A Theory of Corporate Boards and Forced CEO Turnover

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1. Introduction
The seeming inability of corporate boards to monitor the performance of firm management proactively, and thus prevent corporate crises, has become the topic of debate among both practitioners and academics over the last few years in the wake of various corporate scandals. A number of important questions arise in the above context. First, what explains situations where boards do not fire chief executive officers (CEOs), even when a majority of directors are individually convinced that the CEO is of poor quality? In particular, what are the characteristics of boards that fire poor-quality CEOs proactively versus waiting and firing such CEOs only after considerable destruction in shareholder value has occurred? Second, what is the relationship between board size and the effectiveness of board decision making in terms of monitoring the CEO? Third, what is the relationship between board composition (the proportion of outside directors) and the effectiveness of board decision making and forced CEO turnover? Finally, how will the various policy proposals for corporate governance and board reform that have been suggested by academics, practitioners, and policy makers affect the ability of a board to appropriately monitor the CEO and fire him or her as necessary?

Some of the above questions have already been addressed by the existing theoretical literature on corporate boards (reviewed in Section 2). Thus, three of the most prominent models of corporate boards—namely, those of Hermelin and Weisbach (1998), Adams and Ferreira (2007), and Harris and Raviv (2008)—have developed their own answers to some of these questions. However, most of the existing theoretical models treat the board as a monolithic entity. In contrast to these models, our objective in this paper is to address the above and related questions in a simple framework that explicitly models information sharing among directors and subsequent voting, and to use this framework to analyze coordination and communication problems in multiagent boards. We then develop the implications of our model for board passivity, optimal board size, optimal board composition, and various other issues related to board decision making.

We consider a firm with an incumbent CEO, whose quality is uncertain, and a board consisting of a number of directors whose role is to evaluate the CEO and decide whether to fire her or to retain her. Each director receives an independent private signal about the CEO’s quality, after which directors discuss this quality and vote to retain or replace the CEO. Directors care about true CEO ability, since it affects their equity holding values; however, a CEO may impose costs of dissent on a director who votes to fire but fails to oust her. We relate the equilibrium CEO firing decision to board size, board composition, the effect of an imprecise public signal, and the cost and probability of finding a good replacement CEO.
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In the absence of dissent costs, each director reports his private signal truthfully in the discussion round and then votes to fire the CEO if the majority of signals reported in the discussion round are bad and votes to retain her otherwise. Furthermore, in the absence of dissent costs, we show that the board’s firing decision efficiently aggregates all directors’ private signals, in the sense that the board fires the CEO only if, in the opinion of a social planner who has access to all directors’ signals, the incumbent CEO is worse (in terms of expected quality) than a potential replacement; the board retains the CEO if the reverse is true.

We then analyze our basic model with dissent costs. We show that, once such dissent costs are significant, coordination problems arise between directors, so that the board may become suboptimally passive. In particular, the board may choose to retain the CEO even when a majority of directors are privately convinced that the CEO is of poor quality. This is because, even when each director has a bad signal about the CEO’s quality, his probability assessment that enough other directors will vote against the CEO (and thus will be able to successfully fire the CEO and avoid incurring dissent costs) is not large enough, so that the director is better off voting to retain the CEO.\(^3\) In this case, we show that whether each director reports his signal truthfully before the vote and then votes accordingly (in this case we will say that the director discusses and votes “informatively”) depends on how well he is incentivized: if each director’s equity holdings in the firm is large enough, so that his dissent cost to equity loss ratio is low, then he discusses and votes informatively; however, if this ratio is high, he discusses “uninformatively” (i.e., does not reveal his private information truthfully to other directors) and votes to retain the CEO even if he receives a bad signal about the CEO’s quality.

We then analyze the effect of board size on the quality of the board’s decision making. On the one hand, since each director’s signal about the CEO’s quality is independent (conditional on true CEO quality), a larger number of directors increases the amount of information that is potentially available (collectively) to the board. On the other hand, a larger board worsens the coordination problems across directors.\(^3\) The optimal board size emerges from this trade-off.

We then extend our basic model to analyze issues of optimal board composition. Here, we distinguish between insider directors and independent outsiders: while outsiders are similar to the directors in our basic model, insiders are assumed to receive more precise signals but have higher dissent costs than outsiders. The advantage of insider-dominated boards is that insiders have better information about the quality of the CEO; the disadvantage is that, since insiders have higher dissent costs, they are less likely to discuss and vote informatively (according to their own private signals). We show that there is an optimal composition of the board and that, in equilibrium, boards should include as many insiders as possible to take advantage of their superior signal quality but have enough outsiders to ensure that all directors discuss and vote informatively.

We now extend our basic model in a different direction: we study the effect of a public signal on the coordination problem among directors. One might at first think that publicly observable information regarding the CEO’s quality (for example, the accounting performance of the firm under the CEO’s management) is a natural coordination mechanism. To examine this question, we assume here that, in addition to their own private signals, all directors observe a noisy public signal of the CEO’s quality, before voting. We show that having noisy public signals of CEO performance available to the board will not always improve board decision making, since such signals may sometimes make the directors’ coordination problem worse rather than better. The intuition here is that, when a director receives a negative private signal but a positive public signal, he may choose not to vote against the CEO, since, given the positive public signal, he assesses a greater probability that many other directors have also received positive private signals and will vote to retain the CEO, thus allowing the CEO to stay in power and impose dissent costs on any director who voted against her.

Even though our model is set in the context of a board deciding whether or not to force a CEO turnover, the intuition behind our model generalizes to other binary corporate decisions made by the board as well. For example, consider the case of a CEO proposing a takeover of another firm, and the board voting on the CEO’s proposal, with each director having a private signal about the desirability of the takeover from the point of view of shareholder value maximization. While, unlike in the turnover setting, rejecting the CEO’s preferred policy will not rid the firm of her, the basic trade-offs of our model will go through even in a nonturnover setting as long as we make the assumption that the costs imposed by the CEO on a dissenting director will be lower if the majority of directors went against the CEO’s proposal. Here, again, the relationship between dissent costs and equity loss will determine the equilibrium.\(^5\) Larger boards are less likely to sustain informative discussion and voting and there emerges an optimal board size even in this case. Furthermore, there will also be an optimal combination of insiders and outsiders on the board in equilibrium, based on the trade-offs we discussed earlier.

The rest of the paper is organized as follows. Section 2 relates our paper to the existing theoretical and empirical literature. Section 3 describes the setup of
our basic model. Section 4 characterizes its equilibrium and analyzes issues related to optimal board size. Section 5 extends our basic model to analyze optimal board composition. Section 6 extends our basic model to study the effect of an imprecise public signal on the effectiveness of board decision making. Section 7 presents an extension to our basic model where we introduce CEO dismissal costs and relax the assumption in our basic model that the directors’ prior probability assessment about the quality of the incumbent CEO is the same as their probability assessment of finding a good replacement CEO. Section 8 discusses the empirical and policy implications of our model, relating them to the existing empirical literature and the implications of other prominent theoretical models. Section 9 concludes. In the interest of conserving space, the proofs of all propositions are confined to an online appendix, Appendix A. We also provide another online appendix, Appendix B, giving three extensions to our basic model: Section B.1 presents a dynamic extension to our basic model that allows us to analyze the timing of forced CEO turnover, where the turnover decision is no longer dichotomous and the board may choose to delay the CEO turnover decision to another round of voting, until more information is revealed about the CEO’s quality; Section B.2 analyzes voting rules other than majority and compares the equilibrium outcome in these situations to the majority voting rule; and Section B.3 addresses the robustness of our results to removing the equilibrium selection criterion of Pareto (payoff) dominance (or synonymously, Pareto efficiency).

2. Relation to the Existing Literature

There have been several theoretical models of corporate boards in the existing literature, driven by trade-offs different from the ones we analyze here. The first theoretical analysis in this literature was by Hermalin and Weisbach (1998). In their model, board structure is the outcome of negotiation between the CEO and outside directors: CEOs, who generate a surplus for their firms (for whom good substitutes are unavailable), wield considerable influence over their outside directors and use this influence to capture some of the surplus they generate by placing insiders in open board positions. Adams and Ferreira (2007) develop a model in which the CEO’s preferred projects, which yield him control benefits, differ from those of shareholders. The CEO faces a trade-off in disclosing information to the board: if he reveals his information, he receives better advice from it, but may lose control benefits, since an informed board will also monitor him more intensively. They thus show that management-friendly boards, which can precommit not to use the information provided by the CEO to monitor him, may be value maximizing for the firm.

Harris and Raviv (2008) develop a model of a corporate board consisting of both insiders and outsiders and which can profitably use the information held by both insiders and outsiders to make optimal project choices. Insiders have private information relevant to this choice but have private benefits that lead their incentives to be misaligned with those of shareholders; outsiders, whose interests are perfectly aligned with those of shareholders, are initially uninformed but can engage in producing information relevant to project choice at a cost. Control of the board entitles the controlling party (insiders or outsiders) either to make decisions themselves or to delegate decisions to the other party. In this setting, they characterize the conditions under which there will be insider versus outsider control of the board, when each party will delegate decision making to the other party, the extent of communication between the two parties, and the number of outside directors.

Gillette et al. (2003) show both theoretically and experimentally that when agency problems are especially severe, having uninformed outsiders in control of a board can prevent inefficient outcomes.7 Hirshleifer and Thakor (1994) model the interaction between internal (corporate boards) and external (acquisitions) governance mechanisms in the maintenance of managerial quality. Raheja (2005) models a board consisting of both insiders and outsiders in charge of project selection, where insiders have decision-relevant private information and have private benefits that distort their incentives and where outsiders can engage in costly production of information relevant to project choice. Song and Thakor (2006) develop a model in which the CEO generates project ideas and the board of directors screens these ideas for approval. They assume that the board’s screening precision is controlled by the CEO and the board’s career concerns interact with those of the CEO. In this setting, they show that the board’s career concerns cause it to distort its investment recommendations procyclically, while the CEO’s career concerns cause her to sometimes reduce the precision of the board’s information. Baranchuk and Dybvig (2009) use a spatial model of board decision making to analyze bargaining among multiple types of directors.

In summary, several of the models we have discussed above have also addressed three of the central issues regarding corporate boards that we analyze here—namely, board passivity (e.g., Hermalin and Weisbach 1998, Adams and Ferreira 2007, Harris and Raviv 2008), optimal board size, and optimal board composition (e.g., Harris and Raviv 2008). However, as can be seen from our discussion above, most of the above models treat the board as a monolithic entity (or two groups,
such as insiders and outsiders). Our contribution with respect to the above literature is to introduce a simple framework to model voting and information sharing among directors, and to use this framework to analyze coordination and communication problems in multi-agent boards. It is also worth pointing out that none of the above models analyze issues related to the effect of public signals on board decision making and the dynamics (timing) of CEO firing decisions that we study here.9

While there has been little empirical research on the dynamics of boards’ CEO firing decisions, there has been a large empirical literature partially addressing some of the other questions we have raised above. However, the above evidence has been mixed, possibly due to the difficulty in overcomining the endogeneity problems that plague this literature. For example, while some papers argue that smaller boards of directors are more effective and therefore maximize shareholder value (e.g., Yermack 1996), others challenge some of the existing evidence and to develop empiriciststodevelopstrongerempiricalteststhatmay

enhance shareholder value; see, for example, Linck et al. 2008). Similarly, the evidence is also inconclusive on the question of whether a larger proportion of inside directors enhances shareholder value (e.g., Yermack 1996), others challenge this notion, documenting that, depending on various firm characteristics, either large or small board sizes may be appropriate from the point of view of enhancing shareholder value (see, e.g., Coles et al. 2008). Similarly, the evidence is also inconclusive on the question of whether a larger proportion of inside directors enhances shareholder value; see, for example, Linck et al. (2008) or Boone et al. (2007). Our paper provides a number of new testable implications that may allow empiricists to develop stronger empirical tests that may challenge some of the existing evidence and to develop new empirical findings on corporate boards and CEO turnover.

