^* \) that maximize \( c^* \)?
The Golden Rule with population growth

To find the Golden Rule capital stock, express \( c^* \) in terms of \( k^* \):

\[
\begin{align*}
c^* &= y^* - i^* \\
&= f(k^*) - (\delta + n) k^*
\end{align*}
\]

\( c^* \) is maximized when

\[
\text{MPK} = \delta + n
\]

or equivalently,

\[
\text{MPK} - \delta = n
\]

In the Golden Rule steady state, the marginal product of capital net of depreciation equals the population growth rate.
The transition to the Golden Rule steady state

- The economy does NOT have a tendency to move toward the Golden Rule steady state.
- Achieving the Golden Rule requires that policymakers adjust $s$.
- This adjustment leads to a new steady state with higher consumption.
- But what happens to consumption during the transition to the Golden Rule?
Starting with too much capital

If \( k^* > k_{gold} \)
then increasing \( c^* \) requires a fall in \( s \).

In the transition to the Golden Rule, consumption is higher at all points in time.
Starting with too little capital

If $k^* < k_{gold}$ then increasing $c^*$ requires an increase in $s$.

Future generations enjoy higher consumption, but the current one experiences an initial drop in consumption.