How Should a Firm Go Public? A Dynamic Model of the Choice between Fixed-Price Offerings and Auctions in IPOs and Privatizations*

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We analyze the choice between fixed-price offerings and auctions in IPOs and privatizations. We model a firm going public by selling equity in the IPO market. Firm insiders have private information about intrinsic firm value, but outsiders can produce information about this value before bidding for shares. Inducing information production is beneficial for higher intrinsic value firms, because this information, reflected in secondary market prices, yields higher equity prices. We show that auctions and fixed-price offerings have different properties for inducing information production, solve for the equilibrium IPO mechanisms for firms with different characteristics, and explain the “IPO auction” puzzle. (JEL G32, D44, D82, L26, O16)

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Introduction

The optimal mechanism for selling shares in initial public offerings (IPOs) has been widely debated. On the one hand, insights from auction theory indicate that IPO auctions are the best way to sell shares in IPOs under a variety of circumstances (see, e.g., Ausubel 2002). On the other hand, Jagannathan, Jinryi, and Sherman (2015) document that IPO...
auctions have been tried, at one point or the other, in at least 50 countries and are in use only in a few of these, having lost out to either fixed-price offerings or bookbuilding (especially the latter) in most of these countries. In particular, bookbuilding is the predominant IPO mechanism currently in use in the United States. Under bookbuilding, the IPO underwriters build an “order book” based on interactions with institutional investors during the IPO roadshow, extracting information from them about their valuation of the IPO firm (see Benveniste and Spindt (1989) for a theoretical model and Cornelli and Goldreich (2003) for an empirical analysis). However, a mechanism such as bookbuilding, which gives the underwriter considerable discretion over IPO allocations, is subject to abuse, especially in developing countries where the regulatory mechanisms governing IPOs may not be well developed.1 Therefore, several proposals to reform IPO procedures are being used in the United States and in other countries.2

To cite a recent example of the debate at the highest levels over reforming IPO offering procedures in the United State, we consider a quote from the letter sent by Darrell Issa, chairman of the House Committee on Oversight and Government Reform, to Mary Schapiro, former chairman of the SEC in June 2012 (following the IPO of Facebook): “While the Facebook IPO, underwritten by Morgan Stanley, may have generated outsized losses for ordinary investors, Google’s IPO via Dutch auction, also underwritten by Morgan Stanley, reflected free market ideals and provided ordinary investors with a unique opportunity to participate alongside institutions. The Google IPO provided a rare glimpse into a market-driven system that many would argue was fairer than what was done at Facebook.” Despite the above interest in reforming procedures for selling shares in IPOs and in using IPO auctions, in particular, there has been little success in practice in adopting IPO auctions to sell equity of firms going public in the United States, with some rare exceptions like the IPOs of Google and Morningstar. Indeed, an investment banking firm, W. R. Hambrecht & Co., was founded several years ago with the explicit objective of selling IPO firm shares using a uniform-price auction, but only a few companies chose to auction shares in their IPOs in the United States. Further, the auction method of selling IPOs, far from

1 See, for example, Ritter (2011), who discusses the CLAS controversies in bookbuilding IPOs: excessive commissions, laddering, analyst conflicts of interest, and spinning.

2 The three IPO mechanisms commonly used around the world are fixed-price offerings, bookbuilding, and IPO auctions. Of these, the two nonauction mechanisms (fixed-price offerings and bookbuilding) are currently more prevalent than are IPO auctions. In particular, fixed-price offerings are currently extensively used in many countries around the world. Examples of countries in which fixed-price offerings are (or have been) used include Australia, Belgium, Finland, Germany, Hong Kong, Italy, Japan (pre-April 1, 1989), Korea (post-June 1998), Singapore, Sweden, Switzerland, Taiwan, the United States (best-efforts contracts), and the United Kingdom (offers for sale). At one time or another, IPO auctions have been used in Belgium, Brazil, France, Israel, Japan, Korea, Portugal, Singapore, Switzerland, Taiwan, Turkey, and the United Kingdom.
gaining in popularity and replacing fixed-price offerings, has been losing market share worldwide, and is increasingly being replaced by either the fixed-price or the bookbuilding mechanism even in those countries where it was in place.\footnote{In addition to the United States, other countries have considerable interest in reforming the mechanism used for selling shares in IPOs. Mainland China is one country that has experimented with different mechanisms for selling shares in IPOs. See Chemmanur et al. (2017) and Chemmanur, Liu, and Tian (2016) for empirical analyses of the hybrid IPO auction (two-stage) mechanism used to sell shares in firms going public in China.}

That IPO auctions, while theoretically optimal in terms of maximizing proceeds from the IPO (and empirically documented in many equity markets as involving a smaller amount of underpricing), do not dominate fixed-price offerings or other non-auction mechanisms in terms of market share in various countries around the world, has been characterized as a puzzle by several authors.\footnote{For example, Jenkinson and Ljungqvist (1996, p. 40) comment: “Auction-like mechanisms such as tenders in the United Kingdom, the Netherlands, and Belgium, or offres publiques de vente in France, are generally associated with low levels of underpricing;... This is not surprising, given that, unlike fixed-price offers, tenders allow market demand to at least partially influence the issue price. What is curious, though, is that we do not observe a shift towards greater use of auctions.” Derrien and Womack (2003) make similar comments.} One of the objectives of this paper is to develop a resolution to this “IPO auction puzzle,” based on a theoretical analysis rooted in the realities of the IPO market, and characterizing those situations under which using auctions in IPOs and other equity offerings will be successful (as well as those in which they are likely to fail). We argue that there are two problems with the argument that auctions maximize the proceeds from IPOs and therefore are the optimal way of selling shares in IPOs. First, it is based on results from auction theory developed in the context of a monopolist auctioning off various goods in the product market. However, unlike the objective of a monopolist trying to maximize the proceeds from a one-time sale of various items, the objective of a firm in selling shares is not to maximize the proceeds from a one-time sale of stock. This is because companies care very much about the price of their stock in the secondary market (one reason they care about the secondary market price of their stock is that many companies wish to issue more equity 2 or 3 years after an IPO; also, if the stock price continues to languish, companies can be subject to a takeover at an undervalued price). Thus, in practice, companies face a dynamic choice: they want to obtain high proceeds from the sale of stock, but they also care about the secondary market price of their stock after the IPO.

The second problem with existing arguments about the optimality of auctions is that they take the information structure of the problem as given. In other words, in much of auction theory, the information that various bidders have about the value of the object being sold is taken as unalterable, and the focus is often on comparing auctions in terms of...
their ability to extract and aggregate the information available with outsiders into the selling price. However, in many auction situations, bidders can produce information about the true value of the object being sold at a cost. For example, when the government is auctioning off rights to drill for oil or other mineral rights, various participants can spend resources to learn more about the value of the mineral rights (by drilling a test hole in the case of oil rights). In particular, investors in the new issues market may devote time and other resources to learn more about the true value of the firm going public.\(^5\) This is important because different ways of selling various objects have different properties when it comes to inducing information production by outsiders. Here, we show that in many situations, a fixed-price offering can induce more investors to learn about the true value of a firm going public compared to an IPO share auction, with implications for the proceeds obtained by the firm and its insiders from these two mechanisms.

Combining the above two ingredients, we show that, in many cases, a company that wishes to maximize a dynamic objective function (i.e., maximize the cash flow to the firm in the long run, rather than the proceeds from a one-time offering of stock) will in fact choose a fixed-price offering rather than an IPO auction. We consider a setting in which a firm goes public by selling a fraction of its equity in an IPO market characterized by asymmetric information between firm insiders and outsiders. Outsiders, can, however, produce information at a cost before bidding for shares in the IPO. Auctioning off shares in a setting in which outsiders can learn more about the company at a cost will maximize the proceeds from a one-shot offering, but may be detrimental to the company’s long-run value, since not enough investors will choose to produce information about the company. Insiders care about getting a large number of outsiders to produce information, since this information will be reflected in the secondary market price (thereby leading to a higher secondary market price for truly higher intrinsic-value firms). Thus, in equilibrium, truly higher valued firms will prefer to sell their shares in a fixed-price offering (rather than auctioning them off) because the former is the mechanism that will maximize the long-term value of their firm. Since lower intrinsic value firms will also mimic higher intrinsic value firms by setting the same offer price, this price will be such that it induces the optimal extent of information production by outsiders.

There are two important differences between the initial offer price emerging from an IPO auction and the fixed-price offering set by a firm in an IPO. First, the price at which shares are sold in the IPO

\(^5\) Some evidence of this information production by institutional investors participating in IPO auctions can be found in Taiwan. See, for example, Chiang, Qian, and Sherman (2010), who document that returns are higher when more institutional investors enter the IPO auction, suggesting that these investors are informed investors able to generate information about intrinsic firm value.
auction is determined because of competition among various informed bidders. This means that the initial offer price in the auction will aggregate, to a significant degree, the information produced by outsiders, unlike in the case of a fixed-price offering, where the offer price is set by the firm. Second, in common value auctions (such as IPO share auctions), bidders, whose information will be correlated with the true value of the firm (and therefore with that of each other), will compete away much of the surplus from each other. Since each bidder expects to be compensated for the cost of producing information, this means that the initial offer price emerging in an IPO auction will be able to support only a smaller number of informed entrants into the auction compared to the number of investors producing information in a fixed-price offering (where the firm can set the offering price to attract the optimal extent of information production by outsiders).

The above intuition is useful in understanding many of our results. First, if a firm is very well known or otherwise faces lower levels of information asymmetry prior to the IPO (so that outsiders’ cost of information production is small), then our analysis implies that auctioning its equity is optimal, since the number of information producers even in an auction is large enough that the disadvantage of lower information production is offset by the greater price received by higher intrinsic value firms in the IPO. In contrast, if the firm is young, small, or faces a greater extent of information asymmetry for any other reason (so that outsiders’ information production costs are significant), then fixed-price offerings will be the equilibrium choice of the firm. This is because considerations of inducing information production and their impact on the secondary market price become important. Similarly, if the fraction of equity sold by the firm in the IPO is relatively large, then IPO auctions are the equilibrium choice, since, in this case, secondary market considerations are relatively unimportant to firm insiders at the time of the IPO. If, in contrast, the firm is selling only a small fraction of its equity in the IPO (as in the case of many firms going public in the United States), then fixed-price offerings are, again, the equilibrium choice, because, firm insiders place relatively less weight on maximizing the proceeds from a one-shot equity offering, and more weight on the impact of the IPO mechanism on the secondary market price of its equity.

A substantial empirical literature compares the properties of IPOs sold by auction and by fixed-price offerings in various countries (when both mechanisms are available in the same country) or across countries (see, e.g., Derrien and Womack 2003; Jacquillat 1986; MacDonald and Jacquillat 1974; Jenkinson and Mayer 1988; Chen and Yeh 2004; Hsu, Hung, and Shiu 2009; Shiu 2004). This literature documents that, in many countries, the extent of IPO underpricing is lower in IPO auctions compared to nonauction IPO mechanisms (i.e., either fixed-price
offerings or bookbuilding). Our model can explain this empirical finding. We show that it is optimal for younger, smaller, or more obscure firms, which face a significant extent of information asymmetry in the equity market, to use fixed-price offerings and underprice significantly in their IPO, since they are concerned about inducing the optimal amount of information production. In contrast, older, larger, or better-known firms, facing a smaller extent of information asymmetry, optimally use IPO auctions, and have a smaller extent of underpricing in equilibrium, since they are less concerned about inducing information production by outsiders. Given that the bulk of firms going public around the world are smaller, younger, and relatively obscure, our model can simultaneously explain the greater popularity of nonauction IPO mechanisms like fixed-price offerings relative to IPO auctions, and the greater extent of underpricing characterizing those mechanisms relative to that in IPO auctions. In addition to explaining these and other regularities documented by the empirical literature, our model also generates yet untested predictions that may serve as testable hypotheses for future empirical research.

1. Relation to the Existing Literature

Here, our approach differs from that in the bookbuilding literature in two important respects. Following the seminal paper of Benveniste and Spindt (1989), a number of papers in this literature (see, e.g., Benveniste and Wilhelm 1990; Sherman 2005) assume that outside institutional investors have information superior to the firm (and its underwriters), and demonstrate that underpricing is part of the optimal mechanism to induce truth-telling by institutional investors about their own valuation of the firm going public.6

In contrast to the above literature, we assume (standard in much of the IPO and corporate finance literature) that it is in fact firm insiders who have information superior to outsiders about their own firm’s true value (see, e.g., Allen and Faulhaber 1989; Chemmanur 1993; Grinblatt and Hwang 1989; Welch 1989; Chemmanur and Fulghieri 1999). Our view is that, while outsiders may indeed have private information about their own valuation of a certain firm going public (and therefore about their demand schedule for its equity), it is firm insiders who are most likely to have superior information about the intrinsic (long-term) value of their own firm.

The two assumptions discussed above are, however, complementary, in the sense that both kinds of information problems may exist simultaneously in practice. Thus, real-world IPOs must accomplish two different

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6 See also Spatt and Srivastava (1991), who show that a posted-price mechanism augmented by preplay communication and participation restrictions can replicate any optimal auction.
information flows: First, they must ensure that firm-specific information initially available only to insiders is also acquired (at least partially) by enough outsiders prior to them deciding whether or not to invest in the firm’s equity. This makes the IPO successful, both in the short run (in terms of enough outsiders investing in the firm’s IPO) and in the long run (in terms of achieving a high secondary market price).

Second, information regarding their valuation of the firm (and therefore their demand for the firm’s equity) needs to be credibly extracted from institutional investors and other informed outsiders, so that this information can be used for pricing equity in the IPO. The theoretical bookbuilding literature has focused exclusively on the second information flow, ignoring the first. In contrast, we focus on the first information flow, abstracting away from the second. Both nonauction mechanisms used in practice around the world, namely, fixed-price offerings and bookbuilding, accomplish the above two information flows to a greater or lesser degree, even though bookbuilding has an advantage over fixed-price offerings in terms of accomplishing the second information flow. Consequently, while the focus of this paper is on firms’ choice between fixed-price offerings and IPO auctions, our model also has some implications for the choice between bookbuilding and IPO auctions as well (to the extent that we can view bookbuilding also as a posted price mechanism, and firms using bookbuilding are also concerned about ensuring that outsiders can obtain some of the firm-specific information initially available only to insiders).

A second important difference between our approach and the literature comparing IPO auctions to bookbuilding is that, in these papers, the objective of firm insiders is simply to maximize the proceeds from a one-shot equity offering. This means that, in these papers, underpricing is a cost imposed on the firm because of the presence of informed outsiders, so that an important measure of the success of the IPO equity sales mechanism in the above setting is its ability to minimize underpricing. This has significant consequences for the ability of some of these papers to explain the IPO auction puzzle, since they make the prediction that bookbuilding will lead to greater IPO proceeds (and therefore lower underpricing) than IPO auctions (in contrast to the greater underpricing

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7 To see that real-world IPOs (whether bookbuilding or fixed-price offerings) place considerable importance on conveying firm-specific information to outsiders, one need only to look at one of the most important activities associated with IPOs, namely, the “road-show.” It is well known that an important objective of roadshows is for insiders (top corporate officers, accompanied by underwriters) to explain details about the firm to institutional investors and other outsiders, thereby making it easier (cheaper) for these outsiders to acquire firm-specific information (i.e., “understand the firm and its business”).

8 In our setting, the assumption is that insiders always have information superior to outsiders, so that we do not model the mechanism through which issuers extract outsiders’ information in order to use this information in setting the IPO offer price. Consequently, our analysis does not have any implications for the relative efficiency of bookbuilding versus IPO auctions in extracting the information held by outside investors about firm value.
documented in practice in the bookbuilding method relative to that in
IPO auctions in many equity markets around the world). Biais and
Faugeron-Crouzet (2002) and Sherman (2005) are two important papers
comparing bookbuilding with IPO auctions. While an advantage of
bookbuilding over IPO auctions in Biais and Faugeron-Crouzet (2002)
arises from possible collusion among investors in uniform price IPO
auctions. Sherman (2005) argues that bookbuilding is superior to IPO
auctions because it gives the issuer more discretion in terms of under-
pricing and allocating shares (and therefore a greater ability to provide
incentives to outsiders to collect information about their valuation of the
firm) compared to IPO auctions.9

While the insights of our paper and the above bookbuilding models are
complementary in some respects, they contrast significantly in others. In
particular, ours is the only paper to demonstrate why auctions have lost
market share to even fixed-price offerings around the world, even though
auction theorists and other academics have argued that the latter method
is clearly inferior. One example of the many contrasting implications
generated by our model and Sherman (2005) is provided by the IPO
auction adopted by the search engine firm Google Inc. The fact that
Google adopted an IPO auction to go public is consistent with the impli-
cations of our model, given its profitability and business model (about
which it was relatively cheap for outsiders to produce information).
These characteristics of Google at the time of its IPO are in sharp con-
trast to the average private firm that goes public in the United States:
such a firm is typically a few years away from profitability, small, and
obscure (with higher costs of information production for outsiders), so
that our model predicts that a fixed-price offering is more appropriate for
such a firm. In contrast, Sherman (2005) predicts that bookbuilding is
always superior to IPO auctions regardless of firm characteristics (since it
gives the issuer greater discretion in underpricing and allocating shares).
See our empirical implication (i) for more details.10

9 Biais and Faugeron-Crouzet (2002) compare fixed-price offerings, market-clearing uniform-price auc-
tions, and the mise en vente (a procedure similar to bookbuilding used in France) in a setting in which
outsiders have private information about their demand for the firm’s shares and the objective of the firm
is to maximize IPO proceeds. In an analysis along the lines of Wilson (1979) and Back and Zender
(1993), they argue that uniform-price auctions may not be optimal for selling shares if auction partic-
ipants are asked to submit their entire demand functions, since bidders can tacitly collude by submitting
demand functions such that the clearing price is very low. In contrast, in the mise en vente, the price
underreacts to demand and thereby unravels tacit collusion on low prices. See also Biais, Bosserats, and
Rochet (2002), who argue that uniform-price offerings may indeed be optimal if the underwriter has
private information about the demand for IPO shares, institutional investors have private information
about share value, and the underwriter and institutional investors can collude.