3. The Model
Our basic model consists of five dates. It starts at time 0 with an incumbent CEO in place. The CEO can be of either good or bad quality (denoted G for good and B for bad). The board consists of n members, where n \( \geq 5 \) and is odd.9 The board does not include the CEO.10 At time 1, each of n directors receives a signal about the CEO’s quality. The signal that a director i receives can be either good, in which case we say \( e_i = 0 \), or bad, in which case we say \( e_i = 1 \). At time 2, directors meet before the vote and discuss the CEO (we call it the discussion round). Directors decide simultaneously whether to report their private signals about the CEO’s quality truthfully or to lie. With some probability, the CEO will learn what signal each director revealed during the discussion round.11 We denote this probability by \( p \). At time 3, on the basis of the signals they received at time 1 and their discussion at time 2, directors vote whether to retain the current CEO or to replace her.12 If the board fires the CEO, it must hire a replacement from the pool of available candidates. The quality of this pool is indexed by \( \gamma \); \( \gamma \) is the ex ante probability of hiring a high-quality replacement. At time 4, the true quality of the then-incumbent CEO is revealed (either of the CEO existing at time 0 if she was not fired or of the new CEO hired at time 3).13 The game ends and all payoffs are distributed. The timeline of the model is depicted in Figure 1. All agents are risk neutral, and the risk-free rate of return is normalized to zero.

The private signals received by directors are assumed to be conditionally independent across directors and informative. In particular, \( p(e_i = 0 | G) = p(e_i = 1 | B) = \alpha > \frac{1}{2} \). Thus, \( \alpha \) gives the precision of each director’s signal about the CEO’s quality. We assume that all directors have signals of identical precision. We further assume that the prior probability assessment of a director that the incumbent CEO is good is \( \gamma \), the same as the probability of hiring a high-quality replacement. (This assumption is made for analytical simplicity: we relax this assumption in Section 7 of the paper.) The assumption of informative signals implies that \( p(B | 1) > 1 - \gamma > p(B | 0) \); that is, after observing a bad private signal, a director’s private assessment of the incumbent CEO’s quality is lower than that of a potential replacement, and after observing a good private signal, a director’s private assessment of the incumbent CEO’s quality is higher than that of a potential replacement.

3.1. The Board’s Voting Rule
Using Bayes’ rule, each director’s updated probability assessment of the CEO being of bad quality conditional on his private signal, denoted as \( p(B | e_i) \), is given by

\[
\begin{align*}
    p(B | 0) & \equiv p(B | e_i = 0) = \frac{(1 - \gamma)(1 - \alpha)}{(1 - \gamma)(1 - \alpha) + \gamma \alpha}, \\
    p(B | 1) & \equiv p(B | e_i = 1) = \frac{(1 - \gamma)\alpha}{(1 - \gamma)\alpha + \gamma(1 - \alpha)}.
\end{align*}
\]

Figure 1. Timeline of the Basic Model
Following the existing voting literature (see, e.g., Austen-Smith and Banks 1996), we use the following definitions.

**Definition 1.** A voting rule, \( k \), is the minimum number of votes required to fire the CEO.

**Definition 2.** A director is pivotal if his vote determines the outcome of the vote.

Under the assumptions that we have made so far, the CEO’s expected quality is worse than that of a potential replacement if the majority of directors’ private signals are bad and the CEO’s expected quality is better than that of a potential replacement if the majority of directors’ private signals are good. To see this, assume that the number of directors who observed bad private signals about the CEO’s quality, given all directors’ private information, can be written as

\[
p(B|r) = \frac{(1 - \gamma)\alpha^r(1 - \alpha)^{n-r}}{(1 - \gamma)\alpha^r(1 - \alpha)^{n-r} + \gamma(1 - \alpha)^{n-r}}. \tag{3}
\]

The CEO’s expected quality is worse than that of a potential replacement if \( p(B|r) > 1 - \gamma; \) with some algebra, this condition simplifies to

\[
\alpha'(1 - \alpha)^{n-r} > (1 - \alpha)'\alpha^{n-r} \iff 2r > n \quad \text{(since } a > \frac{1}{2}). \tag{4}
\]

Hence, the CEO’s expected quality is worse than that of a potential replacement if there is a majority of bad signals about her quality. Conversely, it can be shown that the CEO’s expected quality is better than that of a potential replacement if a majority of directors’ signals about her quality is good. Since optimal voting rules are not the focus of this paper, we will assume throughout that the board votes according to simple majority. In other words, there must be \((n + 1)/2\) votes against the CEO to fire her (i.e., \( k = (n + 1)/2\)). (We extend our analysis to the case of arbitrary voting rules (i.e., a minority or a supermajority voting rule) in Section B.2 of Appendix B.)

### 3.2. The Directors’ Objective

The objective of each director in making his voting decision is to retain the CEO if she is of good quality and fire her if she is of bad quality (and hire a good-quality replacement if the incumbent CEO is fired by the board), but only to the extent that this affects his own costs and benefits, as described below. Assuming that good-quality CEOs can generate higher long-run cash flows for the firm than bad-quality CEOs, this translates into the assumption that the board wishes to maximize the firm’s long-run value net of personal costs incurred. We assume without loss of generality that the maximum payoff a director can receive is \( 0 \). If it is revealed at time 4 that the CEO chosen by the board is of bad quality, each director suffers a cost, \( q > 0 \). Here, \( q \) is the loss in value to directors from making a bad decision that arises from the reduction in the value of the firm’s equity held by individual directors (due to the mismanagement of the firm by a low-quality CEO). We will refer to \( q \) from now on as the directors’ “equity loss.” Under this interpretation, one can think of the equity loss as being proportional to the difference in the long-run value of the firm’s equity under a good-quality CEO versus a bad-quality CEO.

Thus, one way to increase a director’s equity loss would be to increase the equity fraction in his compensation. Alternatively, \( q \) can be thought of as reputational damage\(^{15}\) or a direct cost imposed by shareholders on the members of a board that retains a bad CEO.\(^{16}\)

In addition to the above cost of choosing a low-quality CEO, each director may suffer a cost of dissent, denoted by \( c, c > 0 \). If a director reports a bad signal about the CEO in the discussion round, the CEO may impose a dissent cost on him if the CEO is not subsequently fired and learns about the signal this director reported before the vote (which will occur with probability \( \rho \)).\(^{17}\) If a director votes against the CEO in the voting round but fails to receive enough support from fellow directors to oust her, he incurs the dissent cost with probability \( 1 \).\(^{18}\) What matters to individual directors is the magnitude of \( c \) relative to \( q \) and the probability \( \rho \) with which the CEO may learn that a director reported a bad signal during the discussion round. We will refer to \( \rho c / q \) as the directors’ expected dissent cost to equity loss ratio.

The above dissent cost can be thought of as a potential punishment imposed by the CEO (such as the CEO’s reluctance to nominate the director in the next election cycle) or as the reputational damage from being perceived as a “troublemaker” by other members of this board or by other boards who refuse to grant him a seat. There is considerable empirical evidence suggesting that such dissent costs are significant in practice. For example, Marshall (2010) studies board disputes and shows that dissenting directors who resign amid disputes with management lose, on the net, 85% of their board seats within five years following the dispute. This means dissenting directors are not able to recover the benefits they may lose associated with relinquishing a board seat in a given firm by obtaining additional board seats at other public firms. Furthermore, Dewally and Peck (2010) document that the vast majority (more than 92%) of directors leave the board unannounced (i.e., they do not announce their resignation and do not criticize management even after they leave the firm). This is consistent with the idea that CEOs can impose substantial retaliatory costs on directors who announce their opposition. Dewally and Peck (2010) also document that, among the resignations that are publicly announced, however, a substantial share
(50%) are reported as being due to conflict-related reasons such as noncooperative management, suggesting that director resignations are frequently caused by conflict with the CEO. In a similar vein, Schwartz-Ziv and Weisbach (2013) find that directors in a sample of Israeli firms unanimously voted to approve the CEO’s proposals in all but 3.3% of the cases. Jiang et al. (2016) find that, in publicly traded Chinese firms, dissent occurs in only about 0.6% of board meetings. Anecdotal evidence on the existence of dissent costs is also provided in Bellow and Minow (1998, p. 117), who cite an unnamed CEO saying, “I always tell my directors that they have two choices: vote ‘yes’ or ‘I resign.’” Finally, Tejada (1997) presents a news account of an outside director of a prominent company being denied nomination for reelection after criticizing management.

Let \( \mu_i \) denote the signal reported by director \( i \) to other directors in the discussion round and \( x_{\mu_{j|i}} \) denote the signals reported by all other directors (excluding director \( i \)). We say that \( \mu_i = 0 \) if during the discussion round director \( i \) says that he has a good signal about the CEO’s quality and \( \mu_i = 1 \) if he says that he has a bad signal about the CEO’s quality. Director \( i \)’s vote is denoted by \( v_i \). We say that \( v_i = 0 \) if director \( i \) votes to retain the CEO and \( v_i = 1 \) if he votes to fire the CEO. Given the above, director \( i \)’s payoff after making his report during the discussion round and his subsequent voting decision is given by

\[
\pi_i = -\left\{ \left( \sum_{j=1}^{n} v_j \geq \frac{n+1}{2} \right) q(1−γ) + \left( \sum_{j=1}^{n} v_j < \frac{n+1}{2} \right) \frac{q}{p} p(B(e_i, x_{\mu_{j|i}})) \right\} - \max\{v_i, p\mu_i\} \left\{ \left( \sum_{j=1}^{n} v_j < \frac{n+1}{2} \right) c_i \right\}, \tag{5}
\]

where \( I \) denotes an indicator function in each term (it takes the value of 1 if the relation in parentheses is satisfied and 0 otherwise), and \( p(B(e_i, x_{\mu_{j|i}})) \) denotes the probability that the current CEO is of bad quality given director \( i \)’s signal and other directors’ reports in the discussion round.

