An entrepreneur, with private information about his firm, contracts over two periods with an outside financier, a venture capitalist (VC) or angel. The financier can reduce his information disadvantage by learning about the firm over time. VC financing is scarce relative to angel financing. Further, unlike an angel, a VC may exert effort, which, together with the entrepreneur’s effort, increases the firm’s success probability. The equilibrium VC financing contract ensures optimal effort-exertion by both entrepreneur and VC. We characterize the firm’s equilibrium choice between VC and angel financing, its equilibrium contractual provisions, and the dynamic evolution of its financing contract. (JEL G24)

It is well known that angel financing is an important source of financing for private firms in the United States. However, beyond the fact that the annual amount of angel financing is much larger than that of venture capital financing, and that angels tend to be individuals who invest much smaller amounts than venture capitalists (VCs) in individual firms, little is known about the important economic differences between venture capital and angel financing.\textsuperscript{1} One of the objectives of this paper is to bridge this
gap in the literature by developing a theoretical analysis of the different roles played by venture capitalists and angels in funding private firms, and to develop an understanding of the situations under which firms make use of each type of financing.

The second objective of this paper is to develop an analysis of the dynamic features of financing contracts in the private equity market. The empirical evidence (as well as descriptions of individual cases) indicates that typically, firms undertake several rounds of private equity financing. Sometimes these different rounds of financing to a firm may come from the same source: for example, the same venture capital firm may provide multiple rounds of financing to a firm. In other situations, these different rounds of private equity financing may come from different sources: thus, a firm may be initially angel-financed and may later switch to venture capital financing; alternatively, a venture capitalist may provide funding initially, but may choose to sell his equity stake and leave the firm. The above situations lead us to ask several questions: First, are there any important differences between venture capital and angel-financing contracts? Second, what motivates firms to switch from one form of private equity financing to another? Third, if firms make use of multiple rounds of financing from the same source, are there (and should there be) any systematic differences in the contracts between the entrepreneur and financier from one round to another (i.e., how do venture capital and angel-financing contracts evolve over time?). We develop answers to these questions here.

Our analysis rests on a few assumptions based on certain stylized facts about the private equity market. First, we assume that in the early stages of a firm, the financier (venture capitalist or angel investor) is able to add value to the firm, at least in some situations. Second, we assume that, while both the venture capitalist and the angel may be able to add value in this way, the venture capitalist is more capable of adding value (or equivalently, the venture capitalist can add value in more situations) than the angel. Third, we assume that while the financier is capable of adding value, he has to engage in costly effort to add this value, so that he has to be given the appropriate incentives to put forth effort optimally on behalf of the firm. Fourth, we assume that prior to the financier getting involved with the firm, the entrepreneur has private information regarding the nature of his own project (including the likelihood of the financier being able to add value to the project). However, if he does provide funding to the entrepreneur’s project (thereby getting involved in its


See, for example, Hellmann and Puri (2002), who find that venture capitalists play a significant role in the professionalization of startup firms in general, and in particular, in the hiring of their top management.
activities), the financier is able to learn more about the project over time, thus eliminating the information asymmetry between the entrepreneur and the financier. Finally, we assume that the entrepreneur’s effort is also required for the project to succeed, so that the contract provided to the financier must be such that the entrepreneur also has the appropriate incentives to make the project a success.

In the above setting, we derive a variety of interesting predictions about entrepreneurs’ equilibrium choice of private equity financing source and the structure of private equity financing contracts. First, we show why, in many situations, firms prefer venture capital financing over angel financing, even though venture capitalists require a greater rate of return from their investment in the firm. Second, we characterize the conditions under which firms switch financing sources across financing rounds (angel to venture capitalist or venture capitalist to angel). Third, we characterize the equilibrium financing contracts between venture capitalists and entrepreneurs on the one hand, and angels and entrepreneurs on the other, thus allowing us to make predictions regarding the differences between the two kinds of contracts. Fourth, we make predictions regarding how the structure of venture capital contracts will evolve over time. Fifth, we develop implications for the composition of projects (early- versus later-stage) financed by venture capitalists and angels, and how this composition varies with changes in the scarcity of venture capital financing relative to angel financing. Sixth, we develop predictions regarding the announcement effects of various forms of financing, and the relationship between the dynamic path of firm financing and the quality of firms’ projects.

An important tradeoff driving our results is that the asymmetric information existing between the entrepreneur and the venture capitalist interferes with the provision of appropriate incentives to the venture capitalist to create value for the firm. The venture capitalist learns more and more about the firm and the entrepreneur as he interacts with them over time. Therefore, the venture capitalist’s informational disadvantage relative to the entrepreneur will be significantly lower in later-stage contracting compared with that in early-stage contracting. This has two important consequences in our setting. First, later-stage contracts between the entrepreneur and the venture capitalist will be better than early-stage contracts at providing incentives to the venture capitalist to create value for the firm (while ensuring that the entrepreneur retains the appropriate incentives to exert effort as well). Second, the contract between the entrepreneur and the venture capitalist in a firm previously financed by the same venture capitalist will provide him with better incentives to create value compared with a contract between a venture capitalist and a previously angel-financed firm. This tradeoff also explains the venture capitalist’s equilibrium choice about the funding stage: whether to start
funding a project at an earlier or at a later stage. On the one hand, starting to fund a project at an earlier stage allows the venture capitalist to create more value for the firm; on the other hand, if the venture capitalist starts funding a project at an earlier stage, there is a greater chance that he is investing in a firm where he cannot create significant value, and therefore has to exit the firm before project completion (earning a return lower than his opportunity cost of capital). Thus, we show that during periods where venture capital funding is relatively scarce (so that the opportunity cost of venture capital is moderate or high), venture capitalists tend to finance more later-stage projects.

1. Relation to the Existing Literature

Our paper is related to several studies in the existing literature. The first is the literature offering various rationales for the use of convertible features in financial contracts. These rationales for the use of convertible features in financial contracts can be grouped into five categories. The first category deals with conflicts between stockholders and bondholders, and the related incentives of insiders to take on excessively risky projects (see, e.g., Green 1984). The second category of papers deals with how the use of convertibles may be driven by asymmetric information between firm insiders and outsiders (e.g., Stein 1992; Constantinides and Grundy 1989; Brennan and Kraus 1987). Both of the above rationales apply to the use of convertibles by public as well as private firms. The third category of papers argues that the use of convertibles in venture capital contracts arises from the incompleteness of contracts between the venture capitalist and the entrepreneur, and the ability of different financial contracts to optimally switch control between the two (see, e.g., Hellmann 1998; Berglof 1994; Schmidt 2003).3 The fourth category of papers argues that convertible features arise from issues related to providing the right incentives to the entrepreneur in a setting of moral hazard (Cornelli and Yosha 2003; Repullo and Suarez 2004).4 Finally, Hellmann (2006) argues that a key feature of convertible securities in venture capital financing contracts is to create different cash-flow rights for acquisitions and IPOs, and demonstrates how the convertible security implements an optimal

3 The incomplete contracting literature builds on the pioneering work of Grossman and Hart (1986). Three important papers in this literature are Aghion and Bolton (1992), Hart and Moore (1998), and Dewatripont and Tirole (1994). Many of the control theories of venture capital contracting make use of a modeling setup similar to one or more of these papers. See also Marx (1998), who argues that when the venture capitalist is risk averse, convertible preferred equity motivates the venture capitalist to intervene in the firm in response to poor performance. Gompers (1997) argues that venture capital convertible-debt contracts are quite different from convertible debt in large public corporations. See also Bergemann and Hege (1998).

4 The need to provide incentives to the entrepreneur to put forth optimal effort was argued in a number of early papers by Sahlman (see, e.g., Sahlman 1988, 1990).
tradeoff between the need to allocate cash flows to the venture capitalist and the desire to make efficient exit decisions.

In contrast to the above literature, in our paper the rationale for the use of convertible features emerges from the need to provide incentives to the venture capitalist to exert effort to add value to the firm while maintaining the incentives of the entrepreneur to exert effort. Further, to the best of our knowledge, none of the above papers have analyzed how the contract between the venture capitalist and the firm should evolve across multiple rounds of financing. Thus, our model makes the novel prediction that, while convertible preferred equity or convertible debt will be used in both early- and later-stage financing, the relative magnitudes of the fixed-income component and the upside (warrant) component will differ across financing rounds: while early-stage financing with a venture capitalist will have more of a fixed-income component and less upside, later rounds of financing with the same venture capitalist will feature a smaller fixed-income component but a larger upside. Further, our analysis predicts that angel financing contracts are less likely to incorporate convertible features compared with venture capital contracts.5

Our paper is also related to other strands of the theoretical and empirical literature on private equity financing. Like our paper, Repullo and Suarez (2004) also study the moral hazard problem on the part of both the venture capitalist and entrepreneur.6 However, unlike in our paper, the driving factor in their paper is the allocation of the refinancing cost of the project across states. Because the firm’s later-stage financier has to buy back the firm’s financial contract from the initial financier, in their setting the optimal contract provides the initial financier a greater payoff when the state is high and a smaller payoff when the state is low, so that projects with smaller positive net present values can be financed. Apart from this difference in driving intuition, in their setting there is no asymmetric information between the entrepreneur and the outside financier; neither do they analyze the firm’s choice between different kinds of financiers (angels and venture capitalists). Casamatta (2003) also studies the moral hazard problem on the part of both venture capitalist and entrepreneur. However, the focus of that paper is on why outside experts such as VCs also need to finance the projects (to induce the experts to exert the optimal level of effort).7

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5 The practitioner literature indicates that both venture capital and angel contracts come in four basic forms: common stock, stock with warrants, convertible equity, and convertible notes (debt). Various additional provisions are added to these basic structures depending on the specifics of a given project or firm. See Bartlett (1995) for details.

6 See also Inderst and Mueller (2002), who use a search model to demonstrate that the composition and the type of financial claims held by the venture capitalist and the entrepreneur depend on the market structure.

7 One paper that discusses the choice between different kinds of financiers is Leschinskii (1999). However, that paper is driven by the assumption that while the venture capitalist can fire the manager, business
Fulghieri and Sevilir (2009) study the size and scope of a VC’s portfolio in a similar double-sided moral hazard setting. In their paper, a small portfolio benefits the VC because of the following two reasons. First, the VC can add more value to each project in the portfolio; second, limited competition for later-stage financing among the projects enables each entrepreneur to keep a larger share of his project’s value and as a result motivates him to work harder in the earlier stage. While like our paper, Fulghieri and Sevilir (2009) also have a double-sided moral hazard feature, they abstract away from the financing contracts between the VC and the entrepreneurs by restricting the contracting possibilities in the earlier-stage financing. The later-stage financing contract is thus determined by the bargaining between the VC and entrepreneurs. The VC’s outside options and scarcity are therefore important at this stage in their setting. In contrast, we assume that the long-term cash flow is contractible, sharing between the earlier-stage financier and the entrepreneur can be achieved by contracting. Therefore, the VC’s outside options and scarcity determine not only the kinds of projects VCs invest in, but also the characteristics of the financial contracts used. Admati and Pfleiderer (1994) study a setting in which a venture capitalist can observe the true state of a firm, unlike other outside investors. They show that optimal investment decisions will be made by the firm in all states if and only if the venture capitalist is given a fixed-fraction equity contract, which eliminates his incentives to misrepresent the state to other outside investors.

Finally, our paper is also related to the growing empirical literature providing detailed evidence on the structure of venture capital contracts in the United States and other countries. Prominent examples of this literature are Sahlman (1990), Gompers (1997), Kaplan and Stromberg (2003), and Bengtsson and Ravid (2011). In a similar vein, Cumming (2000) provides Canadian evidence, Bascha and Walz (2001) provide German evidence, and Parhankangas and Smith (2000) provide evidence from Finland.