10 That IPO auctions initially lost market share to fixed-price offerings rather than to bookbuilding in a
majority of countries indicates that the underlying economics of the IPO auction puzzle has more to do
with the superiority of posted price mechanisms in general over IPO auctions for the bulk of firms going
public (rather than any special advantages of the bookbuilding mechanism). Therefore, we have confined
ourselves to an analysis of a firm’s choice between fixed-price offerings and IPO auctions here.
Ours is also the only paper that provides systematic guidance to empirical researchers around the world for comparing fixed-price offerings and IPO auctions. We account for issuers self-selecting into one IPO mechanism from another. Since, in our setting, the insiders’ goal in pricing equity in the IPO is to maximize their dynamic objective function, and minimizing underpricing is not the objective, firms may adopt fixed-price offerings even when they involve greater underpricing. Further, while in the literature comparing bookbuilding to IPO auctions, bookbuilding usually emerges as the optimal IPO mechanism, in our setting either fixed-price offerings or auctions may emerge as the optimal IPO mechanism (depending on the specific characteristics of the firm going public). Finally, our paper also has unique implications for the relationship between the IPO mechanism used and characteristics of the aftermarket, since we explicitly model the relationship between the IPO offer price and the secondary market price.11

2. The Basic Model

There are three dates in this model. At time 0, a private firm goes public by selling a proportion \( x \in (0, 1) \) of its equity in an IPO, using either a fixed-price offering or an auction.12 The transaction cost to the firm of going public is \( T \): this transaction cost is incurred even if the firm attempts to go public but fails. Outside investors then decide whether to produce information about the value of the issuing firm, and whether to bid in the firm’s IPO. At time 1, the issuing firm’s stock is traded in the secondary market. The firm sells the remaining fraction \( 1 - x \) of its equity to outsiders in a seasoned equity offering at the price prevailing in the secondary market.13 At time 2, cash flows are realized and distributed to shareholders.

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11 In a paper after ours, Habib and Ziegler (2007) compare posted price mechanisms and auctions in the context of selling bonds and assume that outside investors have more information than does the bond issuer. The motivation behind their paper for making use of posted price offerings is to dissuade outsiders from producing information by offering a discount. Further, like Sherman (2005), but unlike ours, theirs is a single-period model, so that they do not model how information is reflected in the price of securities in the secondary market.

12 We do not assume a particular motivation for the firm to go public. Thus, the objective of the IPO may either be for the firm to raise new capital for future investment, or for the entrepreneur to diversify his equity holdings in the firm. For models of the timing and motivation of a firm’s going public decision, see Chemmanur and Fulghieri (1999) and Boot, Gopal, and Thakor (2006).

13 Even in the absence of a seasoned equity offering, our results go through qualitatively as long as firm insiders place some weight on the secondary market price in their objective function, which seems to be the case in practice. Instead of assuming an SEO and that the firm sets its offer price to maximize the combined proceeds, a qualitatively similar objective (suggested by an anonymous referee) would be to assume that insiders have much of their personal wealth invested in the firm’s equity even after the IPO, and the insiders’ objective in setting the IPO offer price is to maximize a weighted average of the IPO proceeds and the secondary market price, net of any transaction cost of attempting to go public.
2.1 The issuing firm’s private information and IPO mechanisms

The issuing firm, which is risk neutral, may be either good (type $G$) or bad (type $B$). The present value of cash flows of a good firm is $v = v_G$, and that of a bad firm is $v = v_B$, where $v_G > v_B \geq 0$. While the issuing firm knows its own type, outside investors observe only the prior probability $\theta$ of a firm being of type $G$. The equity offered in the IPO is divided into $k$ shares. We assume that each investor in the IPO is allowed to bid for a maximum of one share, and the fixed cost of going public, $T$, is less than or equal to $v_B$.

The issuing firm can choose one of the following two IPO mechanisms: a fixed-price offering or an auction. If the issuing firm chooses a fixed-price offering, it sets an offering price $F$ per share prior to bidding by investors, and all buyers pay this price. If the total demand is higher than $k$ shares in the fixed-price offering, there will be rationing of shares, and the $k$ shares will be allocated to bidders randomly, with each bidder having an equal probability of being allocated one share. In the case when the total demand is less than $k$ shares, the IPO fails.

In the case of an IPO auction, there will be no posted price prior to bidding by investors. We do not assume any specific price setting rule for the IPO auction. Instead, we define an IPO auction as any selling mechanism where the $k$ highest bidders will each win a share, and the price paid by each investor is a pre-specified function of his own bid and that of other bidders. We refer to this selling mechanism as a “general” IPO auction: the uniform-price IPO auction (which is the most widely used form of IPO auction in the United States and many other countries), and the discriminatory price IPO auction (used mainly in Japan), are special cases of this general IPO auction. While, throughout this paper, we will derive results for a firm’s choice between fixed-price offerings and general IPO auctions, we will briefly make use of the special case of a uniform price IPO auction as an illustrative example (in Section 3.2.1) and in

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14 Since investors are risk-neutral, if they are allowed to buy at most one share, they will bid for either one share or nothing. So this assumption is equivalent to assuming that every participant is allowed to bid for either one share or nothing. The assumption that each investor can bid for at most one share in the firm is made only for analytical tractability. This assumption is, however, not unduly restrictive, since the number of shares that the equity offered is split up into, $k$, is fully under the control of the firm. We can think of $\alpha/k$ as reflecting the wealth constraint of a typical investor in the IPO market. Thus, when $\alpha$ is large and $k$ is small enough, the value of each share can translate into a significant amount invested by each bidder in the firm’s IPO. On the other hand, when the fraction of equity, $\alpha$, is large, the firm can increase $k$ such that the value of one share ($\alpha/k$) in the IPO remains affordable to each investor in the IPO market.

15 Our results are unchanged if we assume that in the event that the demand for shares exceeds supply in a fixed-price offering, all investors will be allocated fractions of shares on a pro rata basis.

16 We will see later that, in the basic model, the IPO of neither firm type fails in equilibrium when the firm chooses an IPO auction. Even when the firm chooses a fixed-price offering, the type $G$ firm’s IPO never fails in equilibrium; only a type B firm’s IPO has a positive probability of failure.
developing graphical illustrations and numerical examples of our results.

The objective of the issuing firm is to maximize the combined proceeds from the sale of equity in the IPO and in the seasoned equity offering. The issuing firm’s choice of IPO mechanism will affect the amount of information production about the firm, which will in turn determine the secondary market price (and therefore the price at which equity can be sold in the seasoned equity offering). In this sense, the IPO mechanism determines both the proceeds from the IPO and that from the seasoned equity offering. Hence the issuing firm will choose the IPO mechanism (and the offering price in the case of a fixed-price offering) which maximizes its combined proceeds.

2.2 The investors’ information production technology

There are a large number of risk-neutral investors in the market, who do not know the true type of the firm, but have a prior belief that the firm is of type $G$ with probability $\theta$, and of type $B$ with the complementary probability, that is,

$$Pr(v = v_G) = \theta, \quad Pr(v = v_B) = 1 - \theta. \quad (1)$$

In addition to the equity of the issuing firm, there is a risk-free asset in the economy, the net return on which is normalized to 0.

After the issuing firm announces the IPO mechanism (general IPO auction versus fixed-price offering, and the offering price in the latter case), outside investors decide whether or not to produce information about the issuing firm before bidding.\(^{17}\) If an investor chooses to produce information about the issuing firm, he has to pay an information production cost $C$, and will receive a signal about the type of the issuing firm. We assume that each information producer receives a signal, which can be high ($H$), medium ($M$), or low ($L$), with the following probabilities:

$$\text{Prob}(S_i = H|v = v_G) = \text{Prob}(S_i = L|v = v_B) = p;$$
$$\text{Prob}(S_i = M|v = v_G) = \text{Prob}(S_i = M|v = v_B) = 1 - p. \quad (2)$$

\(^{17}\) We implicitly assume here that, both in the IPO auction and in fixed-price offerings, the expected and realized number of information producers is the same (although, in both methods, there may be some random variation in practice because of a lack of coordination among investors in deciding whether or not to produce information). However, the above assumption can be relaxed (at the expense of making the model more complex) by assuming that investors follow a randomized strategy in deciding whether to produce information, with each of the $W$ investors (say) in the IPO market producing information with an equilibrium probability $n / W$, with this probability determined such that the expected payoff from information production is zero. The value of $n$ given by (3) (for IPO auctions) and (10) (for fixed-price offerings) will then denote the expected number of information producers corresponding to all investors obtaining zero expected payoff from information production. All our results will hold in this case as well, as long as the actual number of information producers becomes known before trading commences in the secondary market (Milgrom (1981) adopts a somewhat similar approach to model information acquisition in an auction setting.).
where \( p \in (0, 1) \) is the probability that a signal reveals the true value of the issuing firm;\(^{18}\) note that the probability of receiving a low signal for a type G firm or a high signal for a type B firm is zero. The signals received by different information producers are independent of each other, conditional on the true value of the firm. After receiving the above private signal, each information producer decides whether to bid for one share or not (in the case of a fixed-price offering) or how much to bid (in the case of an IPO auction), using Bayes’ rule where appropriate. Note that, even if an investor decides not to produce information, he may still decide to bid in the IPO if it is optimal for him to do so. We will analyze the investors’ information production decision in detail when we characterize the equilibrium of our model.

3. Market Equilibrium

*Definition of equilibrium:* The equilibrium concept we use in this paper is the perfect Bayesian equilibrium (PBE). Specifically, an equilibrium consists of (1) a choice of IPO mechanism by the issuing firm at time 0 (between fixed-price offering and IPO auction), and the offering price in the case of a fixed-price offering; (2) a system of beliefs formed by investors about the type of the issuing firm after observing the issuer’s IPO mechanism choice; (3) a choice made by each investor regarding whether or not to produce information after seeing the choice of the issuing firm in the IPO; (4) a decision regarding whether or not to bid for one share (in the case of a fixed-price offering) and how much to bid for each share (in the case of an IPO auction) made by each investor; and (5) a price at which the stock of the issuing firm is traded in the secondary market. The above set of prices, choices, and beliefs must be such that (a) the choice of each party maximizes his objective, given the choices and beliefs of others and the expected secondary market price; (b) the beliefs of all parties are consistent with the equilibrium choices of others; further, along the equilibrium path, these beliefs are formed using Bayes’ rule; (c) the number of investors producing information is such that all information producers obtain a zero expected payoff from information production;\(^{19}\) (d) the secondary market price of the firm’s shares is such that no investor can profit from trading after observing the price; and (e) any deviation from his equilibrium strategy by any party is met by

\(^{18}\) The assumption that a signal of \( H \) or \( L \) reveals the true value of the firm is for tractability. Our results go through qualitatively unchanged even if the signals reveal true firm value only imperfectly.

\(^{19}\) In reality, the number of information producers has to be an integer and information producers may have a small positive expected payoff. For tractability, we ignore this integer problem and assume that the expected payoff from producing information exactly equals the cost of doing so (so that (3) and (10) hold as equalities).
beliefs by other parties which yield the deviating party a lower payoff compared to that obtained in equilibrium.20

To facilitate exposition, we present the equilibrium in a reverse order: we first characterize the equilibrium in the secondary market before going on to characterize the equilibrium in the IPO market. We first describe the equilibrium in the IPO auction, then the equilibrium when a fixed-price offering is used in the IPO, and, finally, a firm’s choice between the two IPO mechanisms.

3.1 Equilibrium in the secondary market

We assume that there is no restriction on how many shares an investor can buy or short in the secondary market. Note that the equilibrium secondary market price depends on the actual IPO mechanism (fixed-price offering versus IPO auction) only to the extent that this affects the number of information producers about the firm. In other words, if the number of information producers is the same under the two IPO mechanisms, the expected secondary market price will be the same. At time 0, when the firm chooses between a fixed-price offering and an IPO auction, the actual realization of the signals obtained by outsiders is not known to insiders. Therefore, it is the expected secondary market price that enters the insiders’ objective function. Proposition 1 characterizes the expected equilibrium secondary market price as a function of the number of information producers, \( n \).

Proposition 1. (Equilibrium price in the secondary market) Let the secondary market price reflect all information produced by outsiders in the IPO. Then (i) The expected secondary market price of a type G firm is 
\[
\frac{1}{1-C_0 \theta} \left( \frac{1}{1-C_0 p} \right)^n (v_G - v_B) + v_B,
\]
which increases with the number of information producers, \( n \). (ii) The expected secondary market price of a type B firm is 
\[
\theta (1-p)^n (v_G - v_B) + v_B,
\]
which decreases with the number of information producers, \( n \).

Part (i) of Proposition 1 above characterizes the expected secondary market price for the type G firm, and shows that it is increasing with the number of information producers in the IPO. Part (ii) characterizes the expected secondary market price for the type B firm, and shows that it is decreasing with the number of information producers in the IPO. Intuitively, the secondary market price will reflect all the information.

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20 Given the rich strategy space for firms and investors in our model, we can think of three broad categories of equilibria that may exist (depending on parameter values): (1) Separating equilibria, where the type G and type B firms behave differently in equilibrium, thus revealing their types; (2) Pooling equilibria, where the type B firm always attempts to mimic the equilibrium choice made by the type G firm; and (3) partial pooling equilibria, where the type B firm mimics the type G firm with some probability, while separating with the remaining probability. However, for a wide range of model parameter values (which we study here), it can be shown that neither separating nor partial pooling equilibria exist in our setting. Therefore, we focus only on pooling equilibria, where the type B firm always pools with the type G.
obtained by participants in the IPO. Since there is no limit on how many shares an investor could buy or short in the secondary market, and investors are risk neutral, if an investor finds that the secondary market price is inconsistent with the signal he receives, he will keep trading until the private information is reflected in the price. Given the information production technology of investors, the information (private signals) held by information producers could be one of the following three cases: (a) at least one signal is $H$; (b) at least one signal is $L$; (c) all signals are $M$. In case (a), the secondary market price must be $v_G$ (for the whole firm). Since at least one information producer observes a signal $H$ in the IPO, he knows that the firm is of type $G$. If the secondary market price is less than $v_G$, he has an incentive to demand more shares and drive the price up. Similarly, in case (b) the secondary market price will be $v_B$, since, otherwise, there is an incentive for the investor who observes $L$ to short shares. In case (c), when all information producers receive a signal of $M$, the secondary market price will reflect this information and equal $\theta v_G + (1 - \theta) v_B$.

Since the price system here is fully revealing (i.e., the secondary market price incorporates all the information produced by outsiders), outsiders do not have the incentive to engage in information production once trading begins in the secondary market at time 1. This is because, while the costs of information production are privately incurred, the benefits no longer accrue to individual outsiders. To illustrate, consider the case in which the secondary market price is $\theta v_G + (1 - \theta) v_B$ (i.e., like in case (c) discussed above). Suppose an investor incurs the information production cost $C$ at time 1, and obtains a signal $H$. To profit from this information, he has to buy equity at this time. However, no other investor will be willing to sell him any shares at a price $\theta v_G + (1 - \theta) v_B$, since investors can infer his information from his demand function. A symmetric argument applies if the investor has a signal $L$. Thus, no investor has the incentive to produce information in the secondary market. The equilibrium in the secondary market in our model is thus very similar to that prevailing in Grossman and Stiglitz (1980), in the sense that no investor must produce information once the equity in the IPO firm starts trading in the secondary market.\(^{22,23}\)

\(^{21}\) If the secondary market price is $v_G$ or $v_B$, the true type of the firm is already revealed, so that there is no incentive for further information production by outsiders.

\(^{22}\) In practice, the price system may be only partially revealing (perhaps due to additional uncertainty in the economy not modeled here). The equilibrium in the secondary market may then be a noisy rational expectations equilibrium: in the spirit of Grossman and Stiglitz (1980) model with noise in the supply of the risky security (see page 398 of their paper). The intuition behind our model holds even in this case, since we merely require that outsiders’ incentives to produce information diminish after the start of trading in the equity in the secondary market.