The terms in braces in (5) represent the cost of choosing a bad CEO. The first term is the expected cost of choosing a bad CEO when the current CEO is fired: the corresponding indicator function, \( I(\sum_{j=1}^{n} v_j \geq (n+1)/2) \), takes on the value of 1 when there are enough votes to oust the CEO (i.e., \( \sum_{j=1}^{n} v_j \geq (n+1)/2) \)). In that case, the probability that the replacement is of bad quality is given by \( 1−γ \), and hence the expected equity loss is \( −q(1−γ) \). The second term in braces is the expected equity loss from retaining the incumbent: the corresponding indicator function takes on the value of 1 when there are not enough votes to oust the CEO (i.e., \( \sum_{j=1}^{n} v_j < (n+1)/2) \)). In that case, the probability that the director will suffer an equity loss from retaining the incumbent: the probability that the director will suffer an equity loss is given by \( p(B|e_i, x_{\mu_{j|i}}) \), so that the expected equity loss is \( −q p(B|e_i, x_{\mu_{j|i}}) \). The last term in (5), outside the braces, represents the expected dissent cost. A director incurs it when two conditions are satisfied: (i) there are not enough votes to oust the incumbent (i.e., \( \sum_{j=1}^{n} v_j < (n+1)/2) \), so that the corresponding indicator function takes on the value of 1), and (ii) the director either reported a bad signal about the CEO in the discussion round \( (\mu_i = 1) \) or voted to oust her in the voting round \( (v_i = 1) \). For example, if director \( i \) reports a bad signal and then votes to fire the CEO but fails to attain enough support from other directors, his expected payoff will be \( −q p(B|e_i, x_{\mu_{j|i}}) − c \). If, on the other hand, director \( i \) reports a bad signal but then votes to retain the CEO and the CEO is not fired, his expected payoff will be \( −q p(B|e_i, x_{\mu_{j|i}}) − c \).

4. Equilibrium of the Basic Model

Equilibrium in our model is defined as a set of director strategies and beliefs that constitute an undominated, symmetric perfect Bayesian equilibrium (PBE) in pure strategies, and which is also Pareto (payoff) dominant and involves symmetric conjectures. An undominated PBE is one where no director, at any information set, plays a weakly dominated strategy, conditional on the information available to him at that information set. Pareto (or payoff) dominance is an equilibrium selection criterion that eliminates equilibria that are Pareto inferior in terms of directors’ expected payoffs. Our requirement of symmetric conjectures restricts beliefs by each director off or on the equilibrium path to those placing probability weight only on symmetric strategy profiles by other directors. Thus, we look for a symmetric PBE where no director plays a weakly dominated strategy, which involves symmetric conjectures, and where the equilibrium outcome is not Pareto dominated by any other undominated PBE.

Before we go on to characterize the equilibrium of our model, it is worth noting that our setup is different from much of the standard voting literature, which usually considers only the pivotal voter. In that literature, votes of nonpivotal voters generally do not affect their payoffs. However, in our setting, directors’ votes can affect their payoffs even if they are not pivotal: if a director votes to fire the CEO but there are not enough votes to fire her, he will suffer the dissent cost. Hence, directors will calculate their payoffs not just conditional on being pivotal but also conditional on the probability of being on a particular side of the vote (given how other directors vote and what signals they report); equilibria in our model may therefore be affected by such calculations.
4.1. The Benchmark Equilibrium with No Dissent Costs

Before we solve for the equilibrium of our basic model and its enriched versions (in later sections), we consider the benchmark case in which each director’s dissent cost, \( c \), is zero—i.e., the case where the directors are only concerned with their equity loss. The following proposition describes the equilibrium in this case.

**Proposition 1** (Benchmark Equilibrium). (i) If there is no dissent cost \( (c = 0) \), then the CEO is fired if the majority of directors’ private signals are bad and is retained otherwise.

(ii) In this case, the board efficiently aggregates all directors’ information, in the sense that if the board decides to retain the CEO, her expected quality is better than that of a potential replacement; if the board decides to fire the CEO, her expected quality is worse than that of a potential replacement.

When there are no dissent costs, directors are concerned with their equity loss only. Therefore, it is a weakly dominant strategy for directors to vote for the alternative that will minimize their expected equity loss (the one they would prefer if they were pivotal). The outcome of the vote in this case is equivalent to a social planner aggregating all directors’ private information in making the decision whether to fire the incumbent CEO or to retain her.

**Proposition 2** (Board Size Without Dissent Costs). If dissent costs are zero, then the quality of the board’s decision making is increasing in the number of directors, \( n \).

Proposition 2 demonstrates that when directors are willing to reveal their private signals truthfully in equilibrium (as is the case in the benchmark equilibrium characterized in Proposition 1), it is better to have as many of them as possible, since this increases the amount of information collectively available to the board, thus making the board more likely to fire a bad CEO and retain a good CEO.

4.2. Equilibrium of Our Basic Model with Dissent Costs

We now characterize the equilibrium of our model in the presence of dissent costs.

**Proposition 3** (Equilibrium of the Basic Model). (i) If the expected dissent cost to equity loss ratio \( pc/q \leq \Upsilon(n, \alpha, \gamma|1) \), then each director reports his signal about the CEO’s quality truthfully in the discussion round, and the CEO is subsequently fired if the majority of reported signals are bad and is retained if the majority of reported signals are good.

(ii) If \( pc/q > \Upsilon(n, \alpha, \gamma|1) \), then each director reports a good signal about the CEO’s quality in the discussion round and subsequently votes to retain the CEO, regardless of his private signal.

Here,

\[
\Upsilon(n, \alpha, \gamma|1) \equiv \frac{\{p(B|1) - (1 - \gamma)\} p(\sum_{j=1}^n e_j = (n + 1)/2 | 1)}{p(\sum_{j=1}^n e_j < (n + 1)/2 | 1)},
\]

and all conditional probabilities represent beliefs of a director \( i \) given his private signal.

Proposition 3 shows that there exist two distinct types of equilibria, and depending on the magnitude of the expected dissent costs relative to the equity loss, the board switches from one type of equilibrium to the other. When dissent costs are small relative to the equity loss, as in part (i) of the above proposition, then directors report their signals truthfully in the discussion round and the CEO is subsequently fired if the majority of signals reported in the discussion round were bad. When dissent costs are large relative to the equity loss, as in part (ii) of the above proposition, all directors always report good signals in the discussion round and then vote to retain the CEO in the voting round. From now on, we will refer to the equilibrium described in part (i) of Proposition 3 as an “informative discussion and voting equilibrium” and to the equilibrium described in part (ii) of Proposition 3 as an “uninformative discussion and voting equilibrium.”

Information can be efficiently aggregated in the discussion round if directors report their private signals about the CEO’s quality truthfully, which happens in an informative discussion and voting equilibrium, characterized in part (i) of Proposition 3 and that prevails when \( pc/q \leq \Upsilon(n, \alpha, \gamma|1) \). After an informative discussion round, each director will update his assessment of the CEO’s quality taking into account all information reported in the discussion round, and the CEO will be fired in the voting round if the majority of signals reported in the discussion round are bad and will be retained if the majority of signals reported in the discussion round are good. Consider first the case where the majority of reported signals are bad. In this case, all directors’ updated assessment of the CEO’s quality will be that she is worse than a potential replacement, so that it is Pareto dominant for a majority of directors to vote to fire her. This is because, by firing the CEO, all directors minimize their expected equity loss and avoid incurring any dissent costs as well (since the CEO is replaced). Similarly, when the majority of reported signals about the CEO’s quality are good, the directors’ updated assessment of the CEO’s quality will be that she is better than a potential replacement, so that it is Pareto dominant for a majority of directors to vote to retain her. This is because, by retaining the CEO, all directors minimize their expected equity loss and avoid increasing their dissent cost (even though directors who reported a bad signal in the discussion round suffer a dissent cost with probability \( \rho \)).
We now discuss the trade-off faced by directors in the discussion round, anticipating the outcome of the voting round described above. Directors who reported bad signals in the discussion round will suffer the dissent cost \( c \) with probability \( p \) if the CEO is ultimately not fired. If they report their signals truthfully in the discussion round, they reduce the probability of suffering the equity loss \( q \) because a bad CEO is more likely to be fired. However, if they report their signals truthfully and fail to attain enough support to fire the CEO, they will suffer an expected dissent cost \( pc \) (the cost of dissent scaled by the probability that the CEO learns the identities of directors who reported bad signals in the discussion round), regardless of whether the CEO is good or bad. Hence, directors with bad signals face a trade-off between incurring an equity loss and this expected dissent cost. Directors with good private signals about the CEO’s quality do not face such a trade-off, since they do not face a probability of incurring any dissent costs. They therefore always report their signals truthfully.

If the expected dissent costs are high relative to the expected equity loss (as in part (ii) of Proposition 3), directors will choose to disregard their private signals and will report good signals in the discussion round. Subsequently, they will vote to retain the current CEO since only good signals will be reported in the discussion round. This equilibrium, which we label an uninformative discussion and voting equilibrium, prevails when \( pc/q > \Upsilon(n, a, \gamma|1) \). If, on the other hand, expected dissent costs are low relative to the expected equity loss (i.e., \( pc/q \leq \Upsilon(n, a, \gamma|1) \), as in part (i) of Proposition 3), directors with bad private signals will report their signals truthfully in the discussion round and, making it informative.\(^{27}\) Thus, it is the ratio \( pc/q \) that will determine whether directors report their signals truthfully in the discussion round or not.

The threshold value \( \Upsilon(n, a, \gamma|1) \) of \( pc/q \) below which the informative discussion and voting equilibrium is sustainable is given by Equation (6). Note that \( \Upsilon(n, a, \gamma|1) \) is positively related to the probability that director \( i \) is pivotal, given that he observes a bad private signal. It is negatively related to the probability that a majority of directors observe good private signals. Thus, if director \( i \) believes that only a small number of other directors obtained a bad signal, he understands that it is unlikely that they will have enough votes to oust the current CEO, so that he will not report his true private signal in the discussion round in order to avoid the expected dissent cost \( pc \). Note that \( \Upsilon(n, a, \gamma|1) \) is also positively related to the difference between the quality of an outside replacement and that of the current CEO: when \( p(B|1) - (1 - \gamma) \) is large, the current CEO is perceived to be much worse by directors with bad signals than the outside replacement, so that their expected equity loss from retaining the incumbent is also high. It is thus natural to compare \( \Upsilon(n, a, \gamma|1) \) with the ratio \( pc/q \) to understand the board’s incentives.

It is useful to compare part (ii) of the benchmark equilibrium characterized in Proposition 1 (with zero dissent costs) with part (ii) of Proposition 3. If dissent costs are zero, then the board always fires the CEO if her expected quality (conditional on all directors’ signals) is worse than that of a potential replacement and retains her if the reverse is true. By contrast, as we showed in Proposition 3, part (ii), when directors’ expected dissent costs are large (relative to their equity loss), the board is unable to aggregate individual directors’ signals efficiently, resulting in decision making by the board that is worse than in the benchmark equilibrium characterized in Proposition 1.

**Proposition 4 (Effect of an Increase in Board Size).** Increasing the number of directors lowers the threshold value of the expected dissent costs to equity loss ratio \( pc/q \) below which an informative discussion and voting equilibrium is sustainable.