8 In a single-sided moral hazard setting, Inderst, Mueller, and Munnich (2007) show that it can be beneficial for a VC to finance a large portfolio of earlier-stage projects and commit to continue to finance only the best ones in the later stage. Competition for the limited later-stage financing can add value by forcing the entrepreneurs to work hard in the earlier stage.

9 In general, our model is different from those in an incomplete contract framework such as Aghion and Bolton (1992) and Hart and Moore (1994, 1998). In such models, because contracting is incomplete, additional contracting options such as the right to renegotiate or event-contingent transfers may help to improve financing efficiency. In contrast, these additional contracting possibilities will not help in our setting since long-run cash flow is fully contractible here.
2. The Model

The model has three dates \((t = 0, 1, 2)\) and three kinds of agents (entrepreneur, venture capitalist, and angel), all of whom are risk neutral. The entrepreneur is endowed with a nondivisible project, which needs external financing \(I\). Of the investment \(I\), a minimum amount \(I_0\) is required at time 0; the entrepreneur may raise the remaining amount \((I - I_0)\) either at time 0 or time 1. Thus, if the entrepreneur has invested an amount \(I_0 \geq I_0\) at time 0, he will invest the remaining investment amount \((I - I_0)\) at time 1.

We refer to the first period (time 0 to time 1) as the earlier stage of the firm’s project, and the second period (time 1 to time 2) as the later stage of that project.\(^{10}\) For simplicity, we normalize the risk-free rate of return to be zero.

The entrepreneur has two potential sources of external financing: The venture capitalist (VC, from now on) or the angel. There are two differences between the VC and the angel in our setting. First, the VC contributes not only capital but also effort, which helps in the successful implementation of the project. In contrast, the angel contributes only capital.\(^{11}\) Second, VC financing is scarce relative to angel financing.\(^{12}\) At time 0 and time 1, the entrepreneur chooses between these two sources of financing depending on his private information and other relevant variables in the firm and the economy. We allow for the entrepreneur to refinance his project at time 1 (in case he decides to switch from an angel to a VC or vice versa). In other words, the amount raised from the time 1 financier can be more than the pure investment amount \((I - I_0)\) by

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\(^{10}\) Private equity financing is often categorized into four stages. The first stage refers to firms in the startup, research and development (R&D), testing, and market research stage. The second stage refers to the prototype, further testing, and early expansion stage. The third stage refers to full-scale manufacturing and marketing. Finally, the fourth stage refers to the financing of firms that are profitable. In our model, early-stage (time 0) financing can be thought of as corresponding to the first stage in the above classification. On the other hand, later-stage (time 1) financing corresponds to the second and the third stages in the above classification. In both of these stages, the VC’s effort can add significant value to the firm: the VC may help the firm hire technical as well as managerial talent, develop relationships with suppliers and potential clients, etc.

\(^{11}\) This argument is consistent with the survey evidence of Prowse (1998), who documents that a large proportion of angels tend to be unsophisticated investors, unable to add significant value to a firm. Empirical evidence consistent with this assumption is also provided by Wong (2001), who documents that angels are more passive compared with VCs, and Hochberg (2002), who documents that VCs tend to add more value than other institutional investors. Similar notions about the difference between venture capitalists and angels in terms of their ability to add value to a firm are also prevalent among practitioners: see, e.g., the discussion of the differences between venture capitalists and angels in Quindlen (2000), Chapter 5. See also the case study on Honest Tea described in Gompers (2001), and the discussion there of the differences between venture capitalists and angels. Of course, there are also many “sophisticated” angels capable of adding considerable value to firms. In our setting, such angels can be included in the category of venture capitalists, since the defining characteristics of a venture capitalist in our setting is the ability to add significant value to a firm.

\(^{12}\) We will discuss the economic consequences of these two differences between VCs and angels later in this paper. In practice, this scarcity may arise from the fact that VCs commit not only financial capital but also human capital to firms they are involved in, and the above human capital is limited. See also Lerner (1998).
the amount required to buy out the time 0 financier. Further, we allow for
the entrepreneur to raise more money than the amount the firm needs at
time 0 and 1. In this case, the amount in excess of the firm’s investment
requirement goes to the entrepreneur.13 The cash flows from the project
are realized at time 2. We assume that there are only two possible
outcomes for the project: “highly successful” (high cash flow $X$) or
“less successful” (low cash flow $\bar{X}$), $0 < \bar{X} < I_0 < X$.14

2.1 The entrepreneur’s private information and effort provision
The cash flow from the firm’s project depends on three things: the nature
of the firm’s project (type G or B at time 0, and state $p$ or $n$ at time 1),
the effort provided by the entrepreneur (high or low level of effort), and the
effort provided by the VC (which is a continuous variable, which we will
discuss in more details in Section 2.2).

We assume that the entrepreneur has private information with respect
to outsiders (including venture capitalists and angels) at time 0 and time
1. We model this private information at time 0 by assuming that, while
the entrepreneur observes the type of his own project (G or B), outsiders
observe only the prior probability $\theta$ of a project being of type G.

Similarly, at time 1, we assume that the realization of the state ($p$ or $n$)
is observable by the entrepreneur and the firm’s current financier, but not
by outsiders. Thus the entrepreneur has private information relative to
the outsiders even at time 1, since he observes the state the firm is in at
time 1. However, if a financier was involved with a firm from time 0 itself,
his observes the realization of the state $p$ or $n$ as well, so that he has the
same information about the firm at time 1 as the entrepreneur, so that
any further financing undertaken by that financier at time 1 would not
suffer from asymmetric information. In contrast, if a firm switches finan-
ciers at time 1, its time 1 financing would suffer from information asym-
metry, since the new financier would not observe the true time 1 state of
the firm.15 Note that unlike types B and G (which are endowed by

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13 Even though we allow for this possibility, the entrepreneur typically will not raise more than the required
investment amount as long as this amount is reasonably large. The only scenario under which this occurs
in equilibrium is when the required external financing amount is so small that if the VC provides funding
only to this extent, his financial stake in the firm is not enough to motivate him to exert the optimal
amount of effort to create value for the firm (so that the entrepreneur may choose to raise an amount in
excess of this investment amount to induce the venture capitalist to exert optimal effort). In practice,
there are several cases where entrepreneurs have raised financing in excess of firms’ investment require-
ments, with the excess cash going to the entrepreneurs: see, e.g., “Startup millionaires even before the
IPO,” Business Week, May 9, 2005.

14 The assumption that $I_0 > \bar{X}$ ensures that the project cannot be financed through risk-free debt at time 0.

15 We assume that there are a number of VCs, angels, and projects of all stages, types, and states in the
economy. This implies that each VC or angel is able to select both the stage (time 0, time 1) and the
nature (type G or type B for a time 0 project and state $p$ or state $n$ for a time 1 project) of the project he
wants to invest in, provided it is in the interest of the corresponding entrepreneur to select such financing.
Conversely, each entrepreneur will also have available to him the financing (VC or angel) of his choice,
and the financing will proceed provided that it is in that financier’s interest to invest in such a firm.
nature), the probability of realization of state \( p \) (versus state \( n \)) for a type G firm is affected by the effort provided by the entrepreneur and the VC as well (see Figure 2); this probability is unaffected by effort for a type B firm. We choose our model this way to endogenize the dynamic evolution of asymmetric information within a startup and the interaction between the asymmetric information faced by a VC and the VC’s incentive to provide optimal effort, as discussed in Section 2.2.

We believe the entrepreneur’s effort is an important element of startup financing in the real world.\(^{16}\) We model the entrepreneur’s effort in the following way. In each period, the entrepreneur can exert either a low level of effort (in which case we normalize his cost of effort to be 0), or a high level of effort (cost of effort \( k_i > 0, i = 0, 1 \)). We assume that the entrepreneur’s effort is complementary to that of the VC in each period, as follows. If the entrepreneur exerts only low effort in a particular period, the VC’s effort is not productive at all (i.e., whether or not the VC exerts effort does not affect the probability of project success). If, however, the entrepreneur exerts high effort in a given period, the VC is able to add value in that period, depending on the nature of the firm’s project, as discussed in detail in Section 2.2. The entrepreneur’s objective in choosing between angel and VC financing, as well as in making his effort choice at each date, is to maximize the expected value of his total cash flow net of effort costs.

2.2 The venture capitalist
We assume that the VC is able to add value to the firm’s project, depending on whether the entrepreneur exerts high or low effort, and also on the nature of the firm’s project. At time 0 (beginning of the first period), the firm’s project can be of either type G or type B. Similarly, at time 1 (beginning of second period) the firm’s project can be either in state \( p \) or in state \( n \). We assume that the VC’s effort to add value is complementary to the nature of the project. In short, the VC can add value only to a type G earlier-stage or a state \( p \) later-stage project. We will first discuss value creation by the VC in the second period before going on to discuss how the VC can create value in the first period.

We define state \( p \) projects as those in which the VC’s effort in the second period is “productive,” and state \( n \) projects as those in which the VC’s effort is “not productive.” We model value creation by the VC at time 1 by assuming that, while the probability of the project achieving the high cash flow \( X \) at time 2 is only \( q \) for a state \( n \) project, it will be \( q + f(c_1) \) for a state \( p \) project with VC financing, provided the entrepreneur also exerts high effort; here \( c_1 \) denotes the VC’s effort in the

\(^{16}\) As will be clear later, the entrepreneur’s effort affects VC contracts in an important way. Without the entrepreneur’s effort, the VC can simply buy out the firm, and the entrepreneur can leave the firm.
second period. We assume that the probability of a high cash flow for any project financed by an angel, or if the entrepreneur exerts only low effort, is \( q \), regardless of the state the project is in (see Figure 2).

We model value creation by the VC in the first period in a similar manner. We define a type G project as one in which the VC’s first-period effort is productive, while we define a type B project as one in which this effort is not productive. Thus, while the probability of a project being in state \( p \) in the second period is only \( \lambda \) for a type B project, it increases to \( \lambda + f(c_0) \) for a venture-financed type G project, provided that the entrepreneur exerts high effort in the first period; here \( c_0 \) denotes the VC’s effort in the first period. We assume that the probability of any project being in state \( p \) in the second period is only \( \lambda \) for any project (regardless of type) if financed by an angel, or if the entrepreneur exerts only low effort.

We assume that the VC’s effort, \( c_i, i = 0, 1 \), is a continuous variable, with the VC incurring a private cost of effort that is monotonically increasing in his effort level. For notational simplicity, we will use \( c_i, i = 0, 1 \), to denote not only the VC’s effort level, but also the corresponding effort cost incurred by him in each period. We assume that \( f(c_i) \) is increasing and concave in \( c_i \), with \( f(0) = 0, q + f(\infty) < 1 \), and \( \lambda + f(\infty) < 1 \).\(^{17}\)

Since VC financing is scarce relative to angel financing, the VC requires a minimum (threshold) net present value (NPV), denoted by \( R^2 > 0 \), from investing in a firm’s project in each period (in other words, \( R \) is the net present value of the VC’s alternative investment opportunity over two periods). One can think of \( R^2 \) as the NPV the VC can obtain in each period (i.e., the present value of cash flow net of all costs) from investing in his alternative investment opportunity for one period. This contrasts with the angel, who insists only that the NPV from any investment he makes be positive. Since \( R^2 \) reflects the current level of scarcity of VC financing in the economy, it varies according to the extent of this scarcity: \( R^2 \) will be high when VC financing is very scarce, and low when VC financing is less scarce.\(^{18}\) The objective of the VC in making his effort choices, as well as

\(^{17}\) Our assumption that the venture capitalist’s effort is a continuous variable while the entrepreneur’s effort is a discrete variable is made only for simplicity. Fulghieri and Sevilir (2009) make a similar assumption.