\(^{23}\) Consistent with this, there is considerable evidence that a large majority of small firms attract very little analyst coverage subsequent to their IPOs (see Cliff and Denis 2004). Further, Rajan and Servaes (1997) and Chen and Ritter (2000) document that the extent of analyst coverage following the IPO is increasing in IPO underpricing.
3.2 Equilibrium in the IPO market: The case in which the issuing firm chooses a general IPO auction

In this section, we first analyze the case in which the firm uses a general IPO auction to sell shares in its IPO. We then go on to study the situation where the firm uses a specific form of the general IPO auction, namely, a uniform-price IPO auction (Section 4.2.1).

If the firm chooses a general auction for its IPO, the issuing firm does not need to set a price: the prices winners pay are determined by the bids submitted by investors. Each investor decides whether or not to produce information about the issuing firm based on his probability assessment (given firm strategies) of the firm being of type $G$, the information production cost $C$, and other IPO parameters (this probability is equal to $\theta$ in the pooling equilibrium we analyze here, where both types of firms attempt to go public, so that firm insiders’ choices do not convey any information to outsiders). If he chooses to produce information, each investor observes a private signal and bids according to it. If an investor decides not to produce information, he will decide whether or not to bid in the IPO auction, and if he decides to bid, how much to bid. We will focus here on the case in which the type $G$ firm and the type $B$ firm pool together by choosing the same offering mechanism in the IPO (we will show this to be the case in equilibrium in Section 4.4).

**Lemma 1.** Suppose the information production cost $C$ is not too large so that there are $n_a \geq k + 1$ informed bidders in the general IPO auction. The expected payoff to each uninformed bidder is 0, if there is more than one uninformed bidder.

The intuition behind the above lemma is that uninformed bidders engage in Bertrand undercutting, and will raise their bid until each uninformed bidder earns a zero profit. The equilibrium bid is driven by the following trade off: raising one’s bid will increase the chance of winning, but also increase the price one will pay while he wins. Since all the uninformed investors will place the same bid, an investor who raises his bid by a small amount can win one whole share instead of being rationed with others. In other words, if the payoff is positive, the benefit of raising the bid is constant and independent of the increase in the amount of the bid, while the cost depends on the magnitude of the increase and is negligible when the increase is very small. Therefore, uninformed investors will increase their bid until the expected payoff of each uninformed bidder is zero.

We now study how investors decide whether or not to produce information. Further, while we will not explicitly characterize the bidding behavior of informed investors (since this will depend on the specific form of the auction used), it is easy to see that the bidding strategy of an informed investor will depend on the realization of his information.
signal (otherwise there is no benefit to his engaging in costly information production, so that he will not produce information in the first place). Define by \( E[\pi^a(n_a, \alpha)] \) the expected payoff (before cost of information production) to each information producer in a general IPO auction, when there are a total of \( n_a \) information producers and the firm is selling a fraction \( \alpha \) of its equity in its IPO. Intuitively, it is easy to see that, under most auction mechanisms, as the number of information producers increases, the potential number of bidders increases, so that the expected payoff from winning in the auction decreases (since each bidder’s probability of winning a share decreases as the number of bidders increases). Further, since the value of the equity sold in the IPO is higher as \( \alpha \) increases, the potential profit from winning shares in the IPO auction is also increasing in \( \alpha \). We now formally assume that the general IPO auction we analyze satisfies these two properties (we will show in Section 4.2.1 that these properties indeed hold for uniform-price auctions).

**Assumption 1.** \( E[\pi^a(n_a, \alpha)] \) decreases in \( n_a \) and increases in \( \alpha \), that is, \( \frac{\partial E[\pi^a(n_a, \alpha)]}{\partial n_a} < 0 \) and \( \frac{\partial E[\pi^a(n_a, \alpha)]}{\partial \alpha} > 0 \).

**Proposition 2. (Equilibrium in a general IPO auction):** In an equilibrium where the type \( G \) and type \( B \) firms pool by choosing a general IPO auction: (i) The equilibrium number of information producers in a general IPO auction, \( n_a \), is determined by the following equation:

\[
E[\pi^a(n_a, \alpha)] = C.
\]

(ii) The equilibrium number of information producers decreases in the information production cost, \( C \), and increases in the fraction of equity sold in the IPO, \( \alpha \) (i.e., \( \frac{\partial n_a}{\partial C} \leq 0 \) and \( \frac{\partial n_a}{\partial \alpha} \geq 0 \)); (iii) Both informed and uninformed bidding may exist in equilibrium.

Since the expected payoff from information production decreases with the number of information producers, and there is free entry to information production, the equilibrium number of information producers in an IPO auction is such that all information producers break-even. When the outsiders’ information production cost is large, it takes only a smaller number of information producers to drive the expected payoff (net of cost) to zero, so that the equilibrium number of information producers decreases with the outsiders’ information production cost (and increases with the fraction of equity sold in the IPO). Finally, uninformed bidding may co-exist with informed bidding in equilibrium in an IPO auction.

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24 Bertrand undercutting, similar to that in uninformed bidding, does not occur in informed bidding. Since an informed bidder bids based on the realization of his information signal, if such a bidder raises his bid by a small amount, both the probability of his winning one share and the price he will pay when he wins will only increase by a small amount, and both depend on the magnitude of the increase in his bid.
(unlike in a fixed-price offering, as we will show later). This is because, in addition to choosing between bidding and not bidding (as in a fixed-price offering), investors in an IPO auction can also choose the amount to bid for a share of stock in the IPO (if they choose to bid). Thus, an investor who chooses to bid uninformed can bid in such a way as to break even in equilibrium in an IPO auction even in the face of the adverse selection generated by the presence of informed investors: they bid lower than informed investors (on average), since they are aware of their informational disadvantage relative to informed investors. This, in turn, means that the presence (or otherwise) of uninformed bidders does not affect the expected payoff of informed bidders in an IPO auction, so that the qualitative nature of the equilibrium in an IPO auction is unaffected by these bidders.  

Denote by $E[IR^G(n)]$ the IPO proceeds to the type $G$ firm in a general IPO auction when there are a total of $n_a$ information producers. We now assume that the general IPO auction adopted by the firm satisfies the following property:

**Assumption 2.** $\frac{\partial E[IR^G(n_a)]}{\partial n_a} > 0$, and $E[IR^G(n_a)] \to \nu_G$ as $n_a \to \infty$.

The above assumption states first that the IPO proceeds to the type $G$ firm in an auction increases with the number of information producers. This assumption comes from the intuition that when there are more information producers, the information produced by them collectively will become more precise. The offer price in the auction will partly reflect this information. For a type $G$ firm, this means that the IPO proceeds become higher, and closer to the true firm value. Further, when the number of information producers is sufficiently large, the auction price will become fully revealing, and the IPO proceeds to the type $G$ firm will be close to the true firm value in the IPO, which is $\nu_G$. We will show in Section 4.2.1 that this property indeed holds for uniform-price auctions. We will make use of this assumption when we compare general IPO auctions to fixed-price offerings.

**Proposition 3. (Underpricing in a general IPO auction):** There is underpricing on average if the issuing firm uses a general IPO auction. The average percentage underpricing is given by $\frac{n_aC}{\theta v_G + (1-\theta)v_B} - \frac{n_aC}{\theta v_G + (1-\theta)v_B}$, where $n_a$ is the equilibrium number of information producers in the IPO auction.

In equilibrium, the aggregate information production cost of investors is $n_aC$, which must be borne by the issuing firm through underpricing.

25 The equilibrium here is thus somewhat reminiscent of that in Wilson’s (1969) drainage tract model where both informed and uninformed bidders bid for a tract of oil-producing land. See also Engelbrecht-Wiggans, Milgrom, and Weber (1983).
The expected secondary market value of the equity sold in the IPO is 

\[
\frac{[\theta v_G + (1 - \theta)v_B]x - n_a C}{n_v + (1 - \theta)v_B x - n_a C},
\]

so that the percentage underpricing is 

\[
naC
\]

3.2.1 A revealing example: The case in which the issuing firm chooses a uniform-price IPO auction. In this section, we study a special case of the general IPO auction, namely, a uniform-price IPO auction. In practice, uniform-price IPO auctions have been one of the most widely used IPO auctions around the world. For example, in the United States, both the IPO auctions held by W. R. Hambrecht and the recent IPO auction of the search engine firm Google used a uniform-price IPO auction. In addition to the fact that their wide use makes uniform-price IPO auctions of intrinsic interest, our objective in this section is to examine some of the details of an IPO auction (which require us to specify a particular auction mechanism), and to demonstrate that the two properties we assumed earlier for a general IPO auction (assumptions 1 and 2) are automatically satisfied by a uniform-price IPO auction. To further simplify our analysis in this section, we normalize the true value of a type \(G\) firm to 1 and that of a type \(B\) firm to 0 (i.e., we set \(v_G = 1\) and \(v_B = 0\)).

In a uniform-price IPO auction, the shares are allocated as follows. Investors simultaneously submit sealed bids for shares. The \(k\) highest bidders are each allocated one share, and pay a uniform price, which is the \((k + 1)\)th highest bid (we will refer to this price as the clearing price). If there is a tie, so that there are more than \(k\) bidders above the clearing price, all investors bidding strictly above the clearing price are allocated one share with probability 1, with the remaining shares allocated with equal probability to those who bid at the clearing price. For example, suppose there are two shares for auction, and four bidders, who bid 0.9, 0.8, 0.8, 0.7, respectively. Then the offering price will be 0.8, with the bidder who bids 0.9 being allocated one share, and the two bidders who bid 0.8 having a 50\% chance of being allocated one share.

In the case in which the firm chooses to auction its shares in the IPO, the issuing firm does not need to set a price: the offering price is determined by the bids submitted by investors. Each outsider decides whether or not to produce information based on his prior probability \(\theta\) of the firm being of type \(G\), and if he does, he observes a private signal through the information production technology discussed before, and bids according to the realization of this signal. Suppose the information production cost is not too high so that there are \(n \geq k + 1\) information producers. The equilibrium bidding strategy of informed bidders is as follows (the proof is provided at the beginning of the appendix). Each bidder bids \(\frac{x}{2}\) if he observes \(H\), 0 if he observes \(L\), and a random withdrawal \(b\) from the
interval \((0, \bar{z}]\) with cdf \(M(b; n)\) if he observes \(M\), where \(M(b; n)\) is characterized by the following equation:
\[
\theta(\frac{\bar{z}}{k} - b)[p + (1 - p)(1 - M(b; n))]^{k-1}[(1 - p)M(b; n)]^{n-k-1} = (1 - \theta)b[(1 - p)(1 - M(b; n))]^{k-1}[p + (1 - p)M(b; n)]^{n-k-1}.
\] (4)

Furthermore, the equilibrium number of information producers, \(n\), is characterized by:
\[
\theta p \int_0^{\bar{z}/k} \left(\frac{\bar{z}}{k} - x\right) \binom{n-1}{1} \binom{n-2}{k-1} \times [p + (1 - p)(1 - M(x))]^{k-1}[(1 - p)M(x)]^{n-k-1} dx = C, \tag{5}
\]
where \(m(x)\) is the probability density function associated with \(M(x; n)\), that is, \(m(x) = \frac{dM(x; n)}{dx}\), and \(\binom{n-1}{1}\), \(\binom{n-2}{k-1}\) are binomial probabilities.

Suppose there are \(m \geq k + 1\) uninformed investors bidding in the uniform-price auction. Each uninformed investor will earn zero expected profit, and will bid a value \(b^u\), given by
\[
b^u = \frac{A_1 \bar{z}}{A_1 + A_2 k}, \tag{6}
\]
where
\[
A_1 = \theta \sum_{j=0}^{k-1} \binom{n}{j} [p + (1 - p)(1 - M(b^u))]^j[(1 - p)M(b^u)]^{n-j} \tag{7}
\times \frac{k-j}{m}; \quad \text{and}
\]
\[
A_2 = (1 - \theta) \sum_{j=0}^{k-1} \binom{n}{j} [(1 - p)(1 - M(b^u))]^j[p + (1 - p)M(b^u)]^{n-j} \frac{k-j}{m}.
\] (8)

We now show that the two properties we assumed for general IPO auctions (assumptions 1 and 2, respectively) hold for uniform-price IPO auctions:

**Lemma 2.** (i) Uniform-price IPO auctions satisfy Assumption 1. That is, the expected gross payoff to each information producer, \(E[\pi^u(n, \alpha)]\), is decreasing in total number of information producers, \(n\), and increasing in \(\alpha\), that is, \(\frac{\partial E[\pi^u(n, \alpha)]}{\partial n} < 0\) and \(\frac{\partial E[\pi^u(n, \alpha)]}{\partial \alpha} > 0\).
(ii) Uniform-price IPO auctions satisfy Assumption 2. That is, the IPO proceeds to the type G firm is increasing in the number of information producers, and converges to \( a v_G \) when the number of information producers is sufficiently large, that is, \( \frac{\partial E[I_{RG}(n_a)]}{\partial n_a} > 0 \), and \( E[I_{RG}(n_a)] \to a v_G \) as \( n_a \to \infty \).

The offer price in a uniform-price IPO auction is less informative than the secondary market price. This is because in the IPO auction, investors are not able to fully exploit their information signal, given that each investor can place a bid in the auction only once (in other words, no bid revision is allowed after observing the clearing price set in the auction). Thus, while there is some aggregation of information in the IPO auction, this information aggregation is incomplete, in the sense that the clearing price in IPO auction does not fully reflect the information available with all investors. In contrast, the secondary market price fully aggregates all the information available with outsiders.

### 3.3 Equilibrium in the IPO market: The case in which the issuing firm chooses a fixed-price offering

We now analyze the situation in which the type \( G \) firm goes public using a fixed-price offering, and the type \( B \) firm mimics the type \( G \) by choosing the same offering mechanism (we will demonstrate later that this is indeed what happens in equilibrium). In this case, the type \( G \) firm will set the IPO offer price \( F \) to maximize its combined proceeds from the IPO and the seasoned equity offering, accounting for the effect of the number of information producers in the IPO on the expected secondary market (seasoned equity offering) price. The type \( G \) firm faces a trade-off when setting the optimal offer price \( F \). On the one hand, a higher offer price means greater proceeds from the IPO; on the other hand, a higher offer price means less information producers in the IPO and hence lower proceeds from the seasoned equity offering. The optimal offer price emerges from the above trade-off.

The objective of the type \( G \) firm is

\[
\text{Max}_F kF + (1 - \alpha) \{(1 - (1 - \theta)(1 - p)^{n_f})(v_G - v_B) + v_B\} - T, \quad (9)
\]

s.t. \( E[\pi'(F, n_f, \alpha)] - C = 0 \),

where \( E[\pi'(F, n_f, \alpha)] \) is the expected payoff to each investor from producing information in a fixed-price offering, given an offer price \( F \), when there are \( (n_f - 1) \) other investors producing information.

When both types of firms set the same IPO offer price, the offer price does not convey any information about firm type. Outside investors make one of the following three decisions, based on their prior valuation
of the firm: (1) engage in uninformed bidding, (2) produce information about the firm and then decide whether or not to bid, or (3) ignore the IPO and invest in the risk-free asset. The outsiders’ prior valuation of a share is
\[ q \left( \theta v_G + (1 - \theta) v_B \right). \]
The value of each share is either \( q v_G \) or \( q v_B \), depending on the firm type. Therefore, for \( F \leq \frac{q}{k} v_B \), there is no need for information production, and all investors will bid for one share without producing information. However, for an offer price such that \( \frac{q}{k} v_B < F < \frac{q}{k} \left( \theta v_G + (1 - \theta) v_B \right) \), the outsiders’ choice between informed and uninformed bidding will depend on the cost and precision of the information available to them. We can show that there exists a certain cutoff value of the offer price, \( F \) (which depends on the cost and precision of information available to outsiders), above which the expected payoff from informed bidding is strictly greater than that from uninformed bidding (since in this range of the offer price, informed bidders will break even, while the expected payoff to uninformed bidders is negative). Similarly, we can also show that there exists another cutoff value, \( \bar{F} \), of the offer price, above which neither informed nor uninformed bidding is optimal, so that investors ignore the IPO and invest in the risk-free asset.

**Lemma 3.** Generically, there will be either uninformed bidding or informed bidding in a fixed-price offering, but the two do not coexist.