To understand the intuition behind the above proposition, consider a director who received a bad private signal about the CEO’s quality. The benefit to this director from reporting his signal truthfully is that, if he is pivotal, he can reduce his expected equity loss by making sure that the CEO is fired. The cost to this director from reporting his signal truthfully is that, if he turns out to be in the minority (reporting bad signals) in the discussion round and the CEO is not fired in the voting round, he will incur a dissent cost. Since the probability of any director being pivotal decreases as the size of the board increases, the benefit of truthful reporting also goes down as board size increases. This decrease in the benefit of truthful reporting lowers the threshold value of the expected dissent costs to equity loss ratio \( pc/q \) below which an informative discussion and voting equilibrium is sustainable. This, in turn, implies that an informative discussion and voting equilibrium will prevail only when the expected dissent to equity loss ratio is smaller.

We now address the issue of optimal board size. We define the optimal size of a board as that board size that allows directors to have the maximum amount of information available for decision making.

**Proposition 5 (Optimal Board Size).** If \( pc/q > 0 \), then there exists an interior optimal board size.

The trade-off driving the above proposition is as follows. The advantage of a larger board is that, conditional on sustaining an informative discussion and voting equilibrium, a larger board brings more information into the decision-making process (since, in a larger board, directors collectively will have access to more information). However, as we showed in Proposition 4, the disadvantage of having a larger board is that...
it lowers the threshold value of the expected dissent costs to equity loss ratio \( pc / q \) below which an informative discussion and voting equilibrium is sustainable. As the board size is allowed to grow very large, the threshold value below which an informative discussion and voting equilibrium is sustainable approaches zero. Therefore, for any positive value of the dissent cost to equity loss ratio, increasing the size of the board beyond a certain number will lead to the board switching to an uninformative discussion and voting equilibrium. Clearly, more directors bring additional information into the board’s decision making but only if they report their signals about the CEO’s quality truthfully in the discussion round. Thus, there is an interior optimum for board size for any positive value of the dissent cost to equity loss ratio: this interior optimum board size is the largest board size that allows an informative discussion and voting equilibrium to prevail.\(^{28}\)

5. Board Composition

One of the recurring topics in the literature on corporate boards is the relationship between insiders and outsiders on the board and the optimal board composition from the point of view of shareholder value maximization. In this section we extend our basic model to explore the role that these two types of directors play in the CEO turnover decision.

We distinguish between two types of directors in this section: insiders and outsiders. We assume that there are \( h \) insiders and \( n - h \) outsiders on the board. We define insiders as directors employed by the firm. Since insiders work alongside the CEO, they are likely to have a better ability to judge her quality. At the same time, they are likely to suffer greater dissent costs if they vote against the CEO. (One justification for the assumption that inside directors have larger dissent costs compared to outside directors is that insiders’ careers may be tied closely to that of the CEO; see, for example, Weisbach 1988.) We model these two differences between insiders and outsiders as follows. On the positive side, we assume that insiders’ signals are more precise: \( p(e_{i}^{\text{ins}} = 0|G) = p(e_{i}^{\text{ins}} = 1|B) = \beta > \alpha > \frac{1}{2} \). On the negative side, however, we assume that insiders suffer larger dissent costs if they vote against the CEO but fail to oust her. We denote insiders’ dissent costs by \( b, b > c \). We assume that outsiders have the same characteristics as directors in our basic model: we therefore denote outsiders’ dissent costs by \( c \). All other assumptions remain the same as in our basic model.

In the following, we will refer to the optimal composition of a board as that number of firm insiders versus outsiders, which, for a given board size, allows directors to have the maximum amount of information available for decision making. Furthermore, denote inside director values by the superscript “ins” and outside director values by the superscript “outs.” Define the threshold values \( \Upsilon^{\text{ins}}(n, \alpha, \gamma|1) \), \( \Theta^{\text{ins}}(n, \alpha, \gamma, h) \), and \( \Theta^{\text{outs}}(n, \alpha, \gamma, h) \) as follows:

\[
\Upsilon^{\text{ins}}(n, \alpha, \gamma|1) \equiv \left( p \left( \sum_{j=1}^{n} e_{j}^{\text{ins}} \geq \frac{n + 1}{2} \left| e_{i}^{\text{ins}} = 1 \right) \right) \right) \cdot \left( p \left( \sum_{j=1}^{n} e_{j}^{\text{ins}} < \frac{n + 1}{2} \left| e_{i}^{\text{ins}} = 1 \right) \right) ^{-1}, \quad (7)
\]

\[
\Theta^{\text{ins}}(n, \alpha, \gamma, h) \equiv \left( \sum_{x_{j}^{\text{outs}} \in M^{\text{ins}}} \left( p(B|e_{j}^{\text{outs}} = 1, x_{j}^{\text{outs}}) - (1 - \gamma) \right) \right) \cdot p(x_{j}^{\text{outs}}) \cdot (p(N^{\text{outs}}|e_{j}^{\text{outs}} = 1))^{-1}, \quad (8)
\]

\[
\Theta^{\text{outs}}(n, \alpha, \gamma, h) \equiv \left( \sum_{x_{j}^{\text{outs}} \in M^{\text{ins}}} \left( p(B|e_{j}^{\text{outs}} = 1, x_{j}^{\text{outs}}) - (1 - \gamma) \right) \right) \cdot p(x_{j}^{\text{outs}}) \cdot (p(N^{\text{ins}}|e_{j}^{\text{ins}} = 1))^{-1}, \quad (9)
\]

where

- \( M^{\text{outs}} \equiv \{ x_{j}^{\text{outs}} : p(B|e_{j}^{\text{outs}} = 1, x_{j}^{\text{outs}}) < 1 - \gamma < p(B|e_{j}^{\text{outs}} = 0, x_{j}^{\text{outs}}) \} \) is the set of all possible combinations of other directors’ signals for which a given outsider’s report is pivotal (i.e., the CEO’s expected quality is worse than that of a potential replacement if this outsider’s private signal is bad and is better than that of a potential replacement if his private signal is good),

- \( N^{\text{ins}} \equiv \{ x_{j}^{\text{outs}} : p(B|e_{j}^{\text{outs}} = 1, x_{j}^{\text{outs}}) < 1 - \gamma \} \) is the set of all possible combinations of other directors’ reported signals for which the CEO’s expected quality is better than that of a potential replacement even if this outsider’s private signal about the CEO’s quality is bad,

- \( M^{\text{ins}} \equiv \{ x_{j}^{\text{outs}} : p(B|e_{j}^{\text{ins}} = 0, x_{j}^{\text{outs}}) < 1 - \gamma < p(B|e_{j}^{\text{ins}} = 1, x_{j}^{\text{outs}}) \} \) is the set of all possible combinations of other directors’ signals for which a given insider’s report is pivotal (i.e., the CEO’s expected quality is worse than that of a potential replacement if this insider’s private signal is bad and is better than that of a potential replacement if his private signal is good), and

- \( N^{\text{ins}} \equiv \{ x_{j}^{\text{outs}} : p(B|e_{j}^{\text{ins}} = 1, x_{j}^{\text{outs}}) < 1 - \gamma \} \) is the set of all possible combinations of other directors’ reported signals for which the CEO’s expected quality is better than that of a potential replacement even if this insider’s private signal about the CEO’s quality is bad.

Proposition 6 (Optimal Board Composition). (i) If the expected dissent cost to equity loss ratio for insiders \( pb / q \leq \Upsilon^{\text{ins}}(n, \alpha, \gamma|1) \), then the optimal board consists only of insiders.
(ii) Let the number of directors \( n \) be finite. If the difference in the precision of signals between insiders and outsiders is large enough and the difference in dissent costs between the two is small enough so that there exists a number of insiders \( h \) such that \( \Upsilon^{\text{ins}}(n, a, \gamma|1) < pb/q \leq \Upsilon^{\text{ins}}(n, a, \gamma, h) \) and \( pc/q \leq \Theta^{\text{out}}(n, a, \gamma, h) \), then there exists an interior informative discussion and voting equilibrium involving both insiders and outsiders serving on the board. In this equilibrium, the optimal number of insiders on the board is the largest integer \( h \) for which an informative discussion and voting equilibrium is sustainable, with the remaining \( n-h \) directors being outsiders.\(^{2}\)

The trade-off driving optimal board composition in Proposition 6 is as follows. Given that firm insiders have more precise signals compared with outsiders, having a larger number of firm insiders on the board increases the amount of information available for board decision making, given an informative discussion and voting equilibrium. However, insiders face larger dissent costs and may therefore be unwilling to report their private signals truthfully in the discussion round. Outsiders, on the other hand, possess less precise information about the CEO’s quality but may be more willing to report this information because of their lower dissent costs. The optimal board is therefore such that it retains as many insiders as possible but has enough outsiders to induce truthful reporting of private signals in the discussion round by all directors (i.e., allows the sustenance of an informative discussion and voting equilibrium).

When insiders’ dissent costs are not very high, so that a board that consists exclusively of insiders is able to sustain an informative discussion and voting equilibrium, the board should consist only of insiders because the quality of their information is higher (i.e., their signals are more precise). This case is characterized in part (i) of Proposition 6. It is when a board consisting only of insiders is unable to sustain an informative discussion and voting equilibrium that adding outsiders to the board improves board decision making (part (ii) of Proposition 6). In this case, replacing some insiders with outsiders can make the remaining insiders more willing to report their private signals truthfully (since insiders are aware that outsiders have lower dissent costs and are therefore more likely to report their signals truthfully), thus allowing the sustenance of an informative discussion and voting equilibrium. Consequently, a board with characteristics such that if it were to contain only insiders it would have an uninformative discussion and voting equilibrium will be able to sustain an informative discussion and voting equilibrium if its membership were changed to include an appropriate combination of insiders and outsiders. In such an informative discussion and voting equilibrium, both insiders and outsiders report their private signals truthfully in the discussion round, and all board members vote to fire the CEO in the voting round if her expected quality (conditional on all directors’ private signals) is worse than that of a potential replacement and vote to retain her otherwise.\(^{30}\)

Note that the existence of an optimal board consisting of both insiders and outsiders depends, implicitly, on the relative magnitudes of signal precisions and dissent costs for insiders and outsiders. In particular, the optimal board will consist of both insiders and outsiders when insiders’ dissent costs are large enough to prevent an insiders-only board from voting informatively \((\Upsilon^{\text{ins}}(n, a, \gamma|1) < pb/q)\) while at the same time not too large, so that insiders are willing to vote informatively in the presence of outsiders \((pb/q \leq \Upsilon^{\text{out}}(n, a, \gamma, h))\). If insiders’ dissent costs are very large while outsiders’ dissent costs are not, then only outsiders should be on the board, since, in this case, a board consisting of outsiders alone will be able to sustain an informative discussion and voting equilibrium (while a board consisting of any insiders will be unable to do so). In particular, the board should consist only of outsiders if the conditions stated in part (ii) of Proposition 6 do not hold while at the same time \( pc/q \leq \Upsilon(n, a, \gamma|1) \) (where the value of \( \Upsilon(n, a, \gamma|1) \) is given by Equation (6)).

6. Public Signals
So far, we have been assuming that directors are able to obtain only private information about the CEO’s quality. In this section, we extend our basic model to investigate the implications of adding a low precision (noisy) public signal to our setting. An example of such a public signal may be a rise or fall in the firm’s share price, or in its operating (accounting) performance. Simple intuition might suggest that directors observing a public signal will always improve decision making. Somewhat surprisingly, however, we show that this is not always the case.