\(^{18}\) Scarcity of VC financing is a natural assumption to make in our setting: while a large number of firms may be able to benefit from the value added by a VC’s effort, the supply of VCs capable of adding such value is likely to be much smaller. As Lerner (1998) has pointed out, the supply of venture capitalists is quite inelastic, since the effective oversight of young companies is a highly specialized skill that can only be developed with years of experience. Since the hallmark of venture capital financing is value addition, this means that venture capital firms cannot rapidly increase the supply of such financing by hiring new venture capitalists. Variations in the scarcity of venture capital financing may also be driven by variation in the flow of investment into venture funds, which in turn may be driven by prevailing economic conditions as well as variations in the risk aversion of venture fund investors and in the expected returns from alternative investment opportunities available to them.
his investment decision, is to maximize the expected value of his total cash flows net of his effort costs.

2.3 The angel
The angel is a pure supplier of capital; he cannot affect the probability of project success through his effort. Further, angel financing is abundant, so that the angel invests in all projects that yield him a positive (expected) NPV.

2.4 Information structure and contracting
Since only the entrepreneur and the inside financiers observe the time 1 state, publicly enforceable contracts cannot be written on these states. Thus, we assume that all contracting is done on time 2 cash flow realizations. However, we allow for the possibility that, after they observe the time 1 state, the entrepreneur and inside financier can renegotiate their original contract: in this case the entrepreneur makes a take-it-or-leave-it offer to the financier, and the financier can accept or reject this offer. The sequence of events is summarized in Figure 1, and the project payoff and information structure is depicted in Figure 2.

3. Equilibrium

Definition of equilibrium: The equilibrium concept we use here is that of Pareto dominant or efficient perfect Bayesian equilibrium (PBE).\(^\text{19}\) An equilibrium consists of: (i) the entrepreneur’s time 0 and time 1 financing choices (between angel and VC), the contracts offered to these financiers, and the amounts raised; (ii) the entrepreneur’s choice of effort in each period; (iii) the VC’s choice of effort in each period, if VC financing is chosen by the entrepreneur in that period, and (iv) the decision of the financier (VC or angel) to invest in the firm’s project or not. Each of the above choices must be such that: (a) the choices of each party maximize his objective, given the equilibrium beliefs and choices of others; (b) the belief of each party is consistent with the equilibrium choices of the others; further, along the equilibrium path, these beliefs are formed using Bayes’ rule. Any deviation from his equilibrium strategy by any party is met by beliefs by other parties that yield the deviating party a lower expected payoff compared with that obtained in equilibrium.

To facilitate exposition, we present the equilibrium in reverse order: we first discuss the equilibrium behavior of various parties at time 1 for a

\(^{19}\) Thus we look for perfect Bayesian equilibria that minimize the dissipative costs incurred due to asymmetric information by the higher-quality firm type (i.e., the type G firm at time 0 and state p firm at time 1). See Milgrom and Roberts (1986) for an application of efficient PBE to signaling games.
Figure 1
Sequence of events.

Figure 2
Project cash flow and information structure from an outside financier's point of view.
given financing choice at time 0, and then go on to discuss the overall equilibrium.

3.1 Later-round financing choices and contracts for a previously VC-financed firm

There are two kinds of projects at time 1: those in state $p$ and those in state $n$. If the firm is in state $p$, the VC’s effort will be productive in the firm. Further, since the VC was the firm’s time 0 financier, he has the same information at time 1 as the entrepreneur (given that both agents observe the realized state at time 1). For both of these reasons, it is beneficial for a firm in state $p$ to obtain another round of financing from the same VC who funded it at time 0. In this case, not only can the VC provide the requisite effort to add value to the firm, but it can also be ensured that the contract between the VC and the entrepreneur does not suffer from asymmetric information. This, in turn, means that the contract between the entrepreneur and the VC can provide the latter with stronger incentives to add value to the firm. If, on the other hand, the firm is in state $n$, then the VC’s effort is not productive (i.e., he cannot add value to the firm). Therefore, if the firm is in state $n$, the firm will not choose VC financing, since VC financing is more expensive than angel financing, and a firm in state $n$ cannot obtain any additional benefit from using VC financing.

We now turn to the optimal design of the financial contract between the entrepreneur whose firm is in state $p$, and a VC who is continuing to fund it at time 1. The objective of the contract design here is to ensure that both the VC and the entrepreneur put forth optimal effort. The entrepreneur designs the contract to maximize his objective, subject to: (i) the VC’s incentive compatibility (IC) constraint, which ensures that the VC puts forth the optimal amount of effort; (ii) the entrepreneur’s own incentive compatibility constraint, which ensures that the entrepreneur exerts the optimal amount of effort; (iii) the VC’s individual rationality (IR) constraint, which ensures that the VC obtains adequate compensation for the investment amount he provides to the firm and for his effort cost, and that the VC’s return is also larger than his opportunity cost of capital; (iv) the entrepreneur’s individual rationality constraint, which guarantees that the entrepreneur gets a non-negative expected payoff; (v) limited liability constraints; (vi) the firm’s budget constraint, which ensures that the firm is able to raise the required second-period financing $I - I_0$ from the VC. Let $(a^p_1, b^p_1)$ specify the contract offered by the entrepreneur to the VC, where $a^p_1$ is the share of the total cash flow of the project to the VC if $X$ is realized at time 2 (i.e., the project is highly successful) and $b^p_1$ is the VC’s share if $X$ is realized (i.e., the project is less successful). By limited liability, $0 \leq a^p_1 \leq 1$ and $0 \leq b^p_1 \leq 1$. Denote by $V_p$ the value of the VC’s time 0 financial contract.
at time 1 when the firm is in state \( p \); let \( V_n \) be the value when the firm is in state \( n \).

\[
V_p = a_0(q + f(\hat{c}_1))X + b_0(1 - q - f(\hat{c}_1))X.
\]

(1)

\[
V_n = a_0qX + b_0(1 - q)X.
\]

(2)

Let \((a_0, b_0)\) be the time 0 financial contract between the entrepreneur and the VC, defined similar to \((a_1^p, b_1^p)\). Recall that, at time 0, contracting is done on time 2 cash flow realizations, since the time 1 states cannot be contracted upon. Thus, a time 0 contract cannot distinguish between the case where the cash flow \( X \) is realized by a state \( p \) or a state \( n \) firm. In other words, the time 0 contract between the entrepreneur and the VC is a two-period contract on \( X \) and \( X \), which is renegotiable at time 1. For simplicity, we assume that renegotiation here takes the form of a buyout (or swap) at time 1, where the entrepreneur makes a take-it-or-leave-it offer to the time 0 financier. When the VC continues to fund the firm at time 1, the time 0 financial contract is swapped for a new contract at time 1 (in other words, the time 1 contract would compensate the VC for the value of the time 0 contract, in addition to compensating him for his second-period investment and effort, and ensuring that he receives at least his opportunity cost of capital from his second-period investment).

\( V_p \) is the value of the VC’s time 0 security in state \( p \) if the VC rejects the entrepreneur’s contract offer. In other words, since the VC knows that the firm is in state \( p \), the VC’s reservation value for his security is \( V_p \). As a result, the VC will get \( V_p \) for his time 0 security upon renegotiation. Similarly, if the state is \( n \), the VC will get \( V_n \) for his time 0 security upon renegotiation.

The problem of an entrepreneur having a firm in state \( p \) can therefore be characterized as:

\[
\max_{a_1^p, b_1^p, c_1} \left(1 - a_1^p \right) \left( q + f(c_1) \right) X + \left(1 - b_1^p \right) \left(1 - q - f(c_1) \right) X + p_1^p - (I - I_0) - k_1
\]

(3)

\[
s.t. c_1 \in \arg\max \{a_1^p \left( q + f(c_1) \right) X + b_1^p \left(1 - q - f(c_1) \right) X - c_1 \}.
\]

(4)

\[
(1 - a_1^p)qX + (1 - b_1^p)(1 - q)X \leq (1 - a_1^p)(q + f(c_1))X + (1 - b_1^p)(1 - q - f(c_1))X - k_1,
\]

(5)

\[
(1 - a_1^p)(q + f(c_1))X + (1 - b_1^p)(1 - q - f(c_1))X + p_1^p - (I - I_0) - k_1 \geq 0.
\]

(6)

The qualitative nature of our results does not depend on the specific form of bargaining adopted here. As will become clear later, as long as the time 0 financier can get a share of the increased firm value in state \( p \) and this share can be affected by the time 0 financial contract, our results go through.
\[ d_1^p (q + f(c_1))X + b_1^p (1 - q - f(c_1))X \geq f_1^p + \frac{R}{2} + c_1 + V_p, \]  
(7)

\[ 0 \leq d_1^p \leq 1, 0 \leq b_1^p \leq 1, \]  
(8)

\[ f_1^p \geq I - I_0. \]  
(9)

Here \( f_1^p \) is the amount raised by the entrepreneur from the VC at time 1. In the above, the constraint (4) is the VC’s incentive compatibility constraint. The constraint (5) is the entrepreneur’s incentive compatibility constraint (recall that \( k_1 \) is the entrepreneur’s cost of high effort), which ensures that the entrepreneur has an incentive to exert high effort. The constraint (6) is the entrepreneur’s individual rationality constraint, which ensures that he gets a non-negative payoff.\(^{21}\) The constraint (7) is the VC’s individual rationality constraint. The constraints (8) are the limited liability constraints. The constraint (9) is the firm’s budget constraint, ensuring that the amount raised from the VC at least covers the firm’s second-period investment requirement \((I - I_0)\).

It is clear that the VC’s individual rationality constraint is binding at the optimum (otherwise the entrepreneur increases \( f_1^p \) and improves his objective). Using this to simplify the entrepreneur’s problem, this is equivalent to maximizing \( f(c_1) X - c_1 - k_1 - \frac{R}{2} \) subject to the above constraints. If the VC’s effort \( c_1 \) were contractible, his (first-best) effort level \( c_{fb} \) would be given by the following first-order condition:

\[ f'(c_{fb}) = \frac{1}{\Delta X}, \]  
(10)

where \( \Delta X \equiv X - X \).

However, since the VC’s effort level is not contractible in practice, it is determined by his IC constraint, which yields the first-order condition:

\[ (d_1^p X - b_1^p X)f'(c_1) = 1. \]  
(11)

Rearranging Equation (11), we get:

\[ f'(c_1) = \frac{1}{(d_1^p X - b_1^p X)}. \]  
(12)

Equation (12) defines the VC’s effort level \( c_1 \) as a function of \( P_1^p \equiv d_1^p X - b_1^p X \). \( P_1^p \) can be thought of as measuring the “power” of the

\(^{21}\) We assume here that the entrepreneur’s IR constraint (6) is satisfied; we will verify later that this assumption is indeed satisfied. If this constraint is not satisfied, the entrepreneur will not use VC financing in any case.
contract between the VC and the entrepreneur. Given our assumptions about \( f(c_1) \), the VC’s effort level, \( c_1 \), is a strictly increasing function of \( P_{1}^p \).

The entrepreneur’s incentive compatibility condition (5) can then be simplified and rewritten as follows:

\[
f(c(P_{1}^p))(\Delta X - P_{1}^p) \geq k_1.
\]  

(13)

It immediately follows that \( P_{1}^p < \Delta X \), since otherwise condition (13) will never be satisfied. Therefore, the first-best effort from the VC, which we denote by \( c^{fb} \), can never be achieved here.\(^{22}\) The following proposition summarizes the solution to the above contract design problem.

**Proposition 1. (Later-round financing contracts)**

(i) The equilibrium financing contract between a firm in state \( p \) and the VC has the following features:

(a) The power of the optimal contract, \( P_{1}^{opt} \), is the maximum solution to condition (13) holding as an equality, so that \( \hat{c}_1 \), the VC’s equilibrium effort choice, is given by \( f'(\hat{c}_1)P_{1}^{opt} = 1 \).