For a given offer price (and depending on the cost and precision of information available to outsiders), outsiders will find that either uninformed bidding or informed bidding is optimal: Unlike in an IPO auction, the two kinds of bidding cannot coexist in a fixed-price offering. This is because, unlike in an IPO auction, where investors make a two-dimensional bidding decision (whether or not to bid for a share and the amount to bid), in a fixed-price offering investors make a one-dimensional bidding decision (whether or not to bid for a share at the given offer price \( F \)). This means that, in the presence of informed bidders, the payoff to uninformed bidders will be (generically) nonzero. Recall that, similar to their payoff in an IPO auction, the number of informed investors in a fixed-price offering also will be such that all information producers break even, net of information production costs (since there is free entry to information production). In contrast, the payoff to uninformed bidding will be either strictly positive or strictly negative in the presence of informed bidding. If the issuing firm sets \( F < \bar{F} \) this payoff is strictly positive, and investors will only engage in uninformed bidding (since informed bidding only breaks even). If the issuing firm sets \( F \in [\bar{F}, \tilde{F}] \), this payoff is strictly negative, and investors will engage only in informed bidding. In summary, if the issuing firm finds it optimal to induce information production, it will set the offer price in the range \( F \in [\bar{F}, \tilde{F}] \), and investors will only
engage in informed bidding.\textsuperscript{26} In the rest of the paper, we will assume that information of sufficient precision is available cheaply enough that the issuing firm finds it optimal to induce at least some information production. In this case, the issuing firm will set the offer price in the range $F \in [\frac{F}{2}]$, and investors will choose to engage only in informed bidding.\textsuperscript{27}

**Proposition 4. (Equilibrium in a fixed-price offering):** (i) Suppose the offering price satisfies $F \leq \frac{\theta(1-p)}{\theta(1-p)+1-\theta} \left( \frac{x}{k} v_G - \frac{x}{k} v_B \right) + \frac{1}{k} v_B$ and the information production cost $C$ is not too large so that there are $n_f \geq k + 1$ information producers in the IPO. Then, in an equilibrium where the two types of firms set the same offer price, the equilibrium bidding strategies of information producers are: bid for one share if the signal is $H$ or $M$, and do not bid if the signal is $L$. (ii) The equilibrium number of information producers in a fixed-price offering, $n_f$, is characterized by the following equation:

$$\theta \left( \frac{x}{k} v_G - F \right) \frac{k}{n} - (1-\theta)(1-p)(F - \frac{x}{k} v_B) \sum_{j=k-1}^{n-1} \binom{n-1}{j} \left( 1 - \frac{1}{p} \right)^{n-1-j} \frac{k}{j+1} = C.$$  

(iii) The equilibrium number of information producers in a fixed-price offering decreases with the offering price, that is, $\frac{\partial n_f}{\partial F} < 0$.

Part (i) demonstrates that the bidding strategy for information producers is to bid for one share when the signal is $H$ or $M$, and not bid if the signal is $L$. In equilibrium, we can see that if the issuing firm is of type $G$, all $n$ information producers will bid, since everyone’s signal will be either $H$ or $M$, and both will lead the information producer to bid for one share. Therefore, the type $G$ firm’s IPO never fails in equilibrium. However, if the issuing firm is of type $B$, it is possible that a large number of information producers receive the signal $L$ and do not bid, so that less

\textsuperscript{26} Note that this lack of co-existence of informed and uninformed bidding in a fixed-price offering arises in our setting since all investors have the same cost of information production, and can rationally choose between informed and uninformed bidding resulting in free entry to information production. In contrast, the two kinds of bidding can co-exist in a setting where the number of investors with the ability to produce information at a reasonable cost is limited, and most investors have a prohibitively high cost of information production (similar to Rock (1986)).

\textsuperscript{27} Of course, if outsiders’ cost of producing information is prohibitive, then no firm will find it optimal to induce information production, regardless of whether it chooses an auction or a fixed-price offering for its IPO, so that there will be only uninformed bidding in equilibrium. The focus of this paper is on the choice of IPO mechanism when outsiders have some incentive to produce information, and the case in which information production is prohibitively costly is not of interest here (in this case all investors bid the pooling value of type $G$ and type $B$ firms, and the choice between fixed-price offerings and IPO auctions is irrelevant).
than $k$ investors bid for shares in the IPO. In this case, the possibility of IPO failure arises. We assume that if there are less than $k$ information producers bidding for shares, the IPO fails and the type $B$ firm will be liquidated in the secondary market at its true (full information) value of $v_B$. Since all the choices are made by type $G$ firms, and type $B$ firms only mimic, the possibility of type $B$ firms’ IPO failure does not affect the equilibrium in our model (we will discuss how the transaction cost of attempting to go public, $T$, affects the equilibrium in Section 4.4).

The left side of (10) is the payoff to each information producer in the above equilibrium, and it is a decreasing function of the number of information producers, $n$. Thus, in equilibrium, the payoff to each information producer only covers the cost of doing so, since there is free entry to information production. Further, for a given number of information producers, the payoff to each information producer is higher when the offer price is lower. Therefore, the equilibrium number of information producers decreases in the offering price.

**Proposition 5. (Underpricing in a fixed-price offering):** (i) $F < \left[ \theta v_G + (1 - \theta) v_B \right] \frac{k}{F}$ represents an underpricing equilibrium. (ii) The smaller the fraction of shares the firm sells in the IPO, the greater the degree of underpricing, that is, $\frac{\left[ \theta v_G + (1 - \theta) v_B \right] \frac{k}{F} - F}{F}$ is decreasing in $\alpha$.

Note that the average secondary market price of equity (of the entire firm) is always $\theta v_G + (1 - \theta) v_B$, so that the average secondary market price of each share is $\left[ \theta v_G + (1 - \theta) v_B \right] \frac{k}{F}$. When the offering price is lower than the expected secondary market price, the issue is underpriced. Part (ii) demonstrates that when the firm sells a smaller fraction of its shares in the IPO, it is optimal to underprice more. This is because, in this case, a greater fraction of the combined proceeds to the firm arise from the seasoned equity offering, so that the cost of underpricing is smaller, while the benefit from doing so (in terms of obtaining a higher secondary market price and therefore greater proceeds from the seasoned equity offering) is larger.

### 3.4 The choice between fixed-price offerings and general IPO auctions for going public

We now discuss the overall equilibrium of the model, including the firm’s choice of IPO mechanism between fixed-price offerings and general IPO auctions.

**Proposition 6. (Overall equilibrium):** In equilibrium: (i) the type $G$ firm goes public using the IPO mechanism which maximizes its expected

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28 Since only the type $B$ firm’s IPO can fail in equilibrium, any IPO failure will reveal that the issuing firm is of type $B$. 

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combined proceeds from the IPO and the seasoned equity offering. In the case in which it chooses a fixed-price offering, the details of the offering are as characterized in Section 4.3. In the case in which the firm chooses to auction its shares, the details of the offering are as characterized in Section 4.2. (ii) In equilibrium, the type B firm chooses the same offering mechanism, and in the case of a fixed-price offering, the same offer price, as the type G. (iii) The equilibrium beliefs of investors at time 0 are such that they assign a probability $\theta$ of any issuing firm choosing an equilibrium action being of type $G$. The equilibrium bidding strategies of investors in response to a firm choosing a fixed-price offering or an IPO auction are given in Sections 4.3 and 4.2 respectively. If investors observe a firm choosing an out-of-equilibrium strategy, they assign a probability 0 to that firm being of type $G$. (iv) The equilibrium in the secondary market is as characterized in Proposition 1.

At time 0, the type $G$ firm chooses the IPO mechanism which maximizes the expected value of its combined proceeds from the IPO and the seasoned equity offering. The type $B$ firm will choose to mimic the type $G$ firm in equilibrium. The benefit to the type $B$ from mimicking the type $G$ is that if its IPO succeeds, it can get the higher (pooling) price and therefore the higher IPO proceeds that result from doing so. In contrast, if the type $B$ firm chooses a different IPO mechanism (or a different offering price in the case of a fixed-price offering) than the type $G$ firm, it will be revealed to be of type $B$, thereby obtaining lower expected combined proceeds (equal to $v_B$) compared to that obtained from mimicking the type $G$ firm. The cost to the type $B$ firm from attempting to mimic the type $G$ is that, in this case, there is a positive probability that not enough outsiders will bid for its IPO, so that its IPO fails, in which case it will have to raise alternative financing at its true value $v_B$ (since its type will be revealed in this case). In this eventuality of IPO failure, the type $B$ firm would waste the transaction cost $T$ it has incurred by attempting to go public. However, since the outsiders’ information is noisy, the type $B$ firm has a significant probability that its IPO will succeed even if it attempts to mimic the type $G$ by choosing the same IPO mechanism (and offer price, if the choice is a fixed-price offering). Further, the type $B$ firm will incur the transaction cost $T$ of attempting to go public even if it separates from the type $G$ by choosing a different offering price (or mechanism) for its IPO. Therefore, as long as the valuation of the type $B$ firm in its alternative financing opportunity (say, from a private placement of equity) is equal to its true value $v_B$, the type $B$ always has an incentive to attempt to mimic the type $G$, despite the probability of IPO failure it incurs by
doing so. Consistent with this equilibrium strategy of the two types of firms, outsiders assign a probability $\theta$ that any firm following the above equilibrium strategy is of type $G$.

**Proposition 7. (Choice between fixed-price offerings and general IPO auctions as a function of $C$):** Assume that the information production cost $C$ is less than a certain upper bound $C_{\text{max}}$, and the fraction of equity sold in the IPO is not too large so that $x \leq \bar{x}$. In equilibrium, (i) When the information production cost is low ($C \leq \bar{C}$), the firm will choose a general IPO auction; (ii) When the information production cost is high ($C > \bar{C}$), the firm will choose a fixed-price offering ($\bar{C}, C, \bar{x}$, and $C_{\text{max}}$ are defined in the appendix). If investors observe an out-of-equilibrium move, they put a probability 0 that the firm is of type $G$.

There are two important economic differences between the initial offer price emerging from an IPO auction and the fixed offer price set by a firm in an IPO. First, the price at which shares are sold in the IPO auction is determined as a result of competition among various informed bidders. This means that the initial offer price in the auction will aggregate, to a significant degree, the information produced by outsiders. In contrast, in a fixed-price offering, the offer price is set by the firm, and this offer price is the same for high-value and low-value firms. This means that there is no aggregation of information across investors in the IPO market in a fixed-price offering, and each investor has access to only his own information signal when bidding for shares in the IPO. In summary, the aggregation across outsiders’ information signals occurs earlier in an IPO auction compared to a fixed-price offering.

Second, in an IPO auction, bidders, whose information will be correlated with the true value of the firm (and therefore with that of each other), will compete away much of the surplus from each other. Since

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29 However, in practice, the valuation of the type $B$ firm in its alternative financing opportunity in the event of IPO failure may be smaller than its true value, perhaps because of additional transaction costs involved, or because of a “private placement discount.” Denoting any such additional value losses by $\delta$, the valuation of the firm in its alternative financing opportunity will then be only $(v_B - \delta)$. In this case, the type $B$ firm will attempt to mimic the type $G$ only if the expected benefit from doing so is larger than the expected cost (arising from this value loss $\delta$). This translates into a parametric restriction on $\delta$: the type $B$ firm will attempt to mimic the type $G$ only if $\delta$ is less than a certain upper bound (available to readers on request). If this restriction is satisfied, the equilibrium will continue to be a pooling equilibrium, as characterized above, rather than a separating equilibrium where the type $B$ firm chooses a different IPO mechanism or sets a different IPO offering price (for fixed-price offerings). However, even if the value loss $\delta$ in the alternative financing opportunity is larger (so that the parametric restriction above is violated), it can be shown in a model with multiple ($> 2$) firm types that there will be a pooling equilibrium where several types of firms pool by choosing the same offering mechanism and offer price (for fixed-price offerings) in equilibrium. The only difference in this case will be that now the lowest types will peel off from the pooling equilibrium (and separate by choosing a different mechanism or offering price): for these lowest firm types, the expected cost (arising from the probability of IPO failing and having to raise additional financing at a value loss $\delta$) dominates the expected benefit of mimicking the higher firm types. Thus, the essential nature of the pooling equilibrium remains unchanged even when the value loss $\delta$ is significant, except that the extent of pooling (i.e., the set of firm types which pool) will become smaller as $\delta$ becomes larger.
each bidder expects to be compensated for the cost of producing information, this means that the initial offer price emerging in an IPO auction will be able to support only a smaller number of informed entrants into the auction compared to the number of investors producing information in a fixed-price offering (where the high-value firm can set the offering price low enough to attract the optimal extent of information production by outsiders in order to maximize combined proceeds from the IPO and the SEO).

Thus, on the one hand, for a given amount of information produced by outsiders, the offer price in an IPO auction will reflect much more of the information produced by outsiders compared to that in a fixed-price offering. On the other hand, the aggregate amount of information produced around the IPO can be induced to be significantly larger in a fixed-price offering compared to an IPO auction, and this information will be reflected in the secondary market price of the IPO firm’s equity. Notice that, while the first difference between the two mechanisms favors the firm choosing an IPO auction, the second difference favors a fixed-price offering.

When the outsiders’ information production cost is high, the amount of information production in an IPO auction will be significantly below the level desired by the type G firm in order to maximize the expected value of its combined proceeds from the IPO and the SEO. However, if the firm chooses a fixed-price offering, the issuing firm can underprice in the IPO, inducing the optimal amount of information production (i.e., the amount of information production that maximizes the type G firm’s objective function (9)). This information will be reflected in the secondary market, and gives the type G firm a higher secondary market price, and therefore, greater combined proceeds. Thus, in this case, the advantage of fixed-price offerings in terms of inducing greater information production dominates its disadvantage in terms of poorer aggregation of information in setting the IPO offer price, making fixed-price offerings the equilibrium choice of the firm. In contrast, when the outsiders’ information production cost is low, the extent of information production is close to the optimal level even when an IPO auction is used. At the same time, an IPO auction will ensure a higher offer price (and therefore greater IPO proceeds) for the type G firm than a fixed-price offering: recall that, while the offer price in an auction will partially incorporate the information produced by outsiders, the offer price in a fixed-price offering is completely pooling and does not incorporate the information produced by outsiders even partially. Therefore, when the outsiders’ information production cost is low, the advantage of IPO auctions in terms of greater information aggregation in setting the IPO offer price dominates its disadvantage in terms of inducing information production, yielding higher combined proceeds to the type G firm from the IPO and the secondary market, making it the equilibrium choice.
Proposition 8. (Choice between fixed-price offerings and general IPO auctions as a function of \( \alpha \)): Suppose the information production cost is not too large so that \( C < \bar{C} \). Then in equilibrium: (i) When the firm sells a small fraction of its equity in the IPO \( (\alpha \leq \tilde{\alpha}) \), it will choose a fixed-price offering; (ii) When the firm sells a large fraction of its equity in the IPO \( (\alpha > \tilde{\alpha}) \), it will choose a general IPO auction \((\bar{C}, \tilde{\alpha}, \text{and } \tilde{\alpha} \text{ are defined in the appendix)}\). If investors observe an out-of-equilibrium move, they put a probability 0 that the firm is of type G.

The amount of information production increases in the fraction of equity sold in the IPO, \textit{ceteris paribus}. When the fraction of equity sold in the IPO is low, the amount of information production will be significantly below the level desired by the type-G firm in order to maximize the expected value of its combined proceeds from the IPO and the SEO. In contrast, if the firm chooses a fixed-price offering, the issuing firm can underprice in the IPO, and induce the optimal amount of information production. This information will be reflected in the secondary market and gives the type G firm a higher expected secondary market price and therefore higher combined proceeds. Furthermore, when the fraction of equity sold in the IPO is low, a large fraction of the combined proceeds come from the secondary market, so that the cost of underpricing is low and the benefit from greater information production is high, making a fixed-price offering the equilibrium choice.

In contrast, when the fraction of equity sold in the IPO is large, the extent of information production is close to the optimal level even when an IPO auction is used. At the same time, an IPO auction will ensure a higher offer price (and therefore greater IPO proceeds) for the type G firm than a fixed-price offering: while the offer price in an auction will partially incorporate the information produced by outsiders, the offer price in a fixed-price offering is completely pooling and does not incorporate the information produced by outsiders even partially. Furthermore, when the fraction of equity sold in the IPO is large, a larger fraction of the combined proceeds comes from the IPO market, so that the cost of underpricing is high, and the benefit from greater information production is low. Therefore, when the fraction of equity sold in the IPO is large, the combined proceeds to the type G firm from the IPO and the secondary market will be greater in an IPO auction, making it the equilibrium choice.

3.4.1 The choice between fixed-price offerings and IPO auctions: Graphical illustrations and numerical examples. To develop further intuition regarding a firm’s choice between fixed-price offerings and IPO auctions, we now present some graphical illustrations and numerical examples. In these illustrations and examples, we assume that the specific auction
mechanism used is the widely used uniform-price IPO auction (we need to assume a specific auction mechanism to calculate an IPO offering price and IPO proceeds). Further, since the presence of uninformed bidders does not change the qualitative nature of the bidding strategy of informed investors, we simplify computations by setting the number of uninformed bidders to zero in an IPO auction; we know from Lemma 2 that there will be no uninformed bidding in the presence of informed bidding in a fixed-price offering in any case. We also assume \( v_G = 1 \) and \( v_B = 0 \) to simplify computations. Other parameter values are as specified in each figure.