Here we assume that, in addition to their own private signals, all directors observe a public signal \( S_{\text{public}} \) about the CEO’s quality \( Q \). The public signal can be either \( H \) (high) or \( L \) (low). As in our basic model, we assume that directors are homogeneous (there is no distinction between insiders and outsiders). The informativeness of the public signal is determined by a parameter \( \phi \):

\[
\begin{align*}
p(H|G) &= p(L|B) = \phi; \\
p(L|G) &= p(H|B) = 1 - \phi; \\
\phi &> \frac{1}{2}. \tag{10}
\end{align*}
\]

We also assume that public and private signals are conditionally independent. That is, \( p(e_i, S_{\text{public}}|Q) = p(e_i|Q)p(S_{\text{public}}|Q) \) for \( e_i \in \{0, 1\}, S_{\text{public}} \in \{H, L\} \), and \( Q \in \{G, B\} \).
Clearly, if the public signal is very precise compared with directors’ private signals, directors will disregard their private information and fire the CEO upon observing public signal \( L \) but retain him upon observing public signal \( H \). To prevent such situations and analyze the interesting cases, we assume that \( \alpha > \phi > 1/2 \); that is, the public signal is informative but less informative than directors’ private signals. This is a reasonable assumption, since otherwise the board of directors brings no value to the turnover decision and it would be optimal to simply dismiss the CEO after observing a negative public signal. This assumption implies that

\[
p(B|0, H) < p(B|0, L) < 1 - \gamma < p(B|1, H) < p(B|1, L)
\]

and that

\[
p\left( B \left| \sum_{i=1}^{n} e_i = \frac{n-1}{2}, H \right. \right) < p\left( B \left| \sum_{i=1}^{n} e_i = \frac{n-1}{2}, L \right. \right) < 1 - \gamma
\]

\[
< p\left( B \left| \sum_{i=1}^{n} e_i = \frac{n+1}{2}, H \right. \right) < p\left( B \left| \sum_{i=1}^{n} e_i = \frac{n+1}{2}, L \right. \right). \tag{12}
\]

Condition (11) states that directors with bad private signals continue to perceive the CEO’s expected quality as lower than that of a potential replacement, while directors with good private signals continue to perceive the CEO’s expected quality as higher than that of a potential replacement, even after observing the public signal. Condition (12) states that even after taking into account the information available in the public signal and all directors’ private signals, the CEO’s expected quality is worse than that of a potential replacement if the majority of directors’ private signals are bad and is better than that of a potential replacement otherwise. All other assumptions remain the same as in our basic model.

**Proposition 7** (Equilibrium with a Public Signal). (i) After observing a public signal \( H \),

(a) if the expected dissent cost to equity loss ratio \( \rho c/q \leq \Upsilon(n, \alpha, \gamma|1, H) \), then each director reports his signal about the CEO’s quality truthfully in the discussion round, and the CEO is subsequently fired if the majority of reported signals are bad and is retained otherwise;

(b) if \( \rho c/q > \Upsilon(n, \alpha, \gamma|1, H) \), then each director reports a good signal about the CEO’s quality in the discussion round and subsequently votes to retain the CEO, regardless of his private signal.

Here,

\[
\Upsilon(n, \alpha, \gamma|1, H) = \frac{\{p(B|1, H) - (1 - \gamma)\}p(\sum_{j=1}^{n} e_j = (n + 1)/2 | 1, H)}{p(\sum_{j=1}^{n} e_j < (n + 1)/2 | 1, H)}.
\]

(ii) After observing a public signal \( L \),

(a) if the expected dissent cost to equity loss ratio \( \rho c/q \leq \Upsilon(n, \alpha, \gamma|1, L) \), then each director reports his signal about the CEO’s quality truthfully in the discussion round, and the CEO is subsequently fired if the majority of reported signals are bad and is retained otherwise;

(b) if \( \rho c/q > \Upsilon(n, \alpha, \gamma|1, L) \), then each director reports a good signal about the CEO’s quality in the discussion round and subsequently votes to retain the CEO, regardless of his private signal.

Here,

\[
\Upsilon(n, \alpha, \gamma|1, L) = \frac{\{p(B|1, L) - (1 - \gamma)\}p(\sum_{j=1}^{n} e_j = (n + 1)/2 | 1, L)}{p(\sum_{j=1}^{n} e_j < (n + 1)/2 | 1, L)}.
\]

Notice that, unlike in our basic model, there are two threshold values for the expected dissent cost to equity loss ratio (depending on the type of the public signal that directors observe) that drive the nature of the equilibrium that prevails in the presence of a public signal. This is because, after observing a public signal, each director reassesses the validity of his own private information, conditional on the public information. The quality of decision making by the board depends on the relation between \( \rho c/q \) and these conditional thresholds. As in our basic model, we refer to equilibria in which directors report their private signals truthfully in the discussion round and then vote in the voting round in the direction of the majority of signals reported in the discussion round as informative discussion and voting equilibria. We refer to equilibria in which all directors report good signals in the discussion round and always vote to retain the CEO in the voting round regardless of their private signals as uninformative discussion and voting equilibria.

We now study how the quality of the board’s decision making is affected by the presence of a public signal. In the following proposition, we refer to a public signal as worsening the quality of board decision making if, in the presence of such a signal, an informative discussion and voting equilibrium prevails only for a smaller set of parameter values (\( \rho, c, \) and \( q \)) compared to the setting without a public signal. Conversely, we refer to a public signal as improving board decision making if, in the presence of such a signal, an informative discussion and voting equilibrium prevails for a larger set of the above parameter values.

**Proposition 8** (Quality of Decision Making with a Public Signal). (i) If \( \rho c/q \) is low, such that \( \rho c/q \leq \Upsilon(n, \alpha, \gamma|1, H) \), an informative discussion and voting equilibrium always prevails, regardless of the public signal; that is, the public signal is irrelevant.

(ii) If \( \rho c/q \) is in the intermediate range, such that \( \Upsilon(n, \alpha, \gamma|1, H) < \rho c/q \leq \Upsilon(n, \alpha, \gamma|1, L) \), an informative discussion and voting equilibrium prevails only if there is no
public signal or if the public signal is \( L \); that is, the public signal worsens the board’s decision making.

(iii) If \( \rho c / q \) is high, such that \( \Upsilon(n, \alpha, \gamma|1) < \rho c / q \leq \Upsilon(n, \alpha, \gamma|1, L) \), an informative discussion and voting equilibrium prevails only when there is a public signal and it is \( L \); that is, the public signal improves the board’s decision making.

(iv) If \( \rho c / q \) is very high, such that \( \Upsilon(n, \alpha, \gamma|1, L) < \rho c / q \), an uninformative discussion and voting equilibrium prevails regardless of the public signal realization; that is, the public signal is irrelevant again.

Here,

\[
\Upsilon(n, \alpha, \gamma|1, H) \equiv \frac{\{p(B|1, H)-(1-\gamma)\}p(\sum_{j=1}^{n} e_j=(n+1)/2|1, H)}{p(\sum_{j=1}^{n} e_j<(n+1)/2|1, H)}, \tag{15}
\]

\[
\Upsilon(n, \alpha, \gamma|1) \equiv \frac{\{p(B|1)-(1-\gamma)\}p(\sum_{j=1}^{n} e_j=(n+1)/2|1)}{p(\sum_{j=1}^{n} e_j<(n+1)/2|1)}, \tag{16}
\]

\[
\Upsilon(n, \alpha, \gamma|1, L) \equiv \frac{\{p(B|1, L)-(1-\gamma)\}p(\sum_{j=1}^{n} e_j=(n+1)/2|1, L)}{p(\sum_{j=1}^{n} e_j<(n+1)/2|1, L)}. \tag{17}
\]

Part (i) of Proposition 8 shows that, when the expected dissent cost to equity loss ratio is low (i.e., \( \rho c / q \leq \Upsilon(n, \alpha, \gamma|1, H) \)), the public signal is irrelevant. Here, since \( \rho c / q \) is low, an informative discussion and voting equilibrium would have prevailed in our basic model and also prevails in the presence of a public signal (regardless of whether the realization of the signal is high or low). Hence, the presence of the public signal does not affect the outcome of board decision making. On the other hand, when this ratio goes up but remains below the threshold value \( \Upsilon(n, \alpha, \gamma|1) \) of our basic model (part (ii) of Proposition 8), the presence of a public signal may worsen decision making but never improves it relative to the outcome of the basic model. This happens because, conditional on observing a low public signal or no public signal at all (as in our basic model), an informative discussion and voting equilibrium prevails. Observing a high public signal, however, may prevent directors with bad private signals about the CEO from reporting them truthfully, since they may assess that many other directors may have observed a good private signal about her, given that a high public signal is observed. A favorable public signal may thus make it less likely (compared with the basic model) that there will be enough directors voting to fire the incumbent CEO. In this case, an informative discussion and voting equilibrium will be unsustainable.

Part (iii) of Proposition 8 shows that, when the expected dissent cost to equity loss ratio is higher than the threshold value \( \Upsilon(n, \alpha, \gamma|1) \) of our basic model but is below the threshold \( \Upsilon(n, \alpha, \gamma|1, L) \), the presence of a public signal may improve decision making relative to the outcome of the basic model. This happens because, in this case, the only way for directors to report their private signals truthfully is to observe a low public signal. In other words, for this range of values of the expected dissent cost to equity loss ratio, directors with bad private signals would have reported good private signals about the CEO in our basic model; that is, the uninformative discussion and voting equilibrium would have prevailed in the absence of a public signal. However, upon observing a negative public signal about the CEO’s quality, directors with bad private signals assess a higher probability that enough other directors will also want to oust her. This higher chance of success in ousting the CEO (as assessed by directors with bad private signals about the CEO’s quality) enables the switch from an uninformative discussion and voting equilibrium to an informative discussion and voting equilibrium. Finally, when the expected dissent cost to equity loss ratio \( \rho c / q \) is very high, as in part (iv) of Proposition 8, even a low public signal will not make an informative discussion and voting equilibrium sustainable. In this case, the public signal is again irrelevant.

7. CEO Dismissal Costs and Different Priors About the Incumbent CEO

Quality and Potential Replacement CEO Quality

In this section, we extend our basic model in two directions. First, we assume that firing a CEO imposes a “dismissal cost” on board members, and we denote this cost by \( d, d \geq 0 \). Second, we relax the assumption that directors’ prior probability assessment regarding the quality of the existing CEO is the same as that about a potential outside replacement CEO; instead, we now assume that the probability of hiring a high-quality outside replacement is \( \sigma \), which may be different from the prior probability assessment of directors that the incumbent CEO is good (\( \gamma \)). The directors’ payoff in this extended model becomes

\[
\pi_i = -\left\{ \left( \sum_{j=1}^{n} v_j \geq \frac{n+1}{2} \right) \left( d + q(1-\sigma) + I \left( \sum_{j=1}^{n} v_j < \frac{n+1}{2} \right) qp(B|1, \mu^*) \right) + I \left( \sum_{j=1}^{n} v_j \leq \frac{n+1}{2} \right) \theta_{i, \mu^*} \right\} - \max \left\{ v_i, \rho \mu_i, I \left( \sum_{j=1}^{n} v_j < \frac{n+1}{2} \right) c \right\}. \tag{18}
\]

Unlike in our basic model, in this extension the CEO’s expected quality is not necessarily worse than that of a potential replacement if the majority of directors’ private signals are bad (because \( \sigma \) can be different
from γ and because directors need to account for CEO dismissal costs in their firing decision).\textsuperscript{32} In particular, all directors infer that the CEO’s expected quality is worse than that of a potential replacement, net of dismissal costs, if the expected equity loss from retaining the incumbent CEO exceeds the sum of CEO dismissal costs and the expected equity loss from hiring an outside replacement, or equivalently,

\[ d + q(1 - \sigma) \leq q p \left( B \left\{ \sum_{j=1}^{n} \mu_j \right\} \right), \]  

(19)

where \( \sum_{j=1}^{n} \mu_j \) is the number of directors who reported bad private signals in the discussion round.