(b) The equilibrium financing contract at time 1 is:

\[
a_{1}^{opt} = \frac{1}{X} \left[ I_{1}^{opt} + \frac{R}{2} + \hat{c}_1 + V_p + (1 - q - f(\hat{c}_1))P_{1}^{opt} \right],
\]

(14)

\[
b_{1}^{opt} = \frac{1}{X} \left[ I_{1}^{opt} + \frac{R}{2} + \hat{c}_1 + V_p - (q+f(\hat{c}_1)) \right].
\]

(15)

where \( I_{1}^{opt} \), the equilibrium amount of financing raised by the firm, satisfies \( I - I_0 \leq I_{1}^{opt} \leq X - [\frac{R}{2} + \hat{c}_1 + V_p] - P_{1}^{opt}(q + f(\hat{c}_1)) \).

(c) If

\[
\frac{P_{1}^{opt}}{I - I_0 + \frac{R}{2} + \hat{c}_1 + V_p - P_{1}^{opt}(q + f(\hat{c}_1))} < \frac{\Delta X}{X},
\]

(16)

then \( a_{1}^{opt} < b_{1}^{opt} \). In general, giving the VC equity alone (\( a_{1}^p = b_{1}^p \)) cannot implement the optimal outcome.

(d) In particular, if \( I - I_0 = X - [\frac{R}{2} + \hat{c}_1 + V_p] - P_{1}^{opt}(q + f(\hat{c}_1)) \), then \( b_{1}^{opt} = 1 \).

\(^{22}\) Some readers may wonder if we can solve the “moral hazard in teams” problem by using a large cash flow, \( C \), paid by the investor (VC or angel) to serve as a budget breaker (similar to Holmstrom 1982). We believe that such an approach will not serve to improve effort in a private equity (VC financing) setting. The budget breaker would work only if both the VC and the entrepreneur can commit to throw away \( C \) when \( X \) is realized. In other words, they have to commit to a Pareto-inefficient allocation. We find such a commitment implausible in practice, for two reasons. First, even if such a commitment is made initially, it is not a credible long-term commitment, since the entrepreneur and the VC can renegotiate between themselves to avoid throwing away \( C \) but instead split \( C \) between themselves ex post. Second, there is no evidence of startups contractually agreeing to intentionally destroy firm value in the real world.
(ii) A entrepreneur in state $n$ will use angel financing. Such an entrepreneur is indifferent to a variety of financing contracts to be given to the angel, as long as the value of the contract is $P_1^* + V_n$.

Clearly, in order to induce the VC to put forth optimal effort, his payoff when the firm is highly successful (cash flow $\bar{X}$) has to be greater than when the firm is less successful (cash flow $X$). This is ensured by setting the power of the contract $P_1^d = a_1^d \bar{X} - b_1^d X > 0$. The contract in (i) can be implemented by giving the VC convertible preferred equity (or equivalently preferred equity with warrants), convertible debt (or equivalently debt with warrants). In general, such a contract dominates a contract that involves giving the VC equity alone ($a_1^d = b_1^d$). Recall that the contracting here is between the entrepreneur and the VC undertaking a second round of financing, so that there is no asymmetric information between the two contracting parties (as will be the case in Section 3.2). This, in turn, allows the entrepreneur to provide very strong incentives to the VC, which will not be possible in the presence of asymmetric information between the contracting parties. The equilibrium financing contract ensures that the entrepreneur also has an incentive to exert high effort. The entrepreneur is the residual claimant here, receiving the cash flow left over after paying the VC.

We now come to the design of the financial contract between a firm in state $n$ and its financier. As discussed before, in this case, the entrepreneur chooses angel financing. Let $(a_1^n = b_1^n)$ be the contract given to the angel. The entrepreneur’s problem is now to maximize his objective function (17), subject to his own individual rationality constraint in condition (18), which ensures that he gets a non-negative payoff, and the angel’s individual rationality constraint in condition (19), which ensures that the angel is compensated for the investment amount he provides to the firm, as well as the amount he provides to the entrepreneur for buying out the VC who financed the firm at time 0. Similar to the case of a state $p$ firm, the contract design here also needs to satisfy the limited liability constraints in condition (20) and budget constraint in condition (21). The entrepreneur’s problem is thus:

$$\begin{align*}
\max_{a_1^n, b_1^n, P_1} & \quad (1 - a_1^n)qX + (1 - b_1^n)(1 - q)X + P_1^t - (I - I_0) \\
\text{s.t.} & \quad 0 \leq (1 - a_1^n)qX + (1 - b_1^n)(1 - q)X + P_1^t - (I - I_0), \\
& \quad a_1^n qX + b_1^n (1 - q)X \geq P_1 + V_n,
\end{align*}$$

It should be noted that while we focus on ordinary convertibles (since they are the most commonly used), such contracts can also be implemented using Participating Convertible Preferred (PCP), a relatively new financing instrument (see Bartlett (1995) for institutional details of this relatively new security).
\[ 0 \leq a_i^0 \leq 1, \quad 0 \leq b_i^0 \leq 1, \quad (20) \]

\[ P_i^0 \geq I - I_0. \quad (21) \]

The solution to this contract design problem is summarized in part (ii) of Proposition 1. In this case, the angel cannot add any value through his effort so that, unlike in the case of VC financing, there is no incentive compatibility constraint to be satisfied here. This also means that the form of the contract is irrelevant here as long as the angel is compensated for the amount he provides to the firm. Thus, the entrepreneur is indifferent between providing the angel equity, convertible preferred equity, or convertible debt. Further, the entrepreneur exerts only low effort in equilibrium in this case. In equilibrium, the angel buys out the financing contract of the VC who financed the firm at time 0 at its time 1 full-information value, \( V_n \), so that the total financing raised by the entrepreneur from the angel will be \( P_{1*} + V_n \).

We assume that \( f(\hat{c}_1)\Delta X - k_1 - \hat{c}_1 - \frac{R_2}{2} > 0 \). The assumption translates into the VC being able to add positive value to the firm: the first term above is the marginal value created by the VC and the entrepreneur, the next two terms are the entrepreneur and the VC’s effort costs, and the last term, \( \frac{R_2}{2} \), is the incremental cost of VC financing (in equilibrium) over angel financing. Note that an \( n \)-state firm will not mimic a \( p \)-state firm, which finds it optimal to get time 1 funding from the VC that financed it at time 0. It is not optimal for an \( n \)-state firm to mimic a \( p \)-state firm and attain VC financing, since the VC’s effort is not productive in such a firm. Furthermore, the firm cannot raise any funding from a new VC, either, because such action would reveal itself to be an \( n \)-state firm. Thus, in equilibrium, a firm in state \( n \) will raise time 1 financing from an angel. These insights are embodied in Proposition 2.\(^{24}\)

**Proposition 2. (Later-round financing choice between VCs and angels)** The equilibrium actions of the entrepreneur, the VC and the angel at time 1 can be characterized as follows:

(i) At time 1, a firm in state \( p \) will continue to use VC financing, with the contract specified in Proposition 1.

(ii) A firm in state \( n \) will use angel financing.

(iii) The VC will not continue to provide funding to any firm in state \( n \), but will instead leave the firm, selling his stake to an angel.

\[^{24}\] The out-of-equilibrium belief supporting this equilibrium is that, if a firm is seeking financing at time 1 from a new financier rather than obtaining additional financing from the current VC, then the outsiders infer that it is in state \( n \).
3.2 Later-round financing choices and contracts for a previously angel-financed firm

We now discuss the time 1 financing choice of a firm whose time 0 financing was undertaken by an angel. As in Section 3.1, there are two kinds of projects at time 1: those in state \( p \) and those in state \( n \). Further, in this case, it is also beneficial for a firm in state \( p \) to seek VC financing at time 1, since the VC can add value to the firm through his effort. Here, however, such VC financing is undertaken under asymmetric information: since the time 0 financier was an angel, any VC would be new to the firm and would therefore not have observed the realization of the firm’s state at time 1. As in Section 3.1, the VC’s effort is not productive if the firm is in state \( n \). However, in contrast to the case discussed there, a previously angel-financed firm in state \( n \) has an incentive to mimic a firm in state \( p \), since doing so allows the firm to sell overvalued securities to the VC who provides funding to the firm at time 1. Thus, an equilibrium that involves separation between state \( p \) and state \( n \) firms has to satisfy the incentive compatibility conditions that ensure that a previously angel-financed firm in state \( n \) will not find it profitable to mimic a firm in state \( p \) and vice versa. First, we present the incentive compatibility condition of an entrepreneur having a firm in state \( n \). This is given by:

\[
(1 - a_1^q)q\bar{X} + (1 - b_1^q)(1 - q)\bar{X} + I_1^p - (I - I_0) \geq (1 - a_1^p) \times \quad \quad (22)
\]

The right-hand side of condition (22) gives the state \( n \) entrepreneur’s payoff if he mimics a state \( p \) entrepreneur by obtaining VC financing. The sum of the first two terms on the right-hand side gives the residual cash flow to the entrepreneur. The sum of the next two terms is the money raised by the entrepreneur, \( V_p + I_1^p \). The sum of the last two terms is the cost incurred by the entrepreneur: he has to pay \( V_n \) to buy out the angel and invest \( I - I_0 \). Similar to Section 3.1, here \( V_n \) and \( V_p \) are the reservation values of the angel’s time 0 security in states \( n \) and \( p \), respectively (which are the equilibrium values of the security if the project is in states \( n \) and \( p \), respectively).

\[
V_p = a_0(1 + f(c_1^p))\bar{X} + b_0(1 - q - f(c_1^p))\bar{X}. \quad \quad (23)
\]

\[
V_n = a_0 q\bar{X} + b_0(1 - q)\bar{X}. \quad \quad (24)
\]

where \( c_1^p \) is the VC’s equilibrium effort if the project is in state \( p \). The left-hand side of condition (22) gives the payoff to an entrepreneur if his firm

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is in state $n$, but he does not mimic a firm in state $p$. In this case, the firm will reveal itself to be a state $n$ firm and thus obtain only angel financing. The sum of the first two terms is the residual cash flow to the entrepreneur, $F_1^p$ is the money raised by the entrepreneur, and $I - I_0$ is the firm’s investment requirement. In sum, the entrepreneur’s incentive compatibility constraint (22) ensures that he is better off revealing that his firm is in state $n$ rather than mimicking a state $p$ entrepreneur. The reason why this condition may hold is because, while an $n$-state firm can try to benefit from mimicking a $p$-state firm by attempting to sell overvalued securities to a VC, the cost of capital charged by a VC is greater than that charged by an angel, since VC financing is scarce. Given the fact that the VC’s effort is not productive in such a firm, if VC financing is expensive enough, a state $n$ firm will be willing to reveal itself.

We now consider a firm in state $p$. The incentive compatibility constraint of this firm’s entrepreneur is given by condition (25). If the firm deviates by seeking angel (rather than VC) financing, the angel would believe that this firm is in state $n$. The entrepreneur’s payoff is now given by the right-hand side of condition (25). If, however, it does not deviate, it will reveal itself as a $p$-state firm, and is thus able to obtain VC financing, yielding the entrepreneur a payoff given by the left-hand side of condition (25).

\[
(1 - d_1^n)(q + f(c_1))X + (1 - b_1^n)(1 - q - f(c_1))X + F_1^p - (I - I_0) - k_1 \\
\geq (1 - d_1^n)qX + (1 - b_1^n)(1 - q)X + F_1^p - (I - I_0).
\]

As before, we assume here that $f(c_1^n)\Delta X - k_1 - c_1^n - \frac{\theta}{2} > 0$, that is, the VC is able to add positive value to the firm in equilibrium. We will show in the Appendix that this assumption implies that the above incentive compatibility condition for the $p$-state firm holds. The reason is very intuitive: if a VC can create additional value for the firm compared with an angel, and the entrepreneur can share in this value creation, it is natural for him to prefer VC financing over angel financing. If, in addition, condition (22) also holds, we have the following separating equilibrium.