Figure 1 illustrates a firm’s choice between fixed-price offerings and IPO auctions as a function of outsiders’ information production cost \( C \). We can see from Figure 1 that, when outsiders’ information production cost is high, the type \( G \) firm obtains higher combined proceeds from a fixed-price offering, making it the equilibrium choice. However, when the information production cost is low, the combined proceeds to the type \( G \) firm are greater from an IPO auction, making it the equilibrium choice. Similarly, Figure 2 illustrates a firm’s choice between fixed-price offerings and IPO auctions as a function of the fraction of equity sold in the IPO, \( x \). From Figure 2, we can see that when \( x \) is high, the combined proceeds to the type \( G \) firm is greater when it uses a fixed-price offering, making it the firm’s equilibrium choice; when \( x \) is low, the combined proceeds to the type \( G \) firm is greater when the firm uses an IPO auction, making it the equilibrium choice.

We now illustrate the relative magnitudes of underpricing in fixed-price offerings and in IPO auctions using numerical examples (see Table 1). In addition to the other assumptions mentioned at the beginning of this subsection, in Table 1, we assume that the equity in the IPO is divided into \( k = 13 \) shares. Investors assign a prior probability \( \theta = 0.75 \) that the issuing firm is of type \( G \). If an investor chooses to produce information, he has a probability \( p = 0.005 \) of observing the true value of the firm. The information production cost is \( C = 0.0000144164 \).\(^{30}\) The fraction of equity sold in the IPO ranges from 6% to 40%. Table 1 gives the optimal IPO mechanism and the percentage underpricing as a function of the fraction of equity sold in the IPO, \( x \). We can see that when the firm sells a small fraction of equity in the IPO (i.e., \( x \in [6\%, 22\%] \)), it is optimal for it to use fixed-price offerings, and when it sells a large fraction of equity in the IPO (i.e., \( x \in [24\%, 40\%] \)), it is optimal for it to use uniform-price IPO auctions. Further, we can see that when the firm uses

\(^{30}\) The specific value of \( C \) assumed here ensures that the equilibrium number of information producers is an integer in the numerical example. Further, the value is \( C \) is small because we have normalized the value of the type-\( G \) firm to 1. For example, if the value of the type-\( G \) firm is $100 million, then \( C = 0.0000144164 \) means it costs $1441 to produce a signal, whereas if the value of the type-\( G \) firm is $10 million, it costs around $144.
fixed-price offerings, the degree of underpricing is decreasing with \( \alpha \). For example, when \( \alpha = 6\% \), the underpricing is 39.14%; when \( \alpha = 10\% \), underpricing decreases to 20.05%; when \( \alpha \) increases to 20\%, the underpricing decreases to 8.85%.

From table 1, we can also see that the degree of underpricing is not monotonic in \( \alpha \) when a uniform-price IPO auction is used. When \( \alpha \)
increases from 24% to 34%, underpricing increases monotonically from 6.18% to 7.78%. However, when $z$ increases from 34% to 40%, underpricing decreases monotonically from 7.78% to 7.61%. We know that the total number of information producers is increasing in $z$, so that the dollar amount of underpricing (“money left on the table”) is also increasing with $z$ (since the underpricing is to compensate information production costs of outsiders, more information production is associated with a greater dollar amount of underpricing). However, the average value of shares sold in the IPO, which is $\theta z$ (when $v_G = 1$ and $v_B = 0$) also increases as $z$ increases. So the impact of an increase in $z$ on percentage underpricing is ambiguous: when $z$ is relatively small, the number of information producers depends crucially on the fraction of equity sold in the IPO and increases very fast; for large values of $z$, the number of information producers increases at a lower rate with $z$. Thus the percentage underpricing is first increasing in $z$, and then decreasing in $z$, in the case of IPO auctions. We can also see from the above example that the average underpricing conditional on the firm choosing a fixed-price offering (for $z \in [6\%, 22\%]$) is 17.05%, while that conditional on the firm choosing a uniform-price IPO auction (for $z \in [24\%, 40\%]$) is only 7.42%, so that the average underpricing in the IPO auction is lower.

### 4. Extensions to the Basic Model

In this section, we extend the basic model in two different directions, by relaxing two of its assumptions (one at a time). In the first subsection, we relax the assumption that the fraction of equity sold in the IPO, $z$, is
exogenous. In the second subsection, we relax the assumption that the issuing firm does not set any reservation price in the IPO auction.

4.1 Fixed-price offerings versus general IPO auctions with an endogenous fraction of equity offered

In the basic model, we assumed that the fraction of equity sold in the IPO, \( \alpha \), is exogenous. In reality, the issuing firm may have some degree of freedom in choosing how much equity to sell in the IPO. In this subsection, we explore this possibility by assuming that the issuing firm can endogenously choose the fraction of equity sold in the IPO, subject to the constraint that at least a certain fraction \( \alpha_{\text{min}} \) has to be sold (all other assumptions remain the same as in the basic model). In this case, the problem facing the issuing firm is two-dimensional. It has to choose: (a) an IPO mechanism: either a fixed-price offering (along with an optimal offering price) or a general IPO auction; and (b) the optimal fraction of equity to offer in the IPO.

When a fixed-price offering is used, the issuing firm’s objective is

\[
E[R^G_f (\alpha^*_f)] = \max_{\alpha \geq \alpha_{\text{min}}} kF + (1 - \alpha) \times \{1 - (1 - \theta)(1 - p)\}^a \{v_G - v_B\} - T, \\
\text{s.t. } E[\pi^f (F, n_f, \alpha)] - C = 0. \tag{11}
\]

When a general IPO auction is used, the objective of the issuing firm is

\[
E[R^G_a (\alpha^*_a)] = \max_{\alpha \geq \alpha_{\text{min}}} E[R^G_a (\alpha)] + (1 - \alpha) \times \{1 - (1 - \theta)(1 - p)\}^a \{v_G - v_B\} + v_B - T, \\
\text{s.t. } E[\pi^a (n_a, \alpha)] - C = 0. \tag{12}
\]

The nature of the overall equilibrium remains very similar to that in the basic model (see proposition 6) with the type \( G \) firm choosing the IPO mechanism which maximizes its combined proceeds, and the type \( B \) firm mimicking it. We now study the firm’s choice between fixed-price offerings and IPO auctions (when the fraction of equity sold, \( \alpha \), is endogenous) as a function of the outsiders’ information production cost, \( C \).

**Proposition 9.** (Choice between fixed-price offerings and general IPO auctions when the fraction of equity sold in the IPO is endogenous): Suppose the firm can optimally choose the fraction of equity to sell in the IPO. (i) The issuing firm will choose a fixed-price offering when the information production cost is high, and a general IPO auction when the information production cost is low. (ii) There exist values of \( C \) such that it is optimal
for the issuing firm to choose a general IPO auction, and sell a fraction $\alpha^* > \alpha_{\min}$ of equity in the IPO.

The intuition behind the above proposition can best be discussed making use of the examples illustrated in Figures 3, 4, and 5 (parameter values are as given in each figure; other assumptions as given at the beginning of Section 4.4.1). These figures illustrate the firm’s equilibrium choice of offering mechanism as well as its equilibrium choice of $\alpha$ for three different values of the information production cost, $C$. When $C$ is low, the type $G$ firm will choose the IPO auction and sell the smallest possible fraction, $\alpha_{\min}$, as illustrated in Figure 3. This is because there will be enough information production even when the firm sells the minimal fraction $\alpha_{\min}$. Since the IPO price is not as informative as the secondary market price (for a type $G$ firm, the expected offering price is always lower than the expected secondary market price), the type $G$ firm tries to sell as little as possible in the IPO in this case.

When the information production cost is moderate, the type $G$ firm may choose to auction its shares and sell a fraction of equity in the IPO greater than $\alpha_{\min}$, as illustrated in Figure 4. Since, in this case, the number of information producers in the IPO crucially depends on the fraction of equity sold, the type $G$ firm faces a trade-off when choosing the optimal fraction of equity to offer. On the one hand, a large fraction sold means more information producers in the IPO, so that both the IPO price and the secondary market price will be higher. On the other hand, since the expected IPO price is always lower than the expected secondary market price for the type $G$ firm, a larger fraction of equity sold in the IPO means a lower expected value of combined proceeds to the type $G$ firm.
Depending on this trade-off, for a wide variety of parameter values, the type $G$ firm will choose an optimal $a$ larger than $a_{\text{min}}$ when $C$ is moderate (as is the case in Figure 4).

When the information production cost is high, the type $G$ firm will choose a fixed-price offering, and again sell the minimum fraction $a_{\text{min}}$, as illustrated by Figure 5. The intuition behind the type $G$ firm choosing a fixed-price offering rather than an IPO auction for large values of $C$ is the same as that discussed under the basic model. The type $G$ firm faces a trade-off when choosing the fraction of equity to sell: it can sell a larger fraction of equity in the IPO and underprice less, or sell a smaller fraction in the IPO and underprice more in order to induce the same amount of information production. For a wide variety of parameter values, it is optimal for the type $G$ firm to sell a smaller fraction in the IPO and underprice more, like in Figure 6.

4.2 Fixed-price offerings versus general IPO auctions with an endogenous reservation price

In the basic model, we assume that the issuing firm does not set any reservation price in the IPO auction. This is equivalent to setting the reservation price at $r = \frac{2}{k} v_B$ (since the true value of a share in a type B firm is $\frac{2}{k} v_B$). In this subsection, we relax this assumption by allowing the issuing firm to choose an endogenous reservation price $r$ (optimally) in the general IPO auction (all other assumptions remain the same as in the basic model). We will refer to the price, $p$, paid by the $k$th highest bidder in a general IPO auction as the “market clearing price” (recall that, in a
general IPO auction, the $k$ shares offered for sale go to the $k$ highest bidders). When the reservation price is set at $r$, the IPO auction fails if the market clearing price, $p$, is below $r$. 

Figure 5
Endogenous fraction of equity sold when the information production cost is high

Figure 6
Comparison of the three mechanisms as a function of the information production cost
The issuing firm’s objective in case a general IPO auction is used will now be to choose the optimal $r$ to maximize the combined proceeds:

$$E[R^G_r(r^*)] = \max_r E[R^G_r(r)] + (1 - \alpha)$$

$$\times \left\{ \left[ 1 - (1 - \theta) (1 - p) a \right] (v_G - v_B) + v_B \right\} - T,$$

s.t.

$$E[\pi'(n_a, r, \alpha)] - C = 0.$$  \hspace{1cm} (13)

The issuing firm’s objective when a fixed-price offering is used will remain the same as in the basic model and is given by (9).

The introduction of a reservation price will affect three things in an IPO auction. First, by setting a reservation price, the issuing firm can protect itself from selling at a very low price in the IPO, since the offering price (if the IPO succeeds) will be higher than the reservation price. Second, there is a possibility of IPO failure when the issuing firm sets a high reservation price (even for a type G firm). When most bidders choose to make a low bid, there may be less than $k+1$ bidders bidding above $r$, so that the IPO auction fails. We assume that if the IPO auction fails, the issuing firm is able to obtain financing from an alternative source (say, a private placement of equity). We denote the cash flow to the issuing firm from this alternative source by $R_{\text{fail}}$, which is common to type G and type B firms. Third, when there is a reservation price in the IPO auction, the information producers’ payoff will be less, since they have no chance of obtaining shares at a price lower than the reservation price, so that there will be a smaller number of information producers in the IPO. In summary, the benefit to the issuing firm of having a reservation price is that it can extract some of the surplus obtained by the bidders in the firm’s IPO. The cost of having a reservation price is that it may reduce the number of information producers about the firm and increase the probability of IPO failure. The equilibrium reservation price set by the issuing firm emerges from the above trade-off.\footnote{In the basic model we assume that the proceeds to the type B firm is $v_B$ in the event of IPO failure. In contrast, here we assume that both types get $R_{\text{fail}}$. The reason is that, in the basic model, only the IPO of the type B firm may fail, so that IPO failure will reveal its true type. Here the IPOs of both types may fail, so that IPO failure does not reveal firm type. Our results remain qualitatively unchanged if we assume that $R_{\text{fail}}$ is different for the two types of firms, and is set equal to $v_B$ for the type B firm.}

\footnote{It can be shown that, regardless of how high the reservation price set by the type G, the type B firm always has an incentive to mimic the type G by setting the same reservation price, so that the type G cannot use its reservation price to signal its type, as long as the type B firm can sell its equity at a valuation at least equal to its true value $v_B$ in the event of IPO failure. The intuition here is that if type B sets a different (lower) reservation price, it will reveal its true type, and will therefore be able to obtain a valuation of only $v_B$ in any case in the IPO. On the other hand, if it attempts to mimic the type G by setting the same reservation price, there is a significant chance that its IPO would be successful (given that outsiders’ information signals are noisy), in which case its IPO valuation would be above $v_B$. Further, the above result holds even when the transaction cost $T$ of a firm of attempting to go public is significant: since the type B firm has to incur this transaction cost regardless of whether it mimics the type G (by setting the same reservation price) or separates from it (by setting a lower reservation price), the magnitude of this cost does not drive the nature of the equilibrium in our setting.}
Assumption 3. In a general IPO auction, the price the winner \(i\) pays, \(p_i\), is equal to his own bid or that of an investor who bids strictly below him.

This assumption is not unduly restrictive: both the two commonly used IPO auctions, uniform-price auctions and discriminatory auctions, satisfy this condition. \(^{33}\) We now prove the following lemma, showing that, in general, setting a reservation price will only discourage information production, and never encourage information production.

**Lemma 4.** Suppose Assumption 3 holds. The number of information producers in a general IPO auction is never larger when a reservation price \(r > \frac{2}{k} v_B\) is set, compared to the case when no reservation price is set.

The nature of the overall equilibrium remains very similar here as in the basic model (see proposition 6) with the type \(G\) firm choosing the IPO mechanism which maximizes its combined proceeds, and the type \(B\) firm mimicking it. We now study the firm’s choice between fixed-price offerings and IPO auctions with an endogenous reservation price as a function of the outsiders’ cost of producing information, \(C\), and the fraction of equity sold in the IPO, \(\alpha\).

**Proposition 10.** (Choice between fixed-price offerings and general IPO auctions with an endogenous reservation price): Suppose the issuing firm chooses an optimal reservation price in a general IPO auction. The issuing firm will choose a fixed-price offering when \(C\) is large \((C > \hat{C})\) and \(\alpha\) is small \((\alpha \leq \hat{\alpha})\), and a general IPO auction with an optimal reservation price when \(C\) is small \((C \leq \hat{C})\) and \(\alpha\) is large \((\alpha > \hat{\alpha})\).

The intuition behind the above proposition is as follows. We know from our basic model that when the information production cost \(C\) is small or the fraction of equity sold \(\alpha\) is large, the IPO auction dominates a fixed-price offering. Since setting an optimal reservation price can only increase the proceeds to a type \(G\) firm from an IPO auction (recall that the firm can always set a very low reservation price if it is optimal to do so), the IPO auction continues to dominate a fixed-price offering, making it the equilibrium choice. On the other hand, we know from our basic model that when the outsiders’ information production cost \(C\) is large and the fraction of equity sold in the IPO \(\alpha\) is small, there is only a smaller extent of information production by outsiders when a general IPO auction is used, making fixed-price offerings the dominant mechanism in terms of maximizing proceeds to the type \(G\) firm (and therefore the equilibrium choice). Since, from lemma 4, we know that adding a reservation price cannot enhance information production but will only reduce it, the flexibility to set an optimal reservation price cannot raise the combined proceeds to the type \(G\) firm from using an IPO auction.

\(^{33}\) This assumption is only a sufficient condition for our results, not a necessary condition.
above that of a fixed-price offering, so that fixed-price offerings continue to be the equilibrium choice even in this case.

Figures 6 and 7 (parameter values as given in each figure; other assumptions as in Section 4.4.1) illustrate a firm’s choice between fixed-price offerings and IPO auctions with an endogenous reservation price, as a function of the information production cost $C$ and the fraction of equity sold $\alpha$, respectively; IPO auctions with no reservation price are also compared, represented by dashed lines. It can be seen from Figure 6 that when $C$ is large, fixed-price offerings are the equilibrium choice; on the other hand, when $C$ is small, IPO auctions with an endogenous reservation price are the equilibrium choice. Similarly, it can be seen from Figure 7 that when $\alpha$ is small, fixed-price offerings are the firm’s equilibrium choice; in contrast, when $\alpha$ is large, IPO auctions with an endogenous reservation price are the equilibrium choice.34

5. Empirical and Policy Implications

We highlight some of the empirical and policy implications of our model below. Even though the focus of our model is on a firm’s choice between fixed-price offerings and IPO auctions, it also has some implications for a

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34 The higher the reservation price, the higher the payoff to a type G firm when an IPO auction succeeds. On the other hand, the higher the reservation price, the higher the probability of IPO failure in an IPO auction. The optimal reservation price in an IPO auction emerges from this trade-off. Thus, the optimal reservation price can be shown to be increasing in $R_{fail}$; the amount the firm can raise from other sources in the event of IPO failure: the higher the value of $R_{fail}$, the lower the cost of IPO failure, resulting in a higher equilibrium reservation price.
firm’s choice between bookbuilding and IPO auctions as well (to the extent that we can view bookbuilding also as a posted price mechanism).

(i) The relationship between firm and IPO characteristics and the optimal mechanism for going public: First, our model predicts that if a firm is young, or small, or faces a greater extent of information asymmetry for some other reason (so that outsiders’ information production costs are significant), then fixed-price offerings will be the equilibrium choice of the firm, since, in this case, considerations of inducing information production and their impact on the secondary market price become important. In contrast, if a firm is older, or larger, or has a well-known (reputable) product, or faces a lower level of information asymmetry for some other reason (so that outsiders’ cost of evaluating the firm is smaller), then our analysis implies that it will choose an IPO auction. Second, our model predicts that, firms selling smaller fractions of equity in the IPO will choose fixed-price offerings, while those selling larger fractions of their equity will choose IPO auctions. Evidence consistent with this prediction in the context of Taiwan is provided by Hsu, Hung, and Shiu (2009), who document that the likelihood of a firm using an IPO auction rather than a fixed-price offering is increasing in the size of the firm as well as the fraction of equity sold in the IPO.35,36

In terms of firms going public in the United States, our analysis indicates that IPO auctions have been tried by precisely the wrong kind of firms, namely, small, lesser known firms (possibly because IPO auctions appealed to these firms purely from the point of view of providing savings in investment banking fees); older, larger, or better-known firms are likely to be able to use IPO auctions with considerably more success (though most firms going public clearly do not belong to this category). Two prominent examples of firms in the latter category that went public in the United States using IPO auctions are Google Inc. and Morningstar Inc. (see also footnote 11). However, while our analysis predicts that IPO auctions will tend to be most successful for the IPOs of larger and

35 In Taiwan, the choice faced by firms going public is between a pure fixed price offering and a hybrid mechanism where the firm conducts an IPO auction tranche followed by a fixed-price offering tranche (see Hsu, Hung, and Shiu 2009). Given that the pricing in the hybrid mechanism is driven by the auction tranche, our model implications will be qualitatively unchanged for the choice faced by Taiwanese firms as well.
36 Evidence consistent with this prediction of our model is also provided by Loughran, Ritter, and Rydqvist (1994), who document that when IPO auctions were tried in many European countries, the firms that successfully used such auctions tended to be the larger, better known firms. Evidence consistent with this implication is also provided by Kutsuna and Smith (2004) (see Table 3), who document that, in Japan, older and larger firms choose to go public using IPO auctions, while smaller and lesser known firms use the bookbuilding mechanism.
better-known firms such as Google and Morningstar, it may take several years for such auctions to gain market share in the United States for two reasons. First, investment banks seem to have a considerable incentive to ensure that IPO auctions do not take market share from bookbuilding, since they not only have considerably more power in price setting and share allocation in bookbuilding than in IPO auctions, but also obtain much larger fees in the former mechanism (7% of capital raised in a bookbuilding IPO, against 3 to 4% in IPO auctions): see, “Why IPOs Still Use the Old Way,” Wall Street Journal, July 6, 2005 for a discussion of underwriter incentives. Second, given that IPO auctions have been tried by only a few firms so far, we may need to go through a period of experimentation and minor modifications of the auction mechanism before a widely successful IPO auction method is found (judging by the example of the now well-established Treasury bond auctions in the United States, which took approximately 30 years to completely replace the fixed-price offerings of Treasury bonds): See Garbade (2004) for an interesting study of how auctions came to replace fixed-price offerings in the case of Treasury bonds.

(ii) Direct experimental verification of our model: A direct test of our model is performed in an experimental setting by Trauten and Langer (2012). They use a laboratory experiment to distinguish the effects of the IPO auction mechanism and information production costs from a variety of other factors that may affect investor behavior in IPOs. The results of their experimental analysis broadly support the main predictions of our model. In particular, their experimental findings indicate that, in a fixed price offering, the issuer is able to maintain the desired extent of information production by setting a lower offering price, while in IPO auctions, high information costs lead to a low extent of information production. Their experimental results thus support our explanation of the IPO auction puzzle by documenting that IPO auctions are not the appropriate mechanism for the typical (small, young) firm going public, since these are characterized by high information production costs.

(iii) The relationship between offering mechanism and IPO underpricing: Our model predicts that IPO auctions will exhibit a significantly lower mean and variance of underpricing compared to fixed-price offerings. This is due to the fact that the offering price in an IPO auction aggregates the information produced by outsiders to a significant degree, so that this offering price is greater for higher intrinsic-value firms (and lower for lower-intrinsic-value firms) in IPO auctions than in fixed-price offerings. At the same time, there is less information production in IPO auctions compared to...
fixed-price offerings (where the offering price is set by insiders to induce the optimal degree of information production), so that a lower amount of information is reflected in the opening price in the secondary market in this case. Since the impact of increased information production is to increase the separation between higher and lower intrinsic-value firms in the secondary market, the price jump (either upward or downward) from the IPO to the secondary market is therefore smaller for IPO auctions than for fixed-price offerings, leading to both a lower mean (see Section 4.4.1 for an illustration) and a lower variance of underpricing in IPO auctions. There is evidence from three different countries consistent with this prediction. First, the studies of Jenkinson and Mayer (1987), Davis and Yeomans (1974), and Mayer and Meadowcroft (1985) indicate that the mean and variance of underpricing were higher in IPOs and privatizations in the United Kingdom conducted using fixed-price offerings relative to those conducted using IPO auctions (“tenders”) in the same country. Second, IPOs where shares were auctioned have lower underpricing compared to those using fixed-price offerings in Taiwan (see Lin and Sheu (1997), Liaw, Liu, and Wei (2001), and Ritter (2003)). Further, more recent evidence has indicated that the variance of underpricing was also lower in Taiwanese IPO auctions compared to that of IPOs using fixed-price offerings in the same country (Chen and Yeh (2004)). Finally, Derrien and Womack (2003) document that the mean and variance of French IPOs conducted using “pure” IPO auctions (Pure Offre à Prix Minimal, or POPM) were both lower than the corresponding values for IPOs conducted using “pure” fixed-price offerings (Offre à Prix Ferme, or OPF).

(iv) A Resolution to the IPO Auction Puzzle: Our model is able to explain why IPO auctions do not dominate fixed-price offerings in terms of market share around the world, while simultaneously predicting that fixed-price offerings will exhibit a greater extent of underpricing compared to IPO auctions. If, in practice, the firm insiders’ objective is not merely to maximize the proceeds from a one-shot equity offering, it is indeed optimal for younger and smaller firms, and those selling smaller fractions of equity, to go public using fixed-price offerings.37 Since a large majority of firms

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37 The fact that, on average, firms sell only about one third or less of their equity in IPOs in the United States seems to indicate that insiders may indeed not be focused on maximizing proceeds from a one-shot equity offering here. The average fraction of equity sold in IPOs in many other countries seems to be even lower: for example, Kutsuna and Smith (2004) document this fraction to be 19.36% in Japan, Chen, Liaw, and Leung (2003) document this fraction to be approximately 14% in Taiwan, and Guner, Onder, and Rhoades (2000) document this fraction to be 25.2% in Turkey.
going public in the United States and in most other countries fall into this category, it is not surprising that IPO auctions are not the dominant mechanism in these countries. Our analysis indicates that one setting in which IPO auctions may indeed be optimal is in the privatization of large, well-run government firms (where the government may be selling off its entire equity in the IPO) or in the IPOs of older and larger firms (for instance, firms going public again after being taken private in a leveraged buyout (LBO)). Our analysis also indicates that auctions will be optimal for selling securities in other settings where the outsiders’ information production cost is low (e.g., for selling shares in seasoned equity offerings) or where the security sold is relatively less information sensitive (e.g., for selling corporate bonds).38

(v) **The relationship between offering mechanism and the average number of bidders in the IPO:** Our model predicts that the number of bidders in fixed-price offerings (controlling for the size of the offering, as measured by the total number or value of shares offered for sale) will be significantly larger than that in IPO auctions. Recall that, in our setting, firms set the offering price in a fixed-price offering so as to induce a greater extent of information production in the IPO, resulting in a larger number of bidders in fixed-price offerings.39

(vi) **The benefit from selling equity in tranches and the optimal fraction of equity to sell in the IPO:** First, our analysis indicates the benefits of selling equity in tranches, since the firm may be able to obtain a significantly higher share price in later sales of equity. Second, our results from Section 5.1 indicate that, when the firm needs to raise only a small amount of capital from the IPO (so that the minimum amount of equity the firm needs to sell in the IPO is small), the fraction of equity to be sold in the IPO may have to be decided jointly with the choice of offering mechanism. Thus, when the outsiders’ cost of information production is large, the firm may choose to sell the smallest fraction of equity possible, using a fixed-price offering. At the other extreme, when the outsiders’ cost of information production is very small, the firm may also choose to sell the smallest fraction of equity possible, but using an

38 Consistent with this implication, the privatization of large, well-known government-owned firms seem to be one setting in which IPO auctions have been extensively and successfully used to date (e.g., in privatizations in the United Kingdom and several other countries). Another real-world example consistent with this implication is the use of auctions to sell Treasury bonds, where information asymmetry can be expected to be insignificant.

39 Preliminary evidence by Chen and Yeh (2004) and Mayer (2004) (from Taiwan and the United Kingdom, respectively) indicates that the average number of bidders in fixed-price offerings in these countries is indeed greater than that in IPO auctions.
IPO auction. At moderate levels of the cost of information production, however, the firm may choose to sell a fraction of equity larger than the minimum it needs to sell from the point of view of raising capital alone, with the mechanism chosen depending on the magnitude of outsiders’ cost of information production. This latter result obtains because the number of information producers tends to increase with the fraction of equity sold in IPOs.\footnote{In this context, various “tranches” of equity have the same payoff structure, and refer simply to different fractions of equity offered at prices close to the price prevailing in the secondary market at the time of the offering. The determination of optimal tranche size was an often-debated problem in the privatizations of government firms in many countries. Our analysis provides two pieces of guidance in this case. First, it indicates that, even when the government has no reason to hold on to large fractions of equity after the IPO, it is optimal from the point of view of revenue maximization to sell the equity in tranches. Second, even when there is no minimum capital to be raised from an IPO or privatization, considerations of inducing information production dictate that the government (or firm going public) sell at least a certain minimum fraction of equity.}

(vii) Reforming the procedures used in existing IPO auctions: Our analysis indicates that existing IPO auction procedures may be reformed in several directions to make IPO auctions more competitive with fixed-price offerings. First, our analysis indicates that firms going public may benefit from offering the IPO at a discount to the clearing-price in the IPO auction.\footnote{The IPO auctions in France do seem to set the offer price 2\% to 5\% below the auction-clearing price (see MacDonald and Jacquillat 1974; Biais and Faugeron-Crouzet 2000). Such a discount also seems to have been applied in the IPO auction of Google in the United States. See also Chemmanur et al. (2017), who show that the fixed-price tranche aimed at retail investors in hybrid auction IPO mechanism used in mainland China is usually set at a discount to the market clearing price from the first (IPO auction) tranche aimed at institutional investors.} Second, this discount may be adjusted to account for the characteristics of firms going public: for instance, a greater discount may be offered in the IPOs of younger, smaller, or lesser-known firms. The idea here is to encourage greater information production by outsiders, over and above that “naturally occurring” in auctions.\footnote{For reasons unrelated to ours, Biais and Faugeron-Crouzet (2002) and Parlour and Rajan (2005) also suggest that it may be optimal for the issuer to set the offer price in IPOs below the market-clearing price. In Biais and Faugeron-Crouzet (2002) this is motivated by a need to unravel tacit collusion among investors in uniform-price auctions, but in Parlour and Rajan (2005) the motivation is to mitigate the winner’s curse faced by uninformed investors.}

\footnote{Of course, investors may anticipate a discount and bid more aggressively in the IPO auction, thus partially eliminating the information production benefits of the discount. However, as long as such aggressive bidding does not completely eliminate the additional surplus provided by this discount to those investors who win shares in the IPO auction, the equilibrium number of information producers will be greater in IPO auctions when such a discount is provided. One way to implement this discount is to place an upper limit on the IPO share price (with the IPO price set at the lower of the market-clearing price in the auction and this upper limit), with all bidders above this upper limit having an equal chance of getting a share allocation. It can be shown that the number of information producers under an IPO auction with a discount implemented in the above manner will be strictly greater than that in an IPO auction without a discount.}
6. Conclusion

We have developed a theoretical analysis of the choice of firms between fixed-price offerings and uniform-price auctions for selling shares in IPOs and privatizations. We considered a setting in which a firm goes public by selling a fraction of its equity in an IPO market where insiders have private information about intrinsic firm value. Outsiders could, however, produce information at a cost about the firm before bidding for shares. Firm insiders care about the extent of information production by outsiders, since this information is reflected in the secondary market price, giving a higher secondary market price for higher intrinsic-value firms. We showed that auctions and fixed-price offerings have different properties in terms of inducing information production and in the time at which outsiders’ information signals are aggregated into the firm’s share price. Thus, in many situations, firms prefer to go public using fixed-price offerings rather than IPO auctions in equilibrium. We related the equilibrium choice between fixed-price offerings and IPO auctions to various characteristics of the firm going public. Our model can explain the empirical finding that while underpricing is lower in IPO auctions than in fixed-price offerings, auctions are not the dominant IPO mechanism around the world (referred to as the “IPO auction puzzle”).

References


Appendix

A Proofs of Propositions.

A.1 Proof of the Equilibrium Bidding Strategies in Uniform-Price Auctions

We consider the bidding strategies of informed bidders first. Suppose the other \( n - 1 \) informed bidders, 2, 3, \ldots, \( n \), use the specified strategy. If we prove that it is also optimal for bidder 1 to use the same strategy, then the specified strategy is a symmetric Nash equilibrium. When bidder 1 observes \( H \), he knows that the true value of each share is \( \frac{v}{k} \). Suppose \( i \) bidders receive signal \( H \) other than bidder 1, where \( i \) ranges from 0 to \( n - 1 \).

Case 1: \( i < k \). The \( k \)th highest bid among the other \( n - 1 \) bidders will be a random withdrawal from the interval \((0, \frac{v}{k})\) with cdf \( M(b; n) \), hence the clearing price will be less than \( \frac{v}{k} \) (call it \( m_k \)). If bidder 1 bids \( \frac{v}{k} \), he will win one unit and pay a price of \( m_k \), the payoff is positive; if he bids a value in \([0, m_k)\), he will never win the object and the payoff is zero; if he bids a value in \([m_k, \frac{v}{k})\), he will win one unit and pay a price of \( m_k \), and the payoff is the same as bidding \( \frac{v}{k} \). Therefore, bidding \( \frac{v}{k} \) is always optimal in this case.

Case 2: \( i \geq k \). The \( k \)th highest bid among investors 2, 3, \ldots, \( n \) is \( \frac{v}{k} \). If bidder 1 bids \( \frac{v}{k} \), he will win one unit with probability \( \frac{k}{n} < 1 \), and pay a price of \( \frac{v}{k} \), resulting in a payoff of zero. If he bids a value in \([0, \frac{v}{k})\), he will never win the object and the payoff is zero. Therefore, bidding \( \frac{v}{k} \) is also weakly optimal in this case.

When a participant observes signal \( L \), he knows that \( v = 0 \). Suppose that investors 2, 3, \ldots, \( n \) use the specified bidding strategy. If investor 1 bids 0, the payoff to him is always zero. If he bids above 0, there is a strictly positive probability that he will receive one unit at a price higher than 0. If he bids less than 0, his bid will never be filled so that the payoff is 0. So it is optimal to bid 0 when all other informed bidders use the specified strategy.

We now characterize the bidding strategy of the investors who receive signal \( M \). Define \( M(b) \) as the cdf of a random variable with support on the interval \((0, \frac{v}{k})\), and let \( m(b) \) be the corresponding pdf. Suppose informed bidders 2, 3, \ldots, \( n \) use the specified strategy. If investor 1 observes \( M \), and if he submits a bid \( b \in (0, \frac{v}{k}] \), the expected payoff to him is

\[
\pi(b) = \theta \frac{v}{k} (\frac{v}{k} - x) \left( \frac{n - 1}{1} \right) (1 - p)m(x) \left( \frac{n - 2}{k - 1} \right) [p + (1 - p)(1 - M(x))]^{k-1} \\
\times \left[ (1 - p)M(x) \right]^{n-k-1}dx - (1 - \theta) \frac{v}{k} \left( \frac{n - 1}{1} \right) (1 - p)m(x) \left( \frac{n - 2}{k - 1} \right) \\
\times \left[ (1 - p)M(x) \right]^{k-1} \left[ (1 - p)(1 - M(x)) \right]^{n-k-1} dx.
\]

To understand the above formula, note that if we define \( x \) as the \( k \)th highest bid among bidders 2, 3, \ldots, \( n \), bidder 1 will win one unit at price \( x \) iff \( b > x \). When the firm is of type \( G \),

\[
\left( \frac{n - 1}{1} \right) (1 - p)m(x) \left( \frac{n - 2}{k - 1} \right) [p + (1 - p)(1 - M(x))]^{k-1} [(1 - p)M(x)]^{n-k-1}
\]

is the pdf that the \( k \)th highest bid among investors 2, 3, \ldots, \( n \) is \( x \), and bidder 1’s payoff is \( \frac{v}{k} - x \) if \( x < b \) and 0 otherwise. When the firm is of type \( B \),

\[
\left( \frac{n - 1}{1} \right) (1 - p)m(x) \left( \frac{n - 2}{k - 1} \right)
\]
How Should a Firm Go Public?