Thus, the board’s decision whether to fire or retain the CEO in this extended model is based on the expected quality of the incumbent CEO (conditional on the number of bad signals reported in the discussion round) relative to CEO dismissal costs and the likelihood of hiring a high-quality replacement CEO. Define \( \mu' \) as the minimum number of directors with bad private signals that satisfies (19). Thus, all directors infer that the CEO’s expected quality is worse than that of a potential replacement, net of dismissal costs, if there are at least \( \mu' \) bad private signals about her quality.

**Proposition 9** (Equilibrium of the Model with CEO Dismissal Costs and Different Priors About Incumbent and Replacement CEO Quality). (i) If the expected dissent cost to equity loss ratio \( pc/q \leq \Omega(n, d, a, \gamma, \sigma|1) \), then there exists an informative discussion and voting equilibrium in which each director reports his signal about the CEO’s quality truthfully in the discussion round, and all directors vote to fire the CEO in the voting round if the number of bad private signals reported in the discussion round is above \( \mu' \) and vote to retain her otherwise.

(ii) In the equilibrium described in part (i), increasing the dismissal cost \( d \) or decreasing the probability of hiring a high-quality replacement CEO \( \sigma \) raises the number of bad private signals that need to be reported in the discussion round for directors to vote to fire the CEO in the voting round; that is, \( \mu' \) is increasing in \( d \) and decreasing in \( \sigma \).

(iii) If the expected dissent cost to equity loss ratio \( pc/q > \Omega(n, d, a, \gamma, \sigma|1) \) and \( q p(B|(n + 1)/2) < d + q(1 - \sigma) \), then each director reports a good signal about the CEO’s quality in the discussion round and subsequently votes to retain the CEO, regardless of his private signal.

Here,

\[ \Omega(n, d, a, \gamma, \sigma|1) \equiv \frac{q p(B|\sum_{j=1}^{n} e_j = \mu') - (1 - \sigma) - d/q}{p(\sum_{j=1}^{n} e_j < \mu'|1)}, \]  

(20)

and all conditional probabilities represent beliefs of a director given his private signal.

Part (i) of Proposition 9 describes the case where the expected dissent cost to equity loss ratio is sufficiently small, so that there exists an informative discussion and voting equilibrium. If the number of bad private signals reported in the discussion round exceeds \( \mu' \) in this equilibrium, then all directors infer that the CEO’s expected quality, net of dismissal costs, is worse than that of a potential replacement. In this case, all directors vote to fire the CEO since ousting her minimizes their equity loss as well as their dissent costs (since no director suffers the dissent cost if the CEO is ousted).

Part (ii) of Proposition 9 reports the comparative statics of the equilibrium characterized in Proposition 9(i). It shows that the magnitude of dismissal costs (\( d \)) and the directors’ prior about the probability of finding a high-quality replacement CEO (\( \sigma \)) affect the board’s inference about the incumbent CEO’s quality. Increasing \( d \) or decreasing \( \sigma \) raises the number of bad private signals that need to be reported in the discussion round for all directors to infer that the CEO’s quality is worse than that of a potential replacement, net of dismissal costs. Thus, a higher dismissal cost or a lower likelihood of finding a high-quality replacement CEO makes it less likely that the board will fire the incumbent CEO.

Finally, part (iii) of Proposition 9 describes situations in which the expected dissent cost to equity loss ratio is large. As in our basic model, there are no informative discussion and voting equilibria in this case. Instead, all directors report good signals about the CEO’s quality in the discussion round and vote to retain her in the voting round because the expected dissent cost from reporting bad private signals outweighs the lower expected equity loss from hiring an outside replacement.\textsuperscript{33}

In summary, in this section we have shown that, even in a more general setting than that in our basic model (Sections 3–5), there exist equilibria that are broadly similar to the equilibrium we characterized in Proposition 3. We no longer wish to claim uniqueness, however, in this generalized setting: equilibria with characteristics different from the one we characterized in Proposition 9 may exist as well.

8. Empirical and Policy Implications

We now discuss the empirical and policy implications of our model, and we suggest how these implications may be tested empirically. We will also discuss how our model predictions relate to the existing empirical literature on corporate boards and CEO turnover.\textsuperscript{34} As in the case of theoretical models in other areas of corporate finance, some of our testable predictions are not unique to our model; that is, they have also been made by models of corporate boards in the existing theoretical literature. When this is the case, we will discuss how our predictions relate to those of other models, and
we provide some suggestions about how to empirically distinguish between these two sets of predictions.

i. Passive boards and suboptimal CEO firing decisions: Our model develops a new rationale for why many boards are sometimes suboptimally passive (i.e., they choose to retain the existing CEO for too long). The prediction of passive (or CEO-friendly) boards has also been made by other theoretical models in the existing literature. Thus, less independent boards may arise as a result of bargaining between the CEO and the firm in Hermelin and Weisbach (1998). Friendly boards arise as an inducement for the CEO to share information with the board in Adams and Ferreira (2007). An insider-dominated board that is friendly to the CEO may also arise optimally (as part of the optimal arrangement for maximizing the information available for firm decision making) in Harris and Raviv (2008).

An important difference between our model and the above models is that, while a passive board is shareholder value maximizing in these models, it is not optimal for maximizing shareholder value in our setting (though it may be optimal for individual directors not to vote against the CEO to avoid incurring dissent costs).

Thus, in our setting, when the dissent cost to equity loss ratio is large (for example, when the CEO has a greater ability to dictate the composition of the board), coordination problems between directors prevent the board from firing the CEO even when, collectively, the board has enough information to know that it is optimal for shareholders to fire the existing CEO and hire a new one from outside. Furthermore, our model implies that if the board’s assessment of the quality of the existing CEO relative to the pool of outside CEOs is low, then the board is more likely to fire the existing CEO and hire a new CEO from the outside.

The above has several testable predictions. First, more entrenched CEOs with greater ability to impose dissent costs on directors are less likely to be fired. Second, there will be a positive relationship between a CEO being fired and the quality of the pool of outside candidates for the CEO’s job and a negative relationship between industry-adjusted firm performance and the CEO being fired. Third, firms in industries where directors are able to evaluate CEOs more precisely (i.e., where directors receive more precise signals of the CEO’s quality) are more likely to make better CEO-firing decisions. Thus, boards of firms in more homogeneous industries are likely to make better firing decisions than those in more heterogeneous industries (where it is harder to benchmark the CEO’s performance relative to other firms). Furthermore, boards in which members have, on average, larger stock ownership (larger equity loss in our model) and are institutional or otherwise activist shareholders (lower dissent costs) are more likely to make better decisions in terms of firing the CEO.

Evidence consistent with our first prediction above is provided by Allen (1981), who documents that CEOs having greater power over directors (resulting in their facing larger dissent costs) have longer tenures. Evidence consistent with the third prediction above is provided by Parrino (1997), who documents that the likelihood of forced CEO turnover and outside succession are both greater in homogeneous industries (consisting of similar firms) than in heterogeneous industries.

ii. Optimal board size: The optimal board size in our model is determined by the trade-off between the information requirements of the board in evaluating the CEO, which favors a larger board (the larger the board, the greater the amount of information available to the board as a whole), and the greater coordination problems among directors that arise with larger boards. Thus, our model predicts that, to maximize firm value, firms will have the largest board size consistent with sustaining an informative discussion and voting equilibrium. Thus, our model predicts that larger, multi-divisional, and more complex firms (in the sense that the board requires more information in evaluating the CEO’s performance) will have larger boards than simpler firms. In the latter case, our model predicts that the directors will (optimally) be better incentivized (smaller $pc/q$ either through larger holdings in the firm’s equity (larger equity loss $q$) or through appointing directors (such as activist shareholders) who suffer from smaller dissent costs.

The prediction that optimal board size may be a function of the information requirements of the firm (which, in turn, are determined by firm characteristics), and that smaller boards are not necessarily optimal, has been made by the theoretical analysis of Harris and Raviv (2008). In their model, board size is determined by the relative importance of insider versus outsider information, the agency problem between insiders and outsiders, and the effort costs incurred by outsiders in producing information (the latter affects the optimal number of outsiders on a firm’s board). The evidence that larger and more complex firms are likely to have larger boards may therefore be viewed as being broadly supportive of the predictions of both our model and that of Harris and Raviv (2008). Thus, Mikkelsen et al. (1997) and Boone et al. (2007) document that firms start out with smaller boards at IPO, with board size increasing significantly after the firm has become seasoned (and more complex). The latter paper also documents that board size is related to measures of the scope and complexity of the firm’s operations such as firm size, firm age, and the number of business segments in which the firm operates.

iii. Optimal board composition: Conventional wisdom among practitioners seems to be that a greater level
of board independence (a larger fraction of outside directors) unambiguously increases firm performance. For example, TIAA-CREF, one of the largest pension funds in the world, has stated that it will invest only in companies that have a majority of outside directors on its board; similarly, CALPERS, another large pension fund, recommends that the CEO should be the only inside director on a firm’s board. The empirical evidence, however, has been mixed: for example, Coles et al. (2008) present evidence suggesting that restrictions on board size and insider representation do not necessarily enhance firm value (see also Bhagat and Black 2001).

Our model contributes to the above debate about the optimal proportion of insiders versus outsiders on corporate boards. Our analysis suggests that there may be an optimal mix of insiders and outsiders: the advantage of having insiders arises from the more precise information they possess relative to outsiders, while the advantage of having independent outsiders arises from their lower dissent costs. The model of Harris and Raviv (2008) also suggests that firms should have an optimal mix of insiders and outsiders on their boards. In their model, the optimal mix of insiders and outsiders on a firm’s board is determined by the relative importance of insider versus outsider information for decision making by the firm, the extent of the agency problem between insiders and outsiders, and the effort costs to be incurred by insiders in producing information.

A crucial ingredient of our model that may allow empiricists to distinguish between our model and that of Harris and Raviv (2008) is the dissent cost faced by directors in our setting. Given our assumption that firm insiders have more precise information about CEO quality compared with outsiders, our model suggests that replacing insiders with outsiders will be value enhancing for the firm only when outsiders have significantly lower dissent cost to equity loss ratios compared with firm insiders (so that adding more outsiders alleviates the coordination problem faced by the board). Thus, our model suggests that firms where the CEO has greater influence (as measured by the CEO’s share ownership and job tenure), resulting in higher dissent costs for directors (both insiders and outsiders) will be associated with a smaller fraction of independent outsiders. Evidence consistent with this prediction is provided by Boone et al. (2007), who document such a relationship between CEO characteristics indicative of higher dissent costs for directors and the fraction of independent outsiders on the board.

Our model also suggests that more complex firms that have larger boards should have a greater proportion of outsiders so as to mitigate the greater coordination problem associated with such boards. Furthermore, in such boards, the outside directors should have greater equity holdings so as to reduce their dissent cost to equity loss ratio. Suggestive evidence consistent with the former prediction is provided by Coles et al. (2008), who document that more complex firms, with greater advising requirements than simpler firms, have larger boards with more outside directors. Evidence consistent with the latter prediction that the proportion of independent outside directors will be greater in firms where outsiders have lower dissent cost to equity loss ratio is provided by Boone et al. (2007), who document that the proportion of independent outsiders on a firm’s board is positively related to the equity ownership of outside directors.

iv. The ambiguous effect of low-precision public signals on board performance: Our model demonstrates that, contrary to common intuition, low-precision public signals may in fact worsen the board’s decision making instead of improving it (relative to the case of no public signal). Thus, short-term improvements in accounting performance by a firm may induce a director to vote to retain the current CEO, even though he is privately convinced that the firm is pursuing a wrong long-term strategy under the current CEO. This arises from the fact that such a public signal worsens coordination problems among directors by increasing each member’s fear that other members may vote to retain the CEO (thus increasing his likelihood of having to incur dissent costs arising from voting against the CEO but failing to oust her).

v. Policy implications for corporate governance and board reform: Our model suggests several ways in which corporate boards can be reformed to improve firm performance. Our analysis suggests that reforms that reduce the dissent costs of directors or increase their equity loss will significantly increase the likelihood of boards firing incompetent CEOs. One proposal that would reduce dissent costs would be to grant security of tenure (i.e., guarantee their presence on the board for some length of time) to current directors, thus reducing their dissent costs in the event they vote against the current CEO but end up being unable to oust her. Another reform proposal to reduce dissent costs would make it easier for dissidents to acquire a seat on the board: because of the cost of proxy fights, few dissidents bother to make board challenges. Yet another proposal to reduce dissent costs would be to reduce the involvement of CEOs in the selection of directors and have boards choose directors through nominating committees composed only of independent members of the board (see, e.g., Working Group on Corporate Governance 1995).

A second set of board reform proposals suggested by our analysis would increase the equity loss of directors. One such proposal would be to increase the proportion of directors’ compensation that depends directly on the firm’s long-run share value, for example, through compensating them with shares in the firm that they would
be required to hold for a certain length of time. Another proposal (perhaps harder to implement) would require directors to invest a significant amount of their own wealth in the firm’s equity.

9. Conclusion
In this paper, we develop a theory of corporate boards and their role in forcing CEO turnover. We consider a firm with an incumbent CEO of uncertain management ability and a board consisting of a number of directors whose role is to evaluate the CEO and fire her if this is beneficial in order to maximize long-run shareholder value. Each director receives an independent private signal about the CEO’s ability, after which directors discuss the CEO’s quality and subsequently vote on whether or not to fire her. If the CEO is fired, the board hires a new CEO from the pool of candidates available. The true ability of the firm’s CEO is revealed in the long run; the firm’s long-run share price is determined by this ability. Each director owns some equity in the firm and thus prefers to fire a CEO of poor ability. However, if a director votes to fire the incumbent CEO but the number of other directors also voting to fire her is not enough to successfully oust her, the CEO can impose significant costs of dissent on him. In this setting, we show that the board faces a coordination problem, leading it to retain an incompetent CEO even when a majority of directors receive private signals indicating that she is of poor quality. We show the existence of an optimal board size and that it depends on various board and firm characteristics: one size does not fit all firms. We also analyze the optimal composition of the board between firm insiders and outsiders and the effect of directors observing imprecise public signals in addition to their private signals on board decision making. Finally, we analyze the relation between the cost (and probability) of finding a good replacement CEO and the propensity of the board to fire an inefficient CEO.

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Endnotes
1 Using a structural model, Taylor (2010) estimates that the average board of directors behaves as if replacing the CEO costs shareholders at least $200 million, and that shareholder value would rise by 3% if we eliminated this perceived turnover cost.
2 Mace (1971) discusses anecdotal evidence of CEOs exercising authority in selecting candidates for the board—in effect, handpicking nominees. Similarly, Lorsch and Maclver (1994) report survey evidence indicating that CEOs wield major influence in selecting new directors. Tejada (1997) presents a news account of an outside director of a prominent company being denied nomination for reelection after criticizing management. Practitioners also seem to be aware that the combination of open ballot voting used in most corporate boards and the CEO’s ability to impose dissent costs on directors may significantly affect board decision making. To quote a practitioner-oriented article by Rock (2004, p. 1), “An open election process may underscore the collegial nature of board deliberations. However, it may also paper over board opposition and dissent. In the five companies on whose boards I serve, a secret ballot has never been used. If it had been, would it have changed any individual votes or perhaps even outcomes, as my fellow director suggested? … Would directors be more willing to register their true opinions if voting were done in private?”
3 We show that, even though the directors have the opportunity to discuss the CEO’s quality prior to voting, they are unable to credibly communicate their private signals truthfully to each other if dissent costs are significant.
4 We refer here to a coordination problem among directors in the following sense. To decide whether or not to vote against the CEO, a director who received a bad private signal about the CEO’s quality needs to assess the expected number of other directors who also received bad private signals and therefore the probability of his being in the majority. This is because, if he votes against the CEO but fails to oust her, he will incur dissent costs. This is a coordination problem in the sense that this problem would not exist if directors could directly observe each other’s private signals and coordinate their voting. This coordination problem becomes more severe in larger boards, since, in a larger board, each director is less likely to be pivotal.
5 A prediction of such a broader model would be that, when the CEO can impose significant dissent costs on directors, then even directors who privately disagree with the CEO’s proposals prior to voting are likely to vote with the majority once voting begins (allowing the CEO to get her way). Evidence consistent with this prediction is provided by the empirical analysis of Schwartz-Ziv and Weisbach (2013), who examine the minutes of board meetings of a sample of Israeli firms and find that the board unanimously approved proposals made by the CEO in all but 3.3% of the cases.
6 In this dynamic extension of our model, we show that boards that are better incentivized (smaller dissent cost to equity loss ratio) are more likely to fire CEOs proactively, before considerable destruction in shareholder value has taken place. Evidence consistent with this is provided by Ertrugrul and Krishnan (2011), who document that boards that dismiss CEOs early (i.e., in the absence of significant negative prior stock returns) are those that are characterized by higher equity ownership by directors, higher board equity compensation as a fraction of total compensation, higher institutional ownership, and better firm corporate governance characteristics.
7 Warther (1998) develops a simple model of a two-member board in which the CEO can eject hostile directors. He shows that this results in directors being reluctant to oppose management and votes being often unanimous in favor of management. However, given that he exogenously assumes a two-member board, he is unable to study any of the issues of coordination among directors that we analyze here; neither does he analyze optimal board size, board composition, the effect of public signals, or the dynamics of CEO turnover. Almazan and Suarez (2003) show that passive (weak) boards may be optimal in a setting where severance pay and weak boards are substitutes.
for costly incentive compensation paid to the CEO. In a subsequent paper, Malenko (2014) analyzes a setting with biased directors to study issues of prevote communication among directors.\footnote{Our paper is also distantly related to the literature on voting in committees; see, for example, Crawford and Sobel (1982), Coughlan (2000), Doraszelski et al. (2003), and Austen-Smith and Feddersen (2005), who allow for prevote communication and study a cheap-talk setting. The broader literature on decision making by committees is also distantly related; see, for example, Moldovanu and Shi (2013), who analyze decision making by committees consisting of specialized experts versus generalists, and Albrecht et al. (2010), who analyze a sequential search problem and compare search by committee to the single-agent problem. Our paper is also related to the broader literature on the optimal amount of dissent; see, for example, Acharya et al. (2011), Che and Kartik (2009), and Landier et al. (2009), in the sense that we model how the presence of costs of dissent potentially imposed on corporate directors can stifle dissent, thus leading to inefficient CEO turnover and other decisions by boards. Note, however, that the focus of the above models and the trade-offs driving them are quite unrelated to those in our model.}

\footnote{The requirement that \( n \geq 5 \) is not binding in the vast majority of real-world boards, since the median number of directors in public U.S. firms is about 10, and the 25th percentile is 8 (see, e.g., Coles et al. 2008 or Fedaseyev et al. 2016). In our model, this requirement precludes situations in which an out-of-equilibrium move by a single director can change the outcome of the vote. As we describe in detail below, all directors simultaneously report their private signals about the CEO in the discussion round. If \( n = 3 \) and one of those reports is bad, then any director who has a bad private signal (but who has not reported it truthfully in the discussion round) will infer that directors with bad private signals are in the majority. Such a director will then have an incentive to vote to fire the CEO in the voting round (contrary to his report in the discussion round), as long as this director believes that the deviating director (who reported a bad private signal in the discussion round) will vote to fire the CEO. All of our results will go through qualitatively unchanged even if \( n = 3 \); however, the values of the various thresholds that determine when information sharing between directors is possible will have to be modified, consistent with the idea that it is easier to sustain information sharing in smaller boards.}

\footnote{This assumption is introduced for ease of exposition and is innocuous if one assumes that the CEO always votes to retain herself. Adding the CEO to the board in this case will simply decrease the effective board size to \( n - 1 \).}

\footnote{For example, some directors may be willing to inform the CEO of private board discussions in order to gain favors or simply because some directors are aligned with the CEO.}

\footnote{We assume voting is done by open ballot. However, all of our qualitative results will remain unchanged even if directors vote by secret ballot, as long as there exists a high enough probability that the CEO will eventually come to know the identities of directors who voted against her and impose a dissent cost on them. In practice, most boards vote through an open ballot. For example, to quote the practitioner-oriented article by Rock (2004, p. 1), “In the five companies on whose boards I serve, a secret ballot has never been used.” Furthermore, if the culture of a firm is to have its directors vote in board meetings on a potential CEO turnover by open ballot, any director who suggests voting by secret ballot would be inferred as opposing the CEO. We assume that there is only one round of voting, and the outcome of this vote is final. This is equivalent to assuming that directors can commit themselves not to vote again.}

\footnote{The assumption that the quality of the newly hired CEO is revealed immediately upon hiring can be viewed as a reduced-form outcome of an infinite horizon firing game that ends after a CEO has been in place for one period (the quality of the CEO can possibly be revealed from firm performance under her tenure). In such a game, the board may fire the newly hired CEO (based on directors’ private signals) before the true quality of this new CEO is revealed. Thus, consider the case where we relax the assumption that the quality of a newly hired CEO is revealed immediately and assume instead that it is revealed only if the board decides not to fire the incumbent CEO. Let \( \delta \) denote the stationary probability that a newly hired CEO will be fired in the voting round after she is hired (this probability will depend on the directors’ ex ante probability assessment that there will be enough votes to fire the newly hired CEO, and this probability assessment will depend on the exogenous parameters of the model). If she is hired, a new CEO is hired, who will also be fired (and replaced) in the next round with probability \( \delta \), etc. Thus, with probability \( 1 - \delta \), the CEO hired at time 3 will be retained and her quality will therefore be revealed. With the remaining probability \( \delta \), the CEO hired at time 3 will be fired at time 4. The newly hired CEO (at time 4) will then be retained with probability \( 1 - \delta \), while with the remaining probability \( \delta \) a new CEO will be hired, and so on. Therefore, the expected quality of the CEO who will ultimately run the firm for at least one period is}

\[
(1 - \delta) y + \delta((1 - \delta) y + \delta((1 - \delta) y + \delta((1 - \delta) y + \cdots)))
\]

\[
= (1 - \delta) y + \delta(1 - \delta) y + \delta^2 + (1 - \delta) y + \delta^3 + \cdots
\]

\[
= (1 - \delta) y + \delta^2 + \delta^3 + \cdots
\]

\[
= (1 - \delta) y \frac{1}{1 - \delta} = y.
\]

Thus, the expected quality of the eventual replacement CEO is \( y \), regardless of \( \delta \).

\footnote{This is a normalization. If we assumed a fixed amount in terms of “sitting fees” paid to directors here (e.g., representing the director’s compensation that is independent of firm value), our results would go through qualitatively unchanged.}

\footnote{This “reputation” interpretation of the equity loss is formally modeled in Levit and Malenko (2016). In their paper, directors can acquire a reputation for being “shareholder friendly” or “management friendly,” which, in turn, may affect directors’ rewards in the labor market through being invited to serve on other corporate boards.}

\footnote{In practice, it may not be feasible for boards to ensure a large enough \( a \) to perfectly align the interests of directors with those of the firm, for the following reasons. First, equity holdings that align the interests of all directors may have to be prohibitively large from their standpoint. In particular, new, competent directors may be in short supply, and insisting on their holding large equity positions in the firm may have significant negative effects on their own portfolios, imposing large costs on them. Alternately, firms may compensate directors with equity, but in this case, awarding equity compensation to directors of a large enough magnitude to induce efficient voting may impose huge costs on the firm.}

\footnote{There can exist an equilibrium in which, during the discussion round, all directors with bad signals say that they have good signals while all directors with good signals say that they have bad signals. In the discussion round, all directors will infer this inverse reporting rule and the good signals reported during the discussion round will be correctly interpreted as bad signals about the CEO’s quality, and vice versa. We assume that, in this case, it is directors who report good signals who suffer dissent costs because those signals are (correctly) interpreted by other directors as negative signals about the CEO’s quality.}

\footnote{One reason why it would be sequentially rational for a CEO to impose such dissent costs is for the purpose of acquiring a reputation for toughness. If the CEO imposes significant dissent costs on a director who is known to have voted against her in the current board meeting on CEO turnover, other directors would be afraid to vote against that CEO in subsequent board meetings. We do not choose to
explicitly model this reputation acquisition by CEOs here, since the insights generated by such a multiperiod model are not commensurate with the additional complexity.

A symmetric PBE is one where all directors with the same information choose the same equilibrium strategy. This is a natural requirement in the equilibrium definition in a setting such as ours, where all directors are identical except for their private signals. Note that the requirement of a symmetric PBE does not restrict directors’ out-of-equilibrium strategies. We will, however, impose some restrictions on the out-of-equilibrium beliefs of directors though the additional requirement of symmetric conjectures.

See Fudenberg and Tirole (1991) for a formal definition of PBE. See Benoit and Dubra (2006) for an application of undominated PBE. See also Palfrey and Srivastava (1991) for a discussion of why elimination of weakly dominated strategies is an appropriate refinement of Nash equilibrium in many situations. See Harsanyi and Selten (1988) for a discussion of why Pareto (or payoff) dominance may be an appropriate equilibrium selection criterion in noncooperative games. Pareto dominance (synonymously, Pareto efficiency or payoff dominance) is a realistic equilibrium selection concept in our context, since even a verbal agreement among directors to vote along the lines suggested by a Pareto-dominant Nash equilibrium would make such an equilibrium focal. The notion that Pareto-dominant (Pareto-efficient) equilibria are more robust than inefficient equilibria (involving excessive dissipation of resources) has been used extensively in asymmetric information models that involve signaling; see, for example, Milgrom and Roberts (1986). A number of asymmetric information models in corporate finance use Pareto dominance (Pareto efficiency) as an explicit equilibrium selection criterion; see, for example, the initial public offering (IPO) underpricing model of Grinblatt and Hwang (1989), the dividends and investment model of Miller and Rock (1985), or the product market advertising around new equity issues model of Chemmanur and Yan (2009). Finally, outside corporate finance, Engers (1987) focuses only on the Pareto-dominant separating equilibrium in his analysis of a market with asymmetric information where there are many signals available, and where both the costs of signaling and product value may depend on many privately known characteristics.

Thus, each director, when considering his optimal strategy, places positive probability weights only on strategies by other directors that involve symmetric strategy profiles on and off the equilibrium path—that is, strategy profiles in which other directors with the same information (private signals and information obtained during the discussion round) behave in the same way. This restriction is not necessary for the equilibria that we describe here to prevail. We use it only to eliminate certain implausible equilibria, such as an equilibrium in which even directors with good private signals vote to fire the CEO, even after no additional information was revealed in the discussion round.

We implement the equilibrium selection criterion of Pareto (payoff) dominance at every stage of the game. Thus, in the basic model, we start by choosing the Pareto-dominant undominated PBE in the voting subgame conditional on all information reported in the discussion round. We then move backward to the discussion round and, among all possible undominated PBEs of the entire game, choose the one that is Pareto dominant. In Appendix B (Section B.3), we show that the existence of the equilibrium in our model is robust to removing the requirement of Pareto dominance from the equilibrium definition. We make use of this last requirement only to eliminate some implausible equilibria under the parameter values prevailing in part (i) of Proposition 3 and in some similar situations in other propositions.

However, this result is clearly a consequence of our assumption that individual directors do not have to incur a cost of information production (e.g., because of a need to exert effort) to obtain their signals about the CEO’s quality. Incorporating such costs of information production into our model may create free-riding incentives among directors. Since such free-riding incentives may become worse as the board size gets larger, there may be a limit to board size in such a setting.

We define an informative discussion round as one where all directors truthfully report their private signals about the CEO’s quality. Conversely, we define an uninformative discussion round as one where all directors report the same signal (good or bad) about the CEO’s quality, regardless of their private signals, so that it is not possible to infer a director’s true private signal from his report. With- out loss of generality, we focus on uninformative discussion rounds where all directors report good private signals about the CEO’s quality, since the key feature of an uninformative discussion round is that no information is transferred across directors (regardless of whether all of them report good signals or bad signals).

Note that if the majority of reported signals are good, it is a weakly dominant strategy for directors who received good private signals to vote to retain the CEO. Given this, it is also optimal for directors who received bad private signals to vote to retain the CEO, since they are aware that the CEO will be retained in any case (because directors with good private signals are in the majority) and voting to retain her will therefore prevent them from incurring a dissent cost in the voting round.

Note that there exists an undominated PBE where all directors vote to retain the CEO even when the majority of reported signals are bad. Such a PBE is supported by each director’s belief that all other directors will vote to retain the CEO, so that a given director voting to retain the CEO has no effect on the final outcome while preventing him from incurring dissent costs in the voting round. Such a PBE is, however, Pareto dominated by an undominated PBE in which the majority of the directors vote to fire the CEO, since all directors obtain a higher expected payoff in the latter PBE.

Notice that PBEs in which all directors report truthfully in the discussion round and then vote to fire the CEO if the number of bad signals reported in the discussion round is lower than some threshold value \( k^* \), where \( k^* < (n + 1)/2 \), are not undominated PBEs. This is because, if only a minority of directors report bad private signals (which implies that a majority report good private signals), it is a weakly dominant strategy for directors with good private signals to vote to retain the CEO, since, by doing so, they minimize their expected equity loss while not incurring any dissent costs.

Of course, here we are characterizing optimal board size purely from the point of view of ensuring that the board has the best information for decision making. However, in practice, additional factors such as the fixed costs of employing directors may also limit the size of the board. These fixed costs may be more important (relative to their revenue) for younger and smaller firms, leading to smaller board sizes for such firms.

While we cannot derive explicit expressions for the optimal number of insiders and outsiders on the board, it is easy to show that there exist many parameter values for which both insiders and outsiders should be on the board. For example, let a board that consists of 11 directors have the following parameter values: \( y = 0.7, a = 0.6, \beta = 0.65, \rho_c/q = 0.25 \), and \( pb/q = 0.55 \). Then, to maximize the probability of firing a bad CEO and retaining a good CEO, this board should consist of five insiders and six outsiders. The relevant numerical calculations underlying this example can be obtained from the authors upon request. Note also that the equilibrium described in part (ii) of Proposition 6 may not be unique. In particular, there may exist equilibria where only outsiders report their signals truthfully in the discussion round while outsiders do not, and there may exist other equilibria where only outsiders report their signals truthfully in the discussion round while insiders do not. However, in such equilibria, the board will have less information available for decision making compared with the equilibrium described in Proposition 6(ii).
It can be shown that, if the dissent costs of both types of directors (outsiders as well as insiders) are high enough, an informative discussion and voting equilibrium is never sustainable regardless of board composition. In this case, the proportion of insiders versus outsiders on the board is clearly irrelevant.

In the online appendix, we show in the proof of Proposition 7 that both (11) and (12) hold as long as $\alpha > \varphi > 1/2$.

This situation does not arise in our basic model because when $d = 0$ and $\alpha = \gamma$, then the CEO's expected quality is worse than that of a potential replacement when the majority of directors' signals are bad; see condition (4).

Note that part (iii) of Proposition 9 places an additional restriction on $\sigma$ by requiring that $q(p)((n + 1)/2) < d + q(1 - \sigma)$. This is because if $\sigma$ (the likelihood of finding a high-quality replacement) is very high, then there may be cases where hiring an outside replacement makes all directors better off even if all of them observe bad private signals about the incumbent CEO; the above restriction rules out this extreme scenario.

Note, however, that while we point out empirical evidence broadly consistent with our model’s predictions to allow the reader to judge their empirical relevance, we acknowledge that some of this evidence may be driven by endogeneity, so that the underlying relationships may not be causal.

It can cost hundreds of thousands of dollars or more for an outside contender to run against a director. For example, a campaign for a board seat may involve repeated mailings to every investor. Directors endorsed by firm management, on the other hand, can make use of the company coffers to finance their candidacies. Thus, RiskMetrics, endorsed by firm management, on the other hand, can make use of a potential replacement when the majority of directors’ signals are bad; see condition (4).

Note, however, that while we point out empirical evidence broadly consistent with our model’s predictions to allow the reader to judge their empirical relevance, we acknowledge that some of this evidence may be driven by endogeneity, so that the underlying relationships may not be causal.


Tejada C (1997) Longtime Tandy director quits board, says he was punished for faulting CEO. *Wall Street Journal* (January 16).