**Proposition 3. (Later-round financing choice between VCs and angels)**

(i) A firm in state $p$ at time 1 will obtain its second-round financing from a VC (with the contract specified in Proposition 4) and buy out its time 0 financier, the angel.

(ii) A firm in state $n$ at time 1 will continue to use angel financing in the second round.

We now turn to the optimal design of the financing contract between the entrepreneur and the VC (in the case of a $p$-state firm) and the entrepreneur and the angel (in the case of an $n$-state firm) in the above
separating equilibrium. We first consider the contract between the \( p \)-state entrepreneur and the VC.

The entrepreneur designs the contract to maximize his objective function (26) subject to: (i) the VC’s incentive compatibility constraint in condition (27), which ensures that he puts forth optimal effort; (ii) the entrepreneur’s own incentive compatibility constraints in condition (28), which ensures that he also puts forth optimal effort; (iii) the entrepreneur’s individual rationality constraint in condition (29), which ensures that he gets a non-negative payoff; (iv) the VC’s individual rationality constraint in condition (30), which ensures that the VC obtains adequate compensation for the investment amount he provides to the firm and for his effort cost, and the VC’s return is also larger than his opportunity cost of capital; (v) the firm’s and the VC’s limited liability constraints; (vi) the firm’s budget constraint ensuring that the amount raised from the VC at least covers the firm’s second-period investment requirement \( (I - I_0) \) and; (vii) the incentive compatibility or “truth-telling” conditions, condition (22) and condition (25), ensuring that \( n \)-state and \( p \)-state firms respectively are better off revealing their true type rather than mimicking the other type. Thus the entrepreneur’s contract design problem is given by:

\[
\max_{a_1^p, b_1^p, I_1^p} (1 - a_1^p)(q + f(c_1))X + (1 - b_1^p)(1 - q - f(c_1))X + I_1^p - (I - I_0) - k_1 \quad (26)
\]

subject to:

\[
s.t. \quad c_1 \in \arg\max \{a_1^p(q + f(c_1))X + b_1^p(1 - q - f(c_1))X - c_1\}, \quad (27)
\]

\[
(1 - a_1^p)qX + (1 - b_1^p)(1 - q)X \leq (1 - a_1^p)(q + f(c_1))X + (1 - b_1^p)(1 - q - f(c_1))X - k_1, \quad (28)
\]

\[
(1 - a_1^p)(q + f(c_1))X + (1 - b_1^p)(1 - q - f(c_1))X + I_1^p - (I - I_0) - k_1 \geq 0, \quad (29)
\]

\[
a_1^p(q + f(c_1))X + b_1^p(1 - q - f(c_1))X \geq I_1^p + \frac{R}{2} + c_1 + V_p, \quad (30)
\]

(22), (25), (8), and (9).

Recall that, unlike in Section 3.1, the contracting here is done under asymmetric information between the VC and the entrepreneur, so that we need to impose the incentive compatibility conditions (22) and (25), in addition to the constraints (i)–(vi) imposed on the entrepreneur’s maximization problem in Section 3.1. In summary, the objective of the contract design here is to ensure that both the VC and the entrepreneur put forth optimal effort, while ensuring separation between \( n \)-state and \( p \)-state firms.
Since the VC’s IR condition (30) is binding, the n-state entrepreneur’s incentive compatibility constraint (22) can be simplified to:\(^{26}\)

\[
\frac{R}{2} + c_1 - P_{1}^{\text{p}} f(c_1(P_{1}^{\text{p}})) \geq 0.
\] (31)

It should be clear from the VC’s first-order condition (12) that the VC’s effort level \(c_1\) is a function only of \(P_{1}^{\text{p}}\). The first two terms of the left-hand side of condition (31) represent the cost of having VC financing for the n-state firm. The last term is the n-state firm’s benefit from selling over-priced securities to the VC. Thus condition (31) says that if the cost of having VC financing is greater than the benefit, then the n-state firm will not mimic a p-state firm by seeking VC financing. We now summarize the solution to the above contract design problem in the following proposition.

**Proposition 4. (Later-round financing contracts)**

(i) The equilibrium financing contract between a firm in state \(p\) and the VC has the following features:

(a) The power of the optimal contract \(P_{1}^{\text{p}}\) is the maximum solution to condition (31) holding as an equality. \(c_1^{\text{p}}\), the VC’s equilibrium effort choice, is such that \(f'(c_1^{\text{p}})P_{1}^{\text{p}} = 1\).

(b) The financing contract at time 1 is given by:

\[
a_{1}^{\text{p}} = \frac{1}{X} [P_{1}^{\text{p}} + V_p + (1 - q)P_{1}^{\text{p}}],
\] (32)

\[
b_{1}^{\text{p}} = \frac{1}{X} [P_{1}^{\text{p}} + V_p - qP_{1}^{\text{p}}].
\] (33)

\(I_{1}^{\text{p}}\), the equilibrium investment amount raised by the firm, satisfies

\[I - I_0 \leq I_{1}^{\text{p}} \leq X - V_p + P_{1}^{\text{p}} q\]

(c) If

\[
\frac{P_{1}^{\text{p}}}{I - I_0 + V_p - P_{1}^{\text{p}} q} < \frac{\Delta X}{X},
\] (34)

then, \(a_{1}^{\text{p}} < b_{1}^{\text{p}}\).

(d) In particular, if \(I - I_0 = X - V_p + P_{1}^{\text{p}} q\), then \(b_{1}^{\text{p}} = 1\).

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\(^{26}\) It can be shown that, under certain parameter restrictions, the VC’s IR is binding at the optimum. In the absence of such parameter restrictions, the p-state firm may be able to overpay the VC to signal its type, so that the VC’s IR may not be binding at the optimum. However, allowing the state p firm to signal through “money burning” does not change our basic results here. Interested readers can contact the authors for details.
(e) Compared with the case where the firm is previously VC financed, the VC exerts a lower level of effort in the second-period when the firm is previously angel-financed.

(ii) A firm in state $n$ will continue to use angel financing. An entrepreneur in state $n$ is indifferent to a variety of financing contracts to be given to the angel, as long as the value of the contract is $P^*_n + V_n$.

Notice that the above equilibrium contract provides weaker incentives for the VC to exert effort compared with the case of a previously VC-financed firm (in Section 3.1), where the contracting proceeds under symmetric information between the entrepreneur and the VC. Asymmetric information prevents the provision of stronger incentives to the VC. This is because the greater the incentive given to the VC to exert effort, the greater also is the incentive of the $n$-state firm to mimic the $p$-state firm. This arises from the fact that the benefit to the $n$-state firm from mimicking the $p$-state firm is given by $(a_1^{p*} - b_1^{p*} X) f(c_1^*)$, which is an increasing function of $(a_1^{p*} - b_1^{p*} X)$. This, in turn, means that the power of the contract given to the VC in equilibrium, $P_1^{p*}$, has to be smaller than that in the case of a previously VC-financed firm, thus leading the VC to underinvest in effort here. The above contract can be implemented by giving the VC convertible preferred equity (or equivalently, preferred equity with warrants), convertible debt (or equivalently, debt with warrants), or equity with warrants (equity with warrants can be used, however, only when $a_1^{p*} > b_1^{p*}$). However, the value of the “upside” to the VC will be smaller here compared with the case of a previously VC-financed firm. Controlling for the total value of the securities issued to the VC, this implies that the value of the conversion option (or warrant value when preferred equity or debt with warrants is issued) will be lower than that in the case of a previously VC-financed firm. As in that case, the equilibrium financing contract here also ensures that the entrepreneur is motivated to exert high effort in equilibrium. The design of the financing contract between an entrepreneur in state $n$ and the angel is similar to that in Section 3.1. We will therefore not discuss that contract design in detail here: the solution to that design problem is summarized by part (ii) of Proposition 4.

4. Early-Round Financing and the Overall Equilibrium

We now describe the overall equilibrium of the model. While our primary focus here will be on characterizing the early-round financing choices made by firms and the nature of early-round financing contracts between the firm and its financiers, we will also discuss how these early-round financing choices affect the later-round financing choices described earlier, thus characterizing the overall equilibrium. Early-round financing
choices and contracts depend on the extent of scarcity of VC financing. One can think of three scarcity regimes: First, consider the case where VC financing is very scarce (“high scarcity”). In this case, the threshold NPV, $R_2^h$, required by the VC to invest in projects will be relatively high. At the other extreme, where the VC financing is freely available (“low scarcity”), the threshold NPV, $R_2^l$, required by the VC will be relatively low. Finally, when VC financing is moderately scarce, the threshold NPV, $R_2^m$, will lie between the above two extremes.

At time 0, the VC suffers from asymmetric information, in that he cannot a priori distinguish between type G and type B firms. Recall that the only difference between the two types of firms at time 0 is that the VC can add value through his effort to a type G firm, but cannot do so in the first period for a type B firm. Since both types of entrepreneurs benefit from having the VC finance their firms if it ends up in state $p$ at time 1 (since the VC can add more value in that case through his effort in the second-period), the VC is a more desirable financier than the angel for both types of firms. On the other hand, VC financing is more expensive than angel financing for both types of firms at time 0. Further, since a type B firm cannot benefit from the VC’s effort in the first period, it is less advantageous for a type B firm to use VC financing than for a type G firm. Given this, we will show below that the nature of earlier-stage projects financed by the VC in equilibrium will differ across different-scarcity regimes of VC financing. In particular, we will show that in the high-scarcity regime, VCs will fund only later-stage (time 1) projects in state $p$, leaving all earlier-stage (time 0) projects, as well as later-stage projects in state $n$ to the angel in equilibrium. In the moderate-scarcity regime, VCs will fund only type G earlier-stage projects and later-stage projects in state $p$; they will leave type B earlier-stage projects, as well as later-stage projects in state $n$, to the angel. Finally, in the low-scarcity regime, the VCs will fund all earlier-stage projects (both type G and type B) and later-stage projects in state $p$, leaving later-stage projects in state $n$ to the angel. In the next section, we will formally define the three regimes of VC-financing scarcity in detail, and develop some intuition regarding the relationship between the scarcity regimes of VC financing and the nature of the firms seeking VC (rather than angel) financing. We will then go on to characterize the equilibrium under the moderate-, low-, and high-scarcity regimes in Section 4.2, 4.3, and 4.4, respectively.

4.1 Relationship between the scarcity of VC financing and the nature of firms seeking VC financing

As mentioned above, a VC can add value even to a type B firm because if the firm ends up in state $p$ at time 1, the VC can create value more efficiently (since contracting and therefore value creation can proceed under symmetric information) if the VC has already involved with
the firm at time 0. Formally, the expected benefit is \( \lambda \delta \), where 
\[ \delta \equiv (f(\hat{c}_1)\Delta X - \hat{c}_1) - (f(c^*_1)\Delta X - c^*_1). \]
A type G firm would benefit more from VC financing because the VC can add value to the firm by increasing the probability of a state \( p \) being realized. As a result, a type G firm’s benefit from VC financing is 
\[ f(c^*_0)(f(\hat{c}_1)\Delta X - \hat{c}_1 - R/2 - k_1) - c^*_0 - k_0 + \lambda \delta, \]
where \( c^*_0 \) is the VC’s equilibrium effort. The first term is the value added by the VC: 
\[ f(c^*_0) \] is the incremental probability of state \( p \) as a result of the VC’s and entrepreneur’s effort. The terms in the parentheses give the incremental value of a \( p \)-state firm over an \( n \)-state firm. \( c^*_0 \) and \( k_0 \) are the VC’s and entrepreneur’s effort costs, respectively. Finally, a firm in state \( p \) can benefit the most from VC financing—that is, a \( p \)-state firm’s benefit from VC financing, 
\[ f(c^*_1)\Delta X - c^*_1 - k_1, \]
is the largest. Now we define the “low-scarcity regime” of VC financing as one where the opportunity cost to VC of financing a firm, \( \frac{R}{2} \), is low enough that it satisfies the following condition:

\[
0 < \frac{R}{2} \leq \lambda \delta \leq f(c^*_0)(f(\hat{c}_1)\Delta X - \hat{c}_1 - \frac{R}{2} - k_1) - c^*_0 - k_0 + \lambda \delta \\
\leq f(c^*_1)\Delta X - c^*_1 - k_1. \tag{35}
\]

Condition (35) implies that under a low-VC-financing scarcity regime, both the type G and type B firms will have a positive net benefit from using VC financing at time 0; further, at time 1, a firm in state \( p \) will also have a positive net benefit from using VC financing, while a firm in state \( n \) will use only angel financing because it cannot benefit from VC financing.