First, let's introduce the notation: $p$ is the fraction of the firm's total value that bidder 1 observes; $M(x)$ is the information production cost; and $M(b)$ is the secondary market value of the whole firm. Then we have the expected profit of each informed bidder is $E[p(n)] = \text{Pr}(s_i = H)E[p(n)|H] + \text{Pr}(s_i = M)E[p(n)|M] + \text{Pr}(s_i = L)E[p(n)|L].$ (A.1)

It is obvious that $E[p(n)|L] = 0$. The expected profit for a participant when he observes M is also 0. To see this, plugging Equation (4) into $p(n)$ and we have $p(n) = 0$ for any $b$, which implies $E[p(n)|M] = 0$. The expected profit when bidder 1 observes $H$ is

$$E[p(n)|H] = \int_0^{\frac{x}{k}} \frac{1}{k-1} (n-1)(1-p)m(x)\left(\frac{n-2}{k-1}\right)[p+(1-p)(1-M(x))]^{k-1} dx,$$

where $x$ is the $k$th highest bid among the bids from bidder 2, 3, ..., $n$, and $\left(\frac{n-2}{k-1}\right)(1-p)m(x)$ is the pdf of $x$ conditional on the firm being of type G and $x < \frac{v}{k}$, $(\frac{x}{k}-x)$ is the payoff to bidder 1 in this case. So the expected profit of each informed bidder is

$$E[p(n)] = \text{Pr}(H)E[p(n)|H] + 0 = \left.p\right|_{0}^{\frac{x}{k}} \frac{1}{k-1} (n-1)(1-p)m(x)\left(\frac{n-2}{k-1}\right)[p+(1-p)(1-M(x))]^{k-1} dx.$$

(A.3)

In equilibrium, the number of information producers is such that the profit from information production equals the information production cost $C$, which leads to Equation (5).

To prove the bidding strategy of uninformed bidders, recall that we have proved in Lemma 1 that uninformed bidders earn zero expected profit. Suppose uninformed bidders bid $b^v$, the expected profit of each uninformed bidder is

$$p''(b^v) = A_1\left(\frac{v}{k} - b^v\right) + A_2(0 - b^v);$$

setting this to 0 leads to Equation (6), where $A_1$ and $A_2$ are defined in Equations (7) and (8). Q.E.D.

Proof of Proposition 1: We use $M^n$ to denote the event that all $n$ signals are $M$, and SP the secondary market value of the whole firm. Then we have

$$SP(M^n) = E[v|M^n] = v_G\text{Pr}(v = v_G|M^n) + v_B\text{Pr}(v = v_B|M^n) = \text{Pr}(v = v_G|M^n)(v_G - v_B) + v_B$$

(A.4)

$$= \frac{\text{Pr}(v = v_G)\text{Pr}(M^n|v = v_G)}{\text{Pr}(v = v_G)\text{Pr}(M^n|v = v_G) + \text{Pr}(v = v_B)\text{Pr}(M^n|v = v_B)}(v_G - v_B) + v_B$$

(A.5)
which means that the secondary market price will be $\theta v_G + (1 - \theta)v_B$ if all $n$ signals are $M$.

If the firm is of type $G$, with probability $(1 - p)^n$ all information producers will receive the signal $M$, and the secondary market price is $\theta v_G + (1 - \theta)v_B$; with probability $1 - (1 - p)^n$ at least one information producer will receive a signal $H$, and the secondary market price is $v_G$. So the expected secondary market value of a type-$G$ firm is

$$E[SP^G(n)] = \theta v_G + (1 - \theta)v_B[(1 - p)^n + (1 - (1 - p)^n)v_G = [1 - (1 - \theta)(1 - p)^n](v_G - v_B) + v_B,$$

(A.7)

which increases in $n$ since $\frac{\partial E[SP^G(n)]}{\partial n} = -(v_G - v_B)(1 - \theta)(1 - p)^n \ln(1 - p) > 0$. Similarly, the expected secondary market value of a type-$B$ firm is

$$E[SP^B(n)] = (1 - p)^n[\theta v_G + (1 - \theta)v_B] + (1 - (1 - p)^n)v_B = \theta(1 - p)^n(v_G - v_B) + v_B,$$

(A.8)

which decreases in $n$ since $\frac{\partial E[SP^B(n)]}{\partial n} = (v_G - v_B)\theta(1 - p)^n\ln(1 - p) < 0$. The expected secondary market price for the firm across types is

$$E[SP(n)] = \theta E[SP^G(n)] + (1 - \theta)E[SP^B(n)],$$

(A.9)

Plugging Equations (A.7) and (A.8) into (A.9), we have

$$E[SP(n)] = \theta v_G + (1 - \theta)v_B,$$

(A.10)

which is independent of $n$. Q.E.D.

**Proof of Lemma 1.** Since uninformed bidders have no information and all of them are in the same position ex ante, we consider the symmetric pure strategy equilibrium in which all of them bid the same number, say, $b^\alpha$. First, consider the case with $m > k$ uninformed bidders. The expected payoff to each uninformed bidder is

$$E[\pi^u(n, m, b^\alpha)] = \sum_{j=0}^{k-1} \Pr(j, b^\alpha) \left( \frac{k-j}{m} \right) [E[\pi^u(j)] - p(j, b^\alpha)],$$

where $\Pr(j, b^\alpha)$ is the probability that $j$ informed bidders will bid above $b^\alpha$, $E[\pi^u(j)]$ is the expected value per share when there are $j$ informed bidders bidding above $b^\alpha$, and $p(j, b^\alpha)$ is the price uninformed investors will pay for each share in this case. If the above payoff is strictly negative, the uninformed bidders would be better off not bidding at all, and this contradicts the assumption that $b^\alpha$ is the equilibrium bidding strategy. If the payoff is strictly positive, one uninformed investor can deviate from the equilibrium bidding strategy and bid $b^\alpha + \varepsilon$, where $\varepsilon \rightarrow 0^+$. $\Pr(j, b^\alpha)$ and $p(j, b^\alpha)$ will remain constant when $\varepsilon$ is infinitely small. However, this bidder will be allocated one full share instead of $\frac{k-j}{m} < 1$ share. The payoff would be

$$E[\pi^u(n, m, b^\alpha + \varepsilon)] = \sum_{j=0}^{k-1} \Pr(j, b^\alpha) [E[\pi^u(j)] - p(j, b^\alpha)] > E[\pi^u(n, m, b^\alpha)],$$

which means that the uninformed investor benefits from raising the bid. This contradicts the assumption that $b^\alpha$ is the equilibrium bidding strategy for uninformed investors. Therefore, we must have $E[\pi^u(n, m, b^\alpha)] = 0$ when $m > k$. 

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Now consider the case when $m \leq k$. If $E[\pi^a(n, n, b^x)] < 0$, uninformed investors are better off not bidding at all, and this contradicts the assumptions that $b^x$ is the equilibrium bidding strategy. If $E[\pi^a(n, n, b^x)] > 0$, the uninformed investors who do not bid in the auction will find it optimal to engage in uninformed bidding. This will either drive $E[\pi^a(n, m, b^x)]$ to 0, which completes our proof, or drive the number of uninformed bidders above $k$, in which case we have already proved $E[\pi^a(n, m, b^x)] = 0$. Q.E.D.

**Proof of Proposition 2.** We proved in Lemma 1 that the payoff to uninformed investors is zero. If an investor decides not to produce information, he can either engage in uninformed bidding or not bid at all, and the payoffs to both strategies are 0. The number of information producers depends on the cost and benefits of informed bidding. If the gross payoff to informed bidding, $E[\pi^a(n, a, a)]$, is greater than the information production cost, $C$, more investors will choose to produce information. Since $E[\pi^a(n, a, a)]$ decreases in $n_a$, this process will continue until investors are indifferent between information production and remaining uninformed. Therefore, there could be both informed and uninformed bidding in the general IPO auction, and $n_a$ is characterized by Equation (3), that is, $E[\pi^a(n, a, a)] - C \equiv 0$. Part (ii) of the proposition directly follows from the implicit function theorem and the assumption that $\frac{\partial E[\pi^a(n, a, a)]}{\partial a} > 0$, and $\frac{\partial E[\pi^a(n, a, a)]}{\partial n_a} < 0$. Q.E.D.

**Proof of Proposition 3.** First, note that the value of the fraction of the firm offered in the IPO goes to two sources: part of it goes to the issuing firm, which is the proceeds to the issuing firm in the IPO; part of it goes to the bidders. We have already proved that the uninformed bidders earn zero profit, and the payoff to each informed bidder equals the issuing firm in the IPO; part of it goes to the bidders. We have already proved that $E[\pi^a(n, m, b^x)] = 0$. Q.E.D.

**Proof of Lemma 2.** Each information producer’s expected payoff is

$$E[\pi^a(n, a)] = Pr(s_i = H)E[\pi^a(n, a)|H] + Pr(s_i = M)E[\pi^a(n, a)|M] + Pr(s_i = L)E[\pi^a(n, a)|L]$$

$$= \theta A\frac{\binom{n-1}{k-1}}{n-1} (1-p) m(x) \binom{n-2}{k-1} [p + (1-p)(1 - M(x))]^{k-1}(1 - p) M(x)^{n-k-1} dx. \tag{A.13}$$

If we change it into an integration of $M(x)$, we have
\[ E[\pi^e(n,\alpha)] = \binom{n-1}{1} \binom{n-2}{k-1} p(1-p)^{\alpha_k} \]
\begin{align*}
\int_0^1 & \theta[p + (1-p)(1-M)]^{\alpha_k-1} [(1-p)M^{\alpha_k-1}(1-\theta)(1-p)(1-M)]^{\alpha_k-1} [p + (1-p)M^{\alpha_k-1}] \, dM.
\end{align*}

Therefore, we have
\[ E[\pi^e(n,\alpha)] = p(1-p)^{\alpha_k} \int_0^1 Y(M; n) \binom{n-1}{1} \binom{n-2}{k-1} (1-M)^{k-1} M^{\alpha_k-1} \, dM \]

where
\[ Y(M; n) = \frac{\theta[p + (1-p)(1-M)]^{\alpha_k-1} (1-p)^{\alpha_k-2} (1-\theta)[p + (1-p)M^{\alpha_k-1}]}{\theta[p + (1-p)(1-M)]^{\alpha_k-1} [(1-p)M^{\alpha_k-1} + (1-\theta)(1-p)(1-M)]^{\alpha_k-1} [p + (1-p)M^{\alpha_k-1}]}.
\]

Note that
\[ \int_0^1 \binom{n-1}{1} \binom{n-2}{k-1} (1-M)^{k-1} M^{\alpha_k-1} \, dM = 1 \]
and \( Y(M; n) \) is decreasing in \( n \).

Therefore, \( E[\pi^e(n,\alpha)] \) decreases in \( n \). This completes the proof of part (i).

We can express \( E[IR^e_n(n)] \) as
\[ E[IR^e_n(n)] = \alpha - \alpha \int_0^1 Z(M, n) \binom{n}{1} \binom{n-1}{k} (1-M)^k M^{\alpha_k-1} \, dM, \]

where
\[ Z(M, n) = \]
\begin{align*}
\int_0^1 & (1-\theta)[p + (1-p)M^{\alpha_k-1} (p + (1-p)(1-M)]^{\alpha_k-1} [(1-p) + p/(1-M)] \, dM.
\end{align*}

Note that
\[ \int_0^1 \binom{n}{1} \binom{n-1}{k} (1-M)^k M^{\alpha_k-1} \, dM = 1 \]
and \( Z(M, n) \to 0 \) as \( n \to \infty \).

Therefore, we have
\[ \int_0^1 Z(M, n) \binom{n}{1} \binom{n-1}{k} (1-M)^k M^{\alpha_k-1} \, dM \to 0, \text{ and } E[IR^e_n(n)] \to \alpha \text{ as } n \to \infty. \]

This completes the proof of part (ii). Q.E.D.

**Proof of Lemma. 3** From Proposition 4, we can see that in equilibrium, the payoff to each informed bidder after cost is 0. The payoff to an uninformed bidder in the presence of \( n \) information producers is
\[ E[\pi^f(F, n, \alpha)] = \theta \left( Z_k v_G - F \right) \frac{k}{n+1} - (1-\theta)(F - \frac{\alpha}{k} v_B) \sum_{j=k-1}^{n-1} \binom{n}{j} (1-p)^j p^{n-j} \frac{k}{j+1}. \]
which is generically nonzero. If the above is positive, then uninformed bidding dominates informed bidding and no investor will produce information. If it is negative, then informed bidding dominates uninformed bidding. Therefore, depending on the range of the offer price \( F \), there will be either informed bidding or uninformed bidding in equilibrium, but not both.

**Proof of Proposition 4.** Suppose bidders 2, 3, \ldots, \( n \) use the specified bidding strategy. Because the true value of each share is either \( \frac{2}{k} v_B \) or \( \frac{2}{k} v_G \), the offer price has to be in between; otherwise, either nobody will bid or everyone will bid without producing any information. If bidder 1 observes \( L \), he knows that the firm is a type \( B \) firm. The expected payoff from bidding is

\[
\left( \frac{2}{k} v_B - F \right) \sum_{j=k}^{n-1} \binom{n-1}{j} (1-p)^{p^{n-1-j}} \frac{k}{j+1} < 0,
\]

so he will not bid. If bidder 1 observes \( H \), his payoff from bidding is

\[
\left( \frac{2}{k} v_B - F \right) \frac{k}{n} - (1-\theta)(F - \frac{2}{k} v_B) \sum_{j=k}^{n-1} \binom{n-1}{j} (1-p)^{p^{n-1-j}} \frac{k}{j+1} > 0,
\]

which is nonnegative if

\[
F \leq \frac{\theta(1-p)}{\theta(1-p)+\theta(1-\frac{2}{k} v_B)} \frac{k}{n} - (1-\theta)(F - \frac{2}{k} v_B) + \frac{2}{k} v_B,
\]

so that it is optimal for bidder 1 to bid if he observes \( M \). This means that it is also optimal for bidder 1 to play the specified strategy. Therefore, the specified strategy is a symmetric Nash equilibrium.

When the offering price is set at \( F \), the expected payoff to each participant as a function of total number of information producers, \( n \), is

\[
E[\pi'(F,n,\alpha)] = \theta\left( \frac{2}{k} v_B - F \right) \frac{k}{n} - (1-\theta)(F - \frac{2}{k} v_B) \sum_{j=k}^{n-1} \binom{n-1}{j} (1-p)^{p^{n-1-j}} \frac{k}{j+1}.
\]

(A.15)

Since

\[
\sum_{j=k}^{n-1} \binom{n-1}{j} (1-p)^{p^{n-1-j}} \frac{k}{j+1} = \frac{k}{n} \sum_{i=0}^{n} \binom{n}{i} (1-p)^{p^{n-i}} \approx \frac{k}{n(1-p)^{n-1}},
\]

we have

\[
E[\pi'(F,n,\alpha)] = \left( \frac{\theta v_B + (1-\theta) v_G}{k} - F \right) \frac{k}{n},
\]

so that the expected payoff to each information producer is a decreasing function of \( n \). Since there is free entry in terms of information production, the number of information producers is given by \( \text{Equation (10)} \), which leads to

\[
\left( \frac{\theta v_B + (1-\theta) v_G}{k} - F \right) \frac{k}{n} = C.
\]

By the implicit function theorem, we have

\[
\frac{\partial n_f}{\partial F} = \frac{k/n}{\left( \frac{\theta v_B + (1-\theta) v_G}{k} - F \right) k/n^2} = \frac{-n}{\left( \frac{\theta v_B + (1-\theta) v_G}{k} - F \right) k/n^2} < 0,
\]

(A.16)

which completes the proof. Q.E.D.

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44 We make this approximation since \( \sum_{i=k}^{n} C_i (1-p)^{p^{n-i}} \) is very close to 1. For example, when \( p = 0.01, k = 13, \) and \( n = 14 \) (we assume that \( n \geq k+1 \) in either auctions or fixed-price offerings), we find that \( \sum_{i=k}^{n} C_i (1-p)^{p^{n-i}} = 0.9916 \). For \( n > 14 \), it is even closer to 1: for example, when \( n = 15, \) \( \sum_{i=k}^{n} C_i (1-p)^{p^{n-i}} = 0.9995842 \), and for \( n=16, \sum_{i=k}^{n} C_i (1-p)^{p^{n-i}} = 0.9999835 \). The error is only of a similar magnitude for other values of \( k \).
Proof of Proposition 5. The average secondary market price of the whole firm is \[ \theta v_G + (1 - \theta) v_B \], as we proved in Proposition 1. Since a fraction \( z \) of the firm is sold in the IPO, and that fraction is divided into \( k \) shares, the average secondary market price of each share is \( \frac{\theta v_G + (1 - \theta) v_B}{k} \). If the offering price of each share, \( F < \theta v_G + (1 - \theta) v_B \), it is an underpricing equilibrium.

Proving that \( \frac{\theta v_G + (1 - \theta) v_B}{k} \) decreases in \( z \) is equivalent to proving that \( \frac{\theta v_G + (1 - \theta) v_B}{k} \) increases in \( z \). In Proposition 4 we have proved that the number of participants in a fixed-price offering is \( \left( \frac{\theta v_G + (1 - \theta) v_B}{k} - F \right)^{\frac{k}{z}} = C \), or \( \theta v_G + (1 - \theta) v_B = kF + nC \). The objective of the type \( G \) firm is

\[
M_{\theta x} \quad \text{s.t.} \quad \theta v_G + (1 - \theta) v_B = kF + nC.
\] (A.17)

Solving the above optimization problem, we find that

\[
F = \frac{\theta v_G + (1 - \theta) v_B}{k} = C \frac{\ln\{1 - (1 - \theta)(1 - \theta)\} - \ln\left(1/(1 - \theta)\right)}{\ln\left(1/(1 - \theta)\right)} (A.19)
\]

This means \( \frac{\theta v_G + (1 - \theta) v_B}{k} = \frac{1}{k} - \frac{\ln\{1 - (1 - \theta)(1 - \theta)\} - \ln\left(1/(1 - \theta)\right)}{\ln\left(1/(1 - \theta)\right)} \) is indeed increasing in \( z \). Q.E.D.

Proof of Proposition 6. We first prove that the type \( B \) firm will always mimic the type \( G \) firm by choosing the same IPO mechanism (and the same offer price in the case of a fixed-price offering). If the type \( B \) firm chooses to separate, it will reveal to investors its true value, and its total payoff is \( v_B - T \). If the type \( B \) firm chooses to mimic, with a positive probability, the IPO will be successful and the payoff to the type \( B \) firm will be higher than \( v_B - T \); with the complementary probability, the IPO will fail, and the payoff is \( v_B - T \). In summary, the expected payoff to the type \( B \) firm from mimicking is strictly higher than that from separating so that it will pool with the type \( G \) firm in equilibrium. The specified beliefs are consistent with the equilibrium strategies of all parties. Finally, the secondary market price is also consistent with equilibrium strategies and beliefs of all parties from the proof of Proposition 1. Q.E.D.

Proof of Proposition 7. Assume that \( C \) is not prohibitively large so that there are at least \( k + 1 \) information producers even when a general IPO auction is used, that is, \( C \leq C_{\text{max}} \), where \( C_{\text{max}} = \max\{ C : n_a \geq k + 1 \} \). Use \( E[R_G^C] \) and \( E[R_F^C] \) to denote the combined proceeds to the type \( G \) firm when an auction and a fixed-price offering are used, respectively. We first prove that for any value \( R < v_G - T \), there exists a value \( n \) such that for all \( n_a \geq n \), \( E[R_G^C(n_a)] > R \). The combined proceeds to the type \( G \) firm when an auction is used is \( E[R_G^C(n)] = E[IR_G^C(n)] + (1 - z) E[SP_G^C(n)] - T \), which is increasing in \( n \) since both \( E[IR_G^C(n)] \) and \( E[SP_G^C(n)] \) are increasing in \( n \) as shown in Proposition 1 and Assumption 2. Furthermore, \( E[R_G^C(\alpha, n)] \rightarrow v_G - T \) as \( n \rightarrow \infty \) since \( E[IR_G^C(n)] \rightarrow \alpha v_G \) (from Assumption 2) and \( E[SP_G^C(n)] \rightarrow v_G \) (from Proposition 1) as \( n \rightarrow \infty \). Therefore, there exists a value \( n_\alpha \) such that for all \( n_a \geq n_\alpha \), \( E[R_G^C(n_a)] > R \). In particular, we have
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\[ E[R_G^T] = kF + (1 - z)\{(1 - (1 - \theta)(1 - p)^{\eta_0})(v_G - v_B) + v_B\} - T < v_G - T, \]

since \( F < \frac{z}{\tilde{v}_G} \) and \( 1 - (1 - \theta)(1 - p)^{\eta_0} < 1 \). Thus, there exists a value \( \tilde{n} \) such that for all \( n_\eta \geq \tilde{n} \), \( E[R_G^T] > E[R_B^T] \). At the same time, we proved in proposition 2 that \( \frac{\partial n_\eta}{\partial \tilde{v}_G} < 0 \). Therefore, for a given value of \( z < \tilde{z} \), there exists a critical value of \( C \) such that for all \( C \leq C \), \( n_\eta \geq \tilde{n} \). This proves part (i) of the proposition: when \( C \leq C \), general IPO auction is the equilibrium choice.

To prove part (ii), let us further assume \( C_{\max} \leq \frac{p(1 - z)(1 - p)^{\eta_0} - (1 - \theta)(v_G - v_B)}{v_G - v_B} + v_B \). When \( C = C_{\max} \), \( n_\eta = k + 1 \) by definition. The combined proceeds to the type G firm is

\[ E[R_G^T(k + 1)] = E[R_G^T(k + 1)] + (1 - z)\{(1 - (1 - \theta)(1 - p)^{k + 1})(v_G - v_B) + v_B\} - T < xv_G + (1 - z)\{(1 - (1 - \theta)(1 - p)^{k + 1})(v_G - v_B) + v_B\} - T. \]  

(A.20)

If a fixed-price offering is used, the maximized combined proceeds from using fixed-price offering is at least as high as when \( n_\eta = k + 2 \), that is, \( E[R_G^T(n_\eta')] \geq E[R_G^T(n_\eta = k + 2)] \). Using Equation A.18,

\[ E[R_G^T(n_\eta')] \geq E[R_G^T(n_\eta = k + 2)] = \left[\theta v_G + (1 - \theta)v_B\right]a - (k + 2)C + (1 - \theta)
\]

\[ \times\left\{(1 - (1 - \theta)(1 - p)^{k + 2})(v_G - v_B) + v_B\right\} - T. \]  

(A.21)

Combining Equations (A.20) and (A.21), we have

\[ E[R_G^T] - E[R_G^T] > \left[p(1 - z)(1 - p)^{k + 1} - x\right](1 - \theta)(v_G - v_B) - (k + 2)C. \]

Since \( C \leq C_{\max} \leq \frac{p(1 - z)(1 - p)^{k + 1} - x(1 - \theta)(v_G - v_B)}{v_G - v_B} + v_B \), \( E[R_G^T] - E[R_G^T] > 0 \), that is, it is optimal for the type G firm to choose a fixed-price offering in this case. Note that \( \frac{p(1 - z)(1 - p)^{k + 1} - x(1 - \theta)(v_G - v_B)}{v_G - v_B} + v_B \) decreases in \( x \) so we need \( x < \tilde{z} \) to ensure \( C \leq C \), fixed-price offerings are the equilibrium choice.

Given the out-of-equilibrium belief that any firm that does not follow the equilibrium strategy is a type B firm, if the type B firm chooses to make an out-of-equilibrium move, its total payoff is \( v_B - T \). In contrast, in equilibrium, the type B firm pools with the type G firm with a positive probability, and the payoff to the type B firm will be higher than \( v_B - T \); with the complementary probability, the IPO will fail, and the payoff is \( v_B - T \). Therefore, the type B firm has no incentive to choose the out-of-equilibrium strategy. Q.E.D.

**Proof of Proposition 8.** We already proved in the proof of proposition 7 that there exists a value \( \tilde{n} \) such that for all \( n_\eta \geq \tilde{n} \), \( E[R_G^T] > E[R_B^T] \). At the same time, we proved in proposition 2 that \( \frac{\partial n_\eta}{\partial \tilde{v}_G} < 0 \). Therefore, for a given value of \( C \), there exists a critical value of \( \tilde{z} \) such that for all \( z \geq \tilde{z} \), \( n_\eta \geq \tilde{n} \). This proves part (ii) of the proposition: when \( z \geq \tilde{z} \), general IPO auction is the equilibrium choice.

Define \( z = \min\{z : n_\eta = k + 1\} \), that is, when \( z = \tilde{z} \), \( n_\eta = k + 1 \). Further assume \( z \leq \frac{p(1 - p)^{k + 1} - (k + 2)(1 - \theta)(v_G - v_B)}{1 + p(1 - p)^{k + 1}} \). Note that \( \frac{p(1 - p)^{k + 1} - (k + 2)(1 - \theta)(v_G - v_B)}{1 + p(1 - p)^{k + 1}} \) decreases in \( C \) so we need \( C < \tilde{C} \) to ensure \( \frac{p(1 - p)^{k + 1} - (k + 2)(1 - \theta)(v_G - v_B)}{1 + p(1 - p)^{k + 1}} > 0 \). If a fixed-price offering is used, the maximized combined proceeds is at least as high as when \( n_\eta = k + 2 \), that is,
\[ E[R_f^G(n_f')] \geq E[R_f^G(n_f = k + 2)]. \] Using an argument like that in the proof of proposition 7, we have
\[ E[R_f^G] - E[R_f^G] > (p(1 - \alpha)(1 - p)^{k+1} - \alpha(1 - \theta)(v_G - v_B) - (k + 2)C). \]

We have \( E[R_f^G] - E[R_f^G] > 0 \) since \( \alpha \leq \tilde{\alpha} \), such that when \( \alpha \leq \tilde{\alpha} \), we have \( E[R_f^G] - E[R_f^G] > 0 \). That is, when the firm sells a small fraction of equity in the IPO, fixed-price offerings are the equilibrium choice.

Like in the proof in Proposition 7, because the out-of-equilibrium belief that any firm that does not follow the equilibrium strategy is a type B firm, the type B firm has no incentive to choose the out-of-equilibrium strategy. This is because if the type B firm chooses to make an out-of-equilibrium move, its total payoff is \( v_B - T \). In contrast, in equilibrium, the type B firm pools with the type G firm with a positive probability, and the payoff to the type B firm will be higher than \( v_B - T \); with the complementary probability, the IPO will fail, and the payoff is \( v_B - T \). Q.E.D.

**Proof of Proposition 9.** Denote the maximized payoff to the type G firm when a fixed-price offering is used and \( \alpha \) is endogenous by \( E[R_f^G(\alpha')] \). Clearly, we have \( E[R_f^G(\alpha')] < v_G - T \).

We have already showed in the proof of Proposition 7 that for any value \( R < v_G - T \), there exists a value \( \tilde{n} \) such that for all \( n_u \geq \tilde{n}, E[R_u^G(n_u)] > R \). We have also proved in Proposition 2 that \( \frac{\partial \alpha}{\partial n} \leq 0 \) and \( \frac{\partial \alpha}{\partial n} > 0 \). Therefore, there exists a critical value \( \alpha' \) such that for all \( \alpha \geq \alpha_{min} \) and \( C < C' \), we have \( E[R_u^G(\alpha', C)] > E[R_u^G(\alpha')] \). In particular, we have \( E[R_u^G(\alpha')] > E[R_u^G(\alpha')] \). Now consider the optimal fraction of equity sold in the IPO when auction is used. The derivative of the objective function with respect to \( \alpha \) is
\[
\frac{\partial E[R_u^G(\alpha)]}{\partial \alpha} = \frac{\partial E[I(1 - (1 - \theta)(1 - p)^{\alpha})(v_G - v_B) + v_B]}{\partial \alpha}
- \frac{\partial E[I(1 - (1 - \theta)(1 - p)^{\alpha})(v_G - v_B) + v_B]}{\partial \alpha}
- \frac{\partial E[I(1 - (1 - \theta)(1 - p)^{\alpha})(v_G - v_B) + v_B]}{\partial \alpha}
- \frac{\partial E[I(1 - (1 - \theta)(1 - p)^{\alpha})(v_G - v_B) + v_B]}{\partial \alpha}
- \{[1 - (1 - \theta)(1 - p)^{\alpha}][v_G - v_B] + v_B\}.
\]

By assumption 2, we know that when \( C \) is small (0 < \( C \leq C'' \)) and \( \alpha = \alpha_{min}, n \to \infty, \) and \( \frac{\partial E[R_u^G(n)]}{\partial n} \to 0 \), and \( \frac{\partial E[1 - (1 - \theta)(1 - p)^{\alpha}](v_G - v_B) + v_B]}{\partial n} \to 0 \). Therefore, we have \( \frac{\partial E[R_f^G(\alpha = \alpha_{min})]}{\partial \alpha} < 0 \), and \( \alpha' = \alpha_{min} \) due to the constraint \( \alpha \geq \alpha_{min} \). Since \( \frac{\partial m}{\partial n} < 0, \) and \( \frac{\partial E[I(1 - (1 - \theta)(1 - p)^{\alpha}][v_G - v_B] + v_B]}{\partial n} \to 0 \), there exists a critical value \( \alpha' \) such that for \( C > C'' \), \( \frac{\partial E[R_f^G(\alpha_{min})]}{\partial \alpha} > 0 \). Therefore, for moderate values of \( C (C' < C < C''), \alpha' > \alpha_{min} \). When \( C \) is high, let the maximized payoff to the type G firm when an auction is used be \( E[R_f^G(\alpha')] \). We have already proved in proposition 7 that for given values of \( \alpha, \) fixed-price offerings dominate auctions. In particular, we have \( E[R_f^G(\alpha')] > E[R_f^G(\alpha')] \) in this case. By definition of \( \alpha' \), we have \( E[R_f^G(\alpha')] > E[R_f^G(\alpha')] \). Therefore, we have \( E[R_f^G(\alpha')] > E[R_f^G(\alpha')] \) when \( C \) is large. That is, fixed-price offerings...
are the equilibrium choice when the information production cost is high even when the fraction of equity sold in the IPO is endogenous. Q.E.D.

Proof of Lemma 4. First, we prove that for any \( y \in \left( \frac{1}{k} v_B, \frac{2}{k} v_G \right) \), \( E[\pi^n(n, x)|p < y] \geq 0 \) when there is no reservation price. We prove by contradiction. Suppose that for some \( y \in \left( \frac{2}{k} v_B, \frac{2}{k} v_G \right) \), \( E[\pi^n(n, x)|p < y] < 0 \). Further, suppose that one information producer, after observing a private signal, decides to bid \( b_i < y \). By assumption 3, the market clearing price would be less than \( y \), so that the expected payoff to the bidder is negative since \( E[\pi^n(n, x)|p < y] < 0 \). The bidder is therefore better off by not bidding at all. However, if everyone is bidding above \( y \) or not bidding at all, the market clearing price cannot be \( p < y \), a contradiction.

We can express the expected payoff to each information producer (before information production cost), when a reservation price \( r > \frac{2}{k} v_B \) is set, as

\[
E[\pi^n(n, x, r > \frac{2}{k} v_B)] = \Pr(p \geq r)E[\pi^n(n, x)|p \geq r],
\]

and that without a reservation price as

\[
E[\pi^n(n, x)] = \Pr(p < r)E[\pi^n(n, x)|p < r] + \Pr(p \geq r)E[\pi^n(n, x)|p \geq r].
\]

We just proved \( E[\pi^n(n, x)|p < r] \geq 0 \). Therefore, we have \( E[\pi^n(n, x)] \geq E[\pi^n(n, x, r > \frac{2}{k} v_B)] \). Furthermore, since the equilibrium number of information producers is the \( n \) such that \( E[\pi^n(n)] = C \), and \( \frac{\partial E[\pi^n(n)]}{\partial n} < 0 \), setting \( r > \frac{2}{k} v_B \) will not increase the equilibrium number of information producers. Q.E.D.

Proof of Proposition 10. Because setting the no reservation price is equivalent to setting \( r = \frac{2}{k} v_B \), we treat the results in the basic model as if the firm sets \( r = \frac{2}{k} v_B \). In that sense, we have already proved in the basic model that \( E[R^n_C(r = \frac{2}{k} v_B)] > E[R^n_C] \) when \( C \leq \hat{C} \) and \( x > \hat{z} \). By definition of \( r^* \), we have \( E[R^n_C(r^*)] \geq E[R^n_C(r = \frac{2}{k} v_B)] \). Therefore, we have \( E[R^n_C(r^*)] > E[R^n_C] \) when \( C \leq \hat{C} \) and \( x > \hat{z} \); that is, the general IPO auction with the optimal reservation price will dominate fixed-price offering when \( C \) is small and \( x \) is large. We also already proved in the basic model that when \( C \) is large (\( C > \hat{C} \)) and \( x \) is small (\( x \leq \hat{z} \)), fixed-price offerings dominate IPO auctions: \( E[R^n_C] > E[R^n_C(r = \frac{2}{k} v_B)] \). We also proved in Lemma 4 that setting a reservation price \( r > \frac{2}{k} v_B \) will never increase the number of information producers, and the issuing firm suffers from not enough information production when \( C \) is large (\( C > \hat{C} \)) and \( x \) is small (\( x \leq \hat{z} \)). The firm will set the reservation price as low as possible, that is, \( r^* = \frac{2}{k} v_B \) in this case. Therefore, we have \( E[R^n_C] > E[R^n_C(r = \frac{2}{k} v_B)] = E[R^n_C(r^*)] \). This proves that fixed-price offering dominates IPO auction with optimal reservation price when \( C \) is large and \( x \) is small. Q.E.D.