At the other extreme, we define the “high-scarcity regime” of VC financing as one where the opportunity cost to VC of financing as high that only firms with the greatest benefit from VC financing will have an incentive to do so, with other firms using angel financing. From condition (36), we can see that in a high-scarcity regime, the cost of VC financing, \( \frac{R}{2} \), is higher than the benefit to a type G firm from VC financing at time 0, but lower than the benefit to a state \( p \) firm from VC financing at time 1. Thus, in the high-scarcity regime, only later-stage (time 1) projects in state \( p \) use VC financing in equilibrium; no early-stage projects are funded by the VC. Both types of early-stage projects will now be financed by the angel, thus making the equilibrium at time 0 a pooling one.

\[
\lambda \delta \leq f(c^*_0)(f(\hat{c}_1)\Delta X - \hat{c}_1 - \frac{R}{2} - k_1) - c^*_0 - k_0 + \lambda \delta \leq \frac{R}{2} \\
\leq f(c^*_1)\Delta X - c^*_1 - k_1. \tag{36}
\]

Intuitively, one can think of the high-scarcity regime of VC financing as one where the cost of VC financing is so high that only firms with the greatest benefit from VC financing will have an incentive to do so, with other firms using angel financing. From condition (36), we can see that in a high-scarcity regime, the cost of VC financing, \( \frac{R}{2} \), is higher than the benefit to a type G firm from VC financing at time 0, but lower than the benefit to a state \( p \) firm from VC financing at time 1. Thus, in the high-scarcity regime, only later-stage (time 1) projects in state \( p \) use VC financing in equilibrium; no early-stage projects are funded by the VC. Both types of early-stage projects will now be financed by the angel, thus making the equilibrium at time 0 a pooling one.
We define the moderate-scarcity regime as one where the scarcity of VC financing is between the above two extremes, so that the opportunity cost of VC financing, \( \frac{R}{2} \), satisfies the following condition:

\[
\lambda \delta \leq \frac{R}{2} \leq f(c_0^{G*})(f(\hat{c}_1)\Delta X - \hat{c}_1 - \frac{R}{2} - k_1) - c_0^{G*} - k_0 + \lambda \delta
\]

\[
\leq f(c_{1}^{*})\Delta X - c_{1}^{*} - k_1.
\]  

(37)

c_0^{G*} in condition (36) above and in condition (37) is the equilibrium level of VC effort in a type G firm in a moderate-scarcity regime. Intuitively, the VC’s opportunity cost of capital in a moderate-scarcity regime is low enough that a type G early-stage firm and a later-stage firm in state \( p \) have an incentive to use VC financing; however, it is high enough that an early-stage type B firm will not have an incentive to use VC financing and will instead use angel financing (we will show this to be the case in equilibrium in section 5.2). Notice that regardless of the scarcity regime, we are imposing the parameter restriction that \( 0 < \frac{R}{2} < f(c_{1}^{*})\Delta X - c_{1}^{*} - k_1 \). This ensures that the time 1 equilibrium is not affected by the scarcity regime: while firms in state \( p \) will use VC financing at time 1, those in state \( n \) will use angel financing.\(^{27}\)

We now formally characterize the equilibrium in the three regimes of VC financing defined above. Since we believe that the moderate-scarcity regime is the most interesting and the one most likely to prevail in practice, we will first analyze the equilibrium in the moderate-scarcity regime in detail, and then briefly discuss the equilibria in the low-scarcity and the high-scarcity regimes.

4.2 Early-round financing choices and contracts under moderate scarcity of VC financing

In a moderate-scarcity regime, a separating equilibrium will exist at both time 0 and time 1. In such an equilibrium, a type G firm chooses VC financing and a type B firm chooses angel financing at time 0. At time 1, a state \( p \) firm uses VC financing, while a state \( n \) firm uses angel financing, as characterized in Sections 3.1 and 3.2.

Proposition 5. (Early-round financing choice between VCs and Angels under moderate scarcity)

At time 0:

(i) The type G entrepreneur seeks (and receives) VC financing;

(ii) The type B entrepreneur seeks (and receives) angel financing.

\(^{27}\) The condition that \( \frac{R}{2} > 0 \) follows automatically from the fact that venture capital financing is scarcer than angel financing, as discussed earlier. The last inequality in (37), \( \frac{R}{2} < f(c_{1}^{*})\Delta X - c_{1}^{*} - k_1 \), ensures that the incremental value created by the VC over and above his opportunity cost by funding a firm in state \( p \) is positive. Clearly, if this condition is violated, no VC would choose to start financing a firm at time 1, which is an uninteresting range of values of the VC’s opportunity cost.
A separating equilibrium of the above nature will exist if the benefit of VC financing at time 0 exceeds its cost for a type G firm but not for a type B, so that the former chooses VC financing, while the latter chooses angel financing. To see this, we first consider the type B firm’s incentive to choose VC financing and thus mimic a type G entrepreneur. There are two benefits from mimicking a type G firm: first, if at time 1 the state of the project turns out to be p, the later-round VC financing will be more efficient because there is no asymmetric information. Second, the type B entrepreneur can get cheaper financing because he can sell overpriced securities to the VC, who would mistake him for a type G. The cost, however, is that a VC demands a higher return than an angel because of the VC’s effort cost and the opportunity cost, \( \frac{R}{2} \). So if the cost of VC financing is high enough, a type B entrepreneur would not find it optimal to choose VC financing over angel financing. On the other hand, a type G entrepreneur faces a similar tradeoff. But as argued before, a type G entrepreneur can benefit more from VC financing because the VC can add value to the project directly. So if the cost of VC financing is higher than its benefit to a type B but lower than its benefit to a type G, we will have a separating equilibrium. The incentive compatibility constraints for type B and type G firms are detailed in the Appendix.

We now discuss the design of the time 0 financial contract between the entrepreneur and the VC under moderate scarcity of VC financing. The objective of contract design by the type G entrepreneur at time 0 is to maximize the payoff while ensuring separation between the two types of firms, subject to the VC’s individual rationality condition. The entrepreneur designs the contract to maximize his expected payoff subject to: (i) the VC’s incentive compatibility constraint, ensuring that the VC puts forth the optimal level of effort; (ii) the entrepreneur’s own incentive compatibility constraint, which ensures that he exerts high effort; (iii) the VC’s individual rationality constraint, which ensures that the VC obtains adequate compensation for the investment amount he provides to the firm and his effort cost, and also ensures that the VC’s return is larger than his opportunity cost of capital; (iv) the firm’s and the VC’s limited liability constraints; (v) the firm’s budget constraint, which ensures that the firm can raise at least the required amount of money for its investment; (vi) the incentive compatibility conditions, which ensure that the type G and the type B firms are better off revealing their true type rather than mimicking the other type; and (vii) the entrepreneur’s own

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28 Notice that while we do not allow for contingent contracts based on time 1 states, \( p \) and \( n \) respectively, this is without loss of generality in our setting. This is because in a contract design problem with contingent claims, what matters are the equilibrium values of \( V_{G}^{p} \) and \( V_{G}^{n} \). But for each pair of \( (V_{G}^{p}, V_{G}^{n}) \), we can always find a contract in our setting, \( (a_{G}^{p}, b_{G}^{n}) \), that implements them.
individual rationality constraint, which ensures that his payoff is non-negative. The details of this formal contracting problem are given in the Appendix.

We assume that the type B and the type G entrepreneur’s individual rationality constraints, the limited liability, and budget constraints are not binding at the optimum at time 0. Further, under the equilibrium time 0 contracts, the entrepreneur’s individual rationality, limited liability, and budget constraints are also assumed to be not binding at time 1. The parameter restrictions (A-17) given in the Appendix guarantee this to be the case. The following proposition summarizes the solution to the entrepreneur’s contract design problem under the above assumptions.

**Proposition 6. (Early-round financing contracts under moderate scarcity)**

(i) The equilibrium financing contract between a type G entrepreneur and the VC has the following features:

(a) The power of the optimal contract, $P^*_0$, is the maximum solution to condition (A-15) holding as an equality. Further, $c^*_G$, the VC’s equilibrium effort choice, is such that $f'(c^*_G) (f(c_1)) P^*_0 = 1$.

(b) The equilibrium financing contract at time 0 is given by:

$$
a^*_0 = \frac{1}{X} [I^*_0 + \lambda \delta + P^*_0 (1 - \lambda f(c_1) - q)], \quad (38)$$

$$
b^*_0 = \frac{1}{X} [I^*_0 + \lambda \delta - P^*_0 (\lambda f(c_1) + q)]. \quad (39)$$

$I^*_0$, the equilibrium external financing raised by the firm at time 0, satisfies: $I_0 \leq I^*_0 \leq X + P^*_0 (\lambda f(c_1) + q) - \lambda \delta$.

(c) In particular, if $I_0 = X + P^*_0 (\lambda f(c_1) + q) - \lambda \delta$, then $b^*_0 = 1$.

(d) If

$$
\frac{P^*_0}{I_0 + \lambda \delta - P^*_0 (\lambda f(c_1) + q)} < \frac{\Delta X}{X}, \quad (40)
$$

then $a^*_0 < b^*_0$.

(e) At this earlier stage, the VC exerts a lower level of effort than at the later stage— that is, $c^*_G < c^*_1 < \hat{c}_1$.

(ii) A type B firm will use angel financing. It is indifferent between a variety of financing contracts to be given to the angel, as long as the value of the contract is $I^*_0$, and the conditions $I_0 \leq I^*_0 \leq X + \lambda f(c_1) P^*_0$ and $f(c_1) (\Delta X - P^*_0) - c^*_1 - \frac{R}{2} - k_1 \geq 0$ hold.
The time 1 equilibrium choice of financing, the equilibrium financing contract, and the VC’s choice of effort are as specified in Section 3.1 and 3.2 respectively, depending upon whether the time 0 financing is done by the VC or the angel. The entrepreneur will choose to exert the appropriate level of effort at time 1, depending on the state the firm is in.

The equilibrium VC-financing contract can be implemented by giving the VC convertible debt (or equivalently, debt with warrants), convertible preferred equity (or equivalently, preferred equity with warrants), or equity with warrants. Under the equilibrium contract, the VC exerts a lower level of effort than in later-stage financing (regardless of whether the existing VC is financing a second round or a new VC is entering the firm at time 1). This is because, in earlier-stage contracting, the extent of asymmetric information between the entrepreneur and the VC is higher.

In other words, the benefit to a type B entrepreneur from mimicking a type G entrepreneur is higher than the benefit to a state $n$ entrepreneur from mimicking a state $p$ entrepreneur, since there is an incremental benefit from getting VC financing at an earlier stage (namely the entrepreneur can get more-efficient VC financing at a later stage). In summary, the earlier stage contract between the VC and the entrepreneur has less power than a later-stage contract, since the greater severity of asymmetric information at time 0 precludes the contract from providing stronger incentives to the VC. As a consequence, the VC exerts a lower equilibrium level of effort in the first period (early stage).

4.3 Early-round financing choices and contracts under low scarcity of VC financing

In the low-scarcity regime, the equilibrium is a pooling one at time 0. In equilibrium, both the type G and the type B firms choose VC financing at time 0. At time 1, a state $p$ firm uses VC financing while a state $n$ firm uses angel financing, as characterized in Sections 3.1 and 3.2. We summarize the equilibrium financing contract in the low-scarcity regime in the following proposition.

**Proposition 7. (Early-round financing contracts under low scarcity)**

(i) At time 0, both types of firms seek (and receive) VC financing:

(a) $c_0^*$, the VC’s equilibrium effort choice in both type of firms, is such that $f'(c_0^*)f(c_1^*)P_0^* = 1$. The optimal $P_0^*$ is given in the Appendix.

(b) The financing contract at time 0 is given by:

$$ a_0^* = \frac{1}{X} [I_0^* + \frac{R}{2} + c_0^* + P_0^*(1 - (\lambda + \theta f(c_0^*))f(c_1^*) - q)]. $$

(41)
\[ b_0^* = \frac{1}{X} [I_0^* + R \frac{c_0^*}{2} - (\lambda + \theta f(c_0^*)) f(\hat{c}_1) P_0^* - P_0^* q]. \] (42)

\( I_0^* \), the equilibrium external financing raised by the firm, satisfies \( I_0^* \leq I_0 \leq X + P_0^* (\lambda + \theta f(c_0^*)) f(\hat{c}_1) + q - \frac{R}{2} - c_0^* \).

(c) If

\[ \frac{P_0^*}{L_0 + R \frac{c_0^*}{2} - (\lambda + \theta f(c_0^*)) f(\hat{c}_1) P_0^* - P_0^* q} < \frac{\Delta X}{X}, \] (43)

then \( a_0^* < b_0^* \).

(d) In particular, if \( L_0 = X + P_0^* (\lambda + \theta f(c_0^*)) f(\hat{c}_1) + q - \frac{R}{2} - c_0^* \), then \( b_0^* = 1 \).

(e) At this stage, the VC exerts a lower level of effort than at the later stage—that is, \( c_0^* < \hat{c}_1 \).

(ii) The time 1 equilibrium choice of financing, the equilibrium financing contract, and the VC’s choice of effort are as specified in Section 3.1. The entrepreneur will always choose to exert an appropriate level of effort at time 1, depending on the state the firm is in.

We now briefly discuss the contract design in the low-scarcity regime. The contract design here is similar to that in the moderate-scarcity regime—that is, the objective of contract design is to maximize the type G entrepreneur’s payoff. Since the VC finances both the type G and the type B firms at time 0, this optimization problem is now constrained by his individual rationality constraint, which is different from that in the moderate-scarcity regime, and is given by condition (A-21) in the Appendix. Analogous to condition (A-17) in the other two regimes, the parametric restrictions in condition (A-25) in the Appendix ensures that the contract offered at time 0 by the entrepreneur to the VC is such that the entrepreneur’s individual rationality constraints, limited liability constraints, and the firm’s budget constraints at time 0 and time 1 are satisfied.

Note that in the low-scarcity regime, VCs fund a greater proportion of projects compared with the proportion they fund in the other two regimes, though the average quality of their projects will be lower than in the other regimes. The equilibrium VC-financing contract can be implemented by giving the VC convertible debt (or equivalently, debt with warrants), convertible preferred equity (or equivalently, preferred equity with warrants), or equity with warrants. Under the equilibrium contract, the VC exerts a lower level of effort than in later-stage financing. This is because some of the projects financed by the VC in this regime cannot benefit from the VC’s effort.
4.4 Early-round financing choices and contracts under high scarcity of VC financing

In the high-scarcity regime, the cost of VC financing, $R_2$, is so high that both the type G and the type B firm find it optimal to choose angel financing. That is, the equilibrium at time 0 is now a pooling one. At time 1, a state $p$ firm uses VC financing, while a state $n$ firm uses angel financing, as characterized in Section 3.2. The following proposition characterizes the equilibrium early-stage financing choices and contracts under high scarcity of VC financing.

**Proposition 8. (Early-round financing choices and contracts under high scarcity)**

(i) At time 0, both the type G and the type B entrepreneurs seek (and receive) angel financing.

(ii) An entrepreneur is indifferent between a variety of financing contracts to be given to the angel, as long as the value of the contract is $I_0/C_3$, and the conditions $I_0/C_3 \leq X + \lambda f(c_1^+) P_0^* \quad \text{and} \quad f(c_1^+)(\Delta X - P_0^*) - c_1^+ - \frac{R_2}{2} - k_1 \geq 0$ hold.

(iii) The time 1 equilibrium choice of financing, the equilibrium financing contract, and the VC’s choice of effort are as specified in Section 3.2. The entrepreneur will always choose to exert an appropriate level of effort at time 1, depending on the state the firm is in.

Because in equilibrium the VC does not finance any firm at time 0, both the type G firm and the type B firm have the same probability of being in state $p$ at time 1. The actual form of angel financing contract does not matter, since the angel faces a pool of firms and does not exert effort. Notice that in the high-scarcity regime, the VC chooses to finance only later-stage projects, since his threshold NPV, $R_2$, is higher in this case. By financing only later-stage projects, the VC is guaranteed that his effort will be productive, since at time 1, only $p$-state firms choose VC financing, as discussed before. By doing this, however, the VC is able to add only less value to the project, since the contracting at time 1 now proceeds under a greater degree of asymmetric information, given that the VC enters the firm at only a later stage in its life cycle.

4.5 Composition of projects financed by VCs and Angels

We now characterize the proportion of early- and later-stage projects funded by VCs and angels, respectively, under the moderate- and high-scarcity regimes of VC financing.\(^{29}\)

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\(^{29}\) We believe that it is the high- and moderate-scarcity regimes that reflect the relative scarcity of VC financing versus angel financing in the United States as well as in most other developed economies in practice. We characterize the equilibrium in the low-scarcity regime only for completeness.
Proposition 9. (Composition of VC and angel portfolios)

(i) If $\theta < \frac{1}{2}$, then in periods of moderate or high scarcity of VC financing, a greater proportion of earlier-stage projects are funded by angels than by VCs.

(ii) Under the high-scarcity regime of VC financing, angels finance all earlier-stage projects, while VCs finance only later-stage projects.

As we discussed in previous sections, as the scarcity of VC financing relative to angel financing goes up from low to high, the stage at which the VC first starts financing the firm goes from early to late. The moderate-scarcity regime reflects a situation between these two extremes, where the VC finances later-stage projects as well as a small fraction of earlier-stage projects. As discussed before, when the scarcity of VC financing is greater, the VCs are able to get their required return only by focusing on later-stage projects, where their effort is guaranteed to be able to add value (thus creating a higher return for their investment). In contrast, since angel financing does not face the same scarcity as VC financing, and angels supply only capital but no effort, this choice between earlier and later-stage projects does not apply to them, so that angels finance a larger fraction of early-stage projects in equilibrium, in the high- and moderate-scarcity regimes of VC financing.

4.6 Announcement effects of VC and angel financing

We now characterize the announcement effects of early-round as well as later-round financing of projects by VCs and angels.

Proposition 10. (Announcement effects)

(i) Under conditions of moderate scarcity of VC financing, the announcement of a firm obtaining VC financing at time 0 conveys favorable information to outsiders, leading them to revise their expectation of the firm’s value upward.

(ii) If a firm receives a new round of financing at a later stage from a VC, it conveys further positive information to outsiders, regardless of whether its earlier-stage financing was undertaken by an angel or a VC.

(iii) If a VC exits a firm funded by him earlier, it conveys negative information about the firm to outsiders.

In the above proposition, by “announcement effect” we refer to the revision of a firm’s valuation (upward or downward) by private equity investors (since the securities of the private firms we are analyzing are not publicly traded). If a firm receives financing from the VC, outsiders infer that there is a greater probability of the VC adding value to that firm, compared with the case where the firm is angel-financed. As a result,
outsiders revise their valuation of the firm upward. Similarly, if a firm receives later-stage funding from a VC, it conveys further positive information to outsiders, who infer that the state of the firm is such that the VC will indeed be able to add additional value to the firm. In contrast, if a VC who financed an earlier round exits the firm at a later stage, it conveys to outsiders that the VC’s assessment of his ability to add value to that firm is negative, thus leading them to revise the firm’s value downward.

5. Empirical Implications

We discuss some of the empirical implications of our model below.

(i) Entrepreneurs’ choice between venture capital and angel financing: Our model predicts that venture capitalists will finance firms in industries where the potential for adding value is the greatest, while angels will fund firms in industries where the potential for adding value is more limited. This means that venture capitalists will tend to finance firms in technologically sophisticated and knowledge-intensive industries, where they can create the most value. In contrast, angels will tend to finance projects in industries that are less technologically sophisticated and knowledge-intensive, with fewer opportunities for the financier to add value. By the same token, entrepreneurs who are technologically sophisticated themselves will tend to have self-financing or angel financing, since the incremental value that can be added by the financier will be smaller in such firms.

(ii) Differences between venture capital financing and angel financing contracts: Convertible features (convertible debt or convertible preferred equity) in venture capital financing contracts have two important roles in our setting. First, the fixed-income portion, which offers downside protection to the financier, minimizes the pricing effects of the entrepreneurs’ private information. Second, the “upside” of the convertible (i.e., the warrant component) motivates the venture capitalists to put forth optimal effort to add value to the firm. (In contrast, when convertible debt or convertible preferred equity are used in angel contracts, they have the first role, namely, minimizing the pricing effects of the entrepreneurs’ private information). Thus, our model predicts that angel financing contracts are less likely to have convertible features, and, when present, will have a smaller upside (ceteris paribus) compared with venture capital financing contracts.30

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30 Some preliminary evidence supporting this prediction is provided by Prowse (1998), who presents evidence on the angel market gathered from field research. He comments that angel financing contracts are more likely to involve the use of common stock. When angels do take convertible preferred, they are motivated by the objectives of minimizing downside investment risk and ensuring performance by management, which is also consistent with the rationale developed here. Similar evidence is provided by Wong (2001).
(iii) The dynamic evolution of venture capital contracts: Our model predicts that as the firm ages and engages in several rounds of venture capital financing, the nature of its venture capital financing contract will be altered across these financing rounds. This is because the venture capital contract tradeoff the need to minimize the impact of asymmetric information between the entrepreneur and the venture capitalist and the competing need to provide high-powered incentives to the venture capitalist to put forth effort to add value to the firm. Thus, in the earlier financing rounds (when the extent of asymmetric information is greater), the contract would give much greater downside protection (i.e., the fixed-income component contract will be greater) to the venture capitalist. In later financing rounds, when there is less asymmetric information between the venture capitalist and the entrepreneur, the need to provide incentives to the venture capitalist dominates, so that the fixed-income component of the contract will be less, while the upside (warrant) component of the contract will be more. We predict that this will be true regardless of the extent of scarcity of venture capital financing relative to angel financing.31 Evidence consistent with this is provided by Kaplan and Stromberg (2003), who document that venture capitalists’ state-contingent cash flows increase over financing rounds at the expense of entrepreneurs.32

(iv) Cross-sectional differences in later-round venture capital financing contracts: Our model predicts that there will be significant differences between the venture capital contracts involving firms that were previously angel-funded (so that the venture capitalist is financing the firm for the first time) and those involving firms financed by the same venture capitalist in previous rounds: Ceteris paribus, the financing contract in the latter situation will have a relatively larger upside (warrant) component and a smaller downside (fixed-income) component compared with the former. This is because the former contracting situation is characterized by a greater extent of asymmetric information, so that the equilibrium contract has to focus more on dissipating the effects of this asymmetric

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31 Of course, in practice, as the firm matures further, so that the venture capitalist’s effort becomes less important for creating value relative to that of the entrepreneur’s, the venture capital contract in subsequent financing rounds will again have a smaller “upside” (for example, this may be the case immediately prior to the firm going public), thus ensuring that the entrepreneur has the appropriate incentives to add value at this later stage in the firm’s life (see implication (v)).

32 Our analysis also has implications for the contracting in R&D alliances, where typically a large corporation contracts with a smaller research firm (with the former outsourcing some of its R&D to the latter). Similar to the relationship between a venture capitalist and an entrepreneurial firm, such strategic alliances are also characterized by asymmetric information, with the large corporation (which acts as the financier) being able to contribute to the success of the research firm through its effort. Similar to our implication for venture capital contracting, our analysis predicts that when the research firm is in its earlier stages, the contract between the two firms should have a greater fixed income component; as the research firm matures, the warrant component of the contract should become larger, with the fixed-income component becoming smaller. Evidence consistent with this prediction is provided by Robinson and Stuart (2004) in the context of biotech strategic alliances.
information and less on motivating the venture capitalist to put forth optimal effort.

(v) **Relationship between firm and industry characteristics and venture capital financing contracts:** Our model predicts that in situations where there is more asymmetric information between the venture capitalist and the entrepreneur, the venture capitalist’s financing contract would give him greater downside protection (i.e., the fixed-income component will be greater). Further, the fixed-income component of a venture capitalist’s contract will be increasing in the importance of the entrepreneur’s effort relative to that of the venture capitalist for project success (ensuring that the entrepreneur has incentive to exert more effort).

(vi) **Relationship between the productivity of the venture capitalist and the nature of venture capital contracts:** Our model predicts that, in situations where the venture capitalist is more productive (able to add more value), the financing contract used will have more of an “upside” (warrant component). Therefore, venture capitalists with greater experience will use convertibles with more upside than those with less experience. Further, a given venture capitalist will use convertibles that have less of an upside in situations where he is less productive.

(vii) **Financing of early- versus late-stage projects:** Our model predicts that there will be systematic differences in the nature of projects financed by venture capitalists and angels (stage of financing as well as project quality). Thus, in periods of high scarcity of such financing, venture capital firms will fund only later-stage projects, while angels will fund early-stage projects. In periods of moderate scarcity of venture capital funds however, venture capitalists will fund later stage projects and a small fraction of early-stage projects, leaving the rest to angel financing. In summary, our model makes two predictions regarding the stage of financing of projects by venture capitalists and angels. First, our model predicts that angels will fund the majority of early-stage projects (see Wong 2001 for evidence consistent with this). Second, as the scarcity of venture capital financing relative to angel financing goes up from low to high (as measured by lower flows into venture capital funds), the fraction of early-stage projects funded by venture capitalists becomes smaller. Primack (2002) provides evidence consistent with the latter prediction of our model. He documents that from 1995 to 2001, when venture capital financing was relatively abundant, early-stage investing accounted for about a quarter of all venture capital investments. In contrast, this

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33 This implication follows from Proposition 4.

34 What matters for our results is the magnitude of the venture capitalists’ opportunity cost of capital relative to the average return of projects available for investment. Thus the proportion of early-stage projects funded by venture capitalists will also become smaller if, holding venture capital availability constant, the average productivity of the projects available for investment falls.
fraction dropped to 18% in the third quarter of 2001 (when venture capital financing became quite scarce) and down to 10% in the fourth quarter of the same year. In terms of project quality, our prediction is that the average quality of projects funded by venture capitalists will be greater than those funded by angels. Further, the average quality of projects funded by venture capitalists will go up during periods of high scarcity of venture capital financing (as measured by lower flows into venture capital funds).

(viii) The concentrated nature of venture capitalists' portfolios relative to that of angels: Our model predicts that (despite the risk-reducing advantages of diversification) venture capital firms will take concentrated positions in a few firms, while angels will invest significantly smaller amounts in a number of firms. Recall that the defining feature of venture capital financing here is value-addition, and if the venture capital firm makes an investment in a firm below a certain threshold amount, it will not be incentive compatible for the firm to allocate any venture capitalist to that firm (this implication requires the additional assumption of the indivisibility of human capital). Since angels do not engage in significant value addition, they do not face such a minimum-investment constraint, thus enabling them to take small positions in a number of firms. Further, we predict that venture capitalists will tend to concentrate their investment portfolios in a limited number of industries, since venture capitalists tend to develop expertise in a few industries, which allows them to add value. In contrast, since angels are not as concerned with value addition, their investments tend to be more dispersed across industries.

(ix) The financing path of firms, announcement effects, and the probability of successful exit: Our model makes the following predictions regarding the relationship between the financing path of firms and firm quality, which, in turn, affects their probability of successful exit (IPO or acquisition versus writeoff).35 Firms that are venture-financed at their early stages, and continue to attract further venture financing, are of the highest quality (most likely to go public or be acquired). Firms that are angel-financed initially and attract subsequent rounds of venture financing are of lower quality (and will therefore be less likely to have a successful exit compared with those which are continuously venture financed). Finally, firms that are initially as well as subsequently angel-financed, or those that start out as venture-backed firms but attract only angel financing in subsequent rounds, will be of the lowest quality (and therefore least likely to go public or be successfully acquired). Consistent with this, our model also predicts that if a firm successfully obtains venture capital financing in

35 The IPO literature indicates that, in an environment of asymmetric information, the highest intrinsic value firms are the ones that are most likely to have successful IPOs (see, e.g., Chemmanur 1993, Allen and Faulhaber 1989, or Welch 1989).
its early stages, this will convey favorable information to outside private-equity investors about that firm, who will revise its value upward. Further rounds of venture capital financing will serve as additional favorable signals to outsiders (as are later rounds of venture capital financing for a firm that is initially angel-financed). Finally, exit by venture capitalists from initially venture-backed firms will serve as a negative signal to outsiders.

6. Conclusion

We have developed a theoretical analysis of an entrepreneur’s choice between venture capital (VC) and angel financing at various stages in a private firm’s life, and characterized the dynamic evolution of the firm’s contract with its financier (VC or angel). In our model, an entrepreneur had information superior to a potential financier about his own firm; however, this information advantage diminished as the financier interacted with the firm over time. Venture capitalists and angels differed in two ways in our model. First, venture capitalists could add value to the firm by exerting effort, which, together with the entrepreneur’s effort, increased the chance of project success; the angel was unable to add significant value. Second, venture capital financing was scarce relative to angel financing. The equilibrium VC contract in our setting maximized value addition by ensuring that both the entrepreneur and the VC exerted optimal effort. We developed the following results in the above setting. First, we characterized the optimal financing path of the firm: depending on firm characteristics, a firm may use angel financing in its early stages and switch to VC financing in later stages, or vice versa. Second, we showed that VC-financing contracts resemble convertible debt or convertible preferred security, while angel-financing contracts may resemble a variety of financial securities, including common stock. Third, we showed that for firms that use venture financing in earlier as well as later-rounds, earlier-round financing contracts will have more of a fixed-income component and less of a warrant (“upside”) component compared with later-round financing contracts. Fourth, we showed that later-round financing contracts between an entrepreneur and a VC who financed it in an earlier-round will have a greater warrant component compared with such a contract between a VC and a previously angel-financed firm. Fifth, we characterized how the structure of VC-financing contracts relate to: (i) the experience and productivity of the VC; (ii) the nature of the firm’s industry; and (iii) the scarcity of VC financing. Finally, we developed predictions for the relationship between the financing path of a firm and the probability of its having a successful exit (IPO or acquisition), and for differences in the compositions of VCs’ and angels’ investment portfolios.
Appendix A. Proofs of propositions

A.1. Proof of Proposition 1

Part (i): Simplifying the objective function using the VC’s IR, the problem becomes that of maximizing \( f(\hat{c}_1)\Delta X - c - k_1 - \frac{\rho}{2} \). Because \( \hat{c}_1 < e^{\rho} \), as we have shown in the main text, the entrepreneur’s IC condition (13) has to be binding. Otherwise, the entrepreneur can increase \( P_i^e \) to \( P_i^0 \) by increasing \( a_i^e \) and fixing \( b_i^e \) to induce the VC to exert more effort \( c_1', c_1 > \hat{c}_1 \). At the same time he can increase \( P_i^e \) such that the VC’s IR is just satisfied. The new contract improves the value of his objective from \( f(\hat{c}_1)\Delta X - \hat{c}_1 \) to \( f(c_1')\Delta X - c_1'(f(c_1')\Delta X - c_1' > f(\hat{c}_1)\Delta X - \hat{c}_1 \), because \( \hat{c}_1 < c_1' < e^{\rho} \). Solving condition (13) as an equality and choosing the maximal solution gives us the optimal \( P_i^e \). Thus we have shown (a). The optimal \( b_i^e \) is solved by setting the VC’s IR condition (7) as an equality and then substituting \( a_i^e X = b_i^e X + P_i^e \) into the equation.

For limited liability to be not binding, we need \( b_i^e \leq 1 \) and \( b_i^e \geq 0 \). \( b_i^e \leq 1 \) is equivalent to \( P_i^e \leq X - \frac{\rho}{2} + \hat{c}_1 + V_p - P_i^e (q + f(\hat{c}_1)) \leq X \). Together with the budget constraint, we get \( I - I_0 \leq P_i^e \leq X - \frac{\rho}{2} + \hat{c}_1 + V_p - P_i^e (q + f(\hat{c}_1)) \). \( b_i^e \geq 0 \) is equivalent to \( P_i^e \leq q + \hat{c}_1 + V_p - P_i^e (q + f(\hat{c}_1)) \), which requires that \( P_i^e \geq P_i^e (q + f(\hat{c}_1)) - \frac{\rho}{2} - \hat{c}_1 - V_p \). This constraint is never binding since the entrepreneur can always increase \( P_i^e \) to satisfy it. Part (c) follows because \( a_i^e < b_i^e \) if and only if \( \frac{P_i^e}{\frac{\rho}{2} + \hat{c}_1 + V_p - P_i^e (q + f(\hat{c}_1))} < \frac{P_i^e}{I - I_0 + \frac{\rho}{2} + \hat{c}_1 + V_p - P_i^e (q + f(\hat{c}_1))} \).

(d) follows because this condition implies that the above inequalities become equalities.

For part (ii), it is clear that the angel’s IR has to be binding. But if that is the case, the entrepreneur’s objective becomes \( q\tilde{X} + (1 - q)\Delta X - (I - I_0) - V_n \), which does not depend on the contract anymore. We assume and verify later that the entrepreneur’s IR (which implies limited liability and budget constraints here) is satisfied in equilibrium.

A.2. Proof of Proposition 2

We prove this proposition using the optimal contract characterized in Proposition 1. If the state \( p \) firm continues to use financing from the same VC, by Proposition 2, the entrepreneur gets

\[
q\tilde{X} + (1 - q)\tilde{X} + f(\hat{c}_1)\Delta X - \hat{c}_1 - R - V_p - (I - I_0) - k_1. \tag{A-1}
\]

On the other hand, if the firm deviates by using angel financing, let the contract be \( (a_i^f, b_i^f) \), and the firm raises \( P_i^f \) from the angel. The entrepreneur gets

\[
(1 - a_i^f)q\tilde{X} + (1 - b_i^f)(1 - q)\tilde{X} + P_i^f + V_n - V_p - (I - I_0). \tag{A-2}
\]

The first two terms are the entrepreneur’s residual cash flow. The next two terms are the total money raised by the entrepreneur from the angel. The entrepreneur has to pay the VC \( V_p \) and invest \( (I - I_0) \). Simplifying expression (A-2) by using the angel’s IR, which is binding, \( a_i^f q\tilde{X} + b_i^f (1 - q)\tilde{X} = P_i^f + V_n \), it becomes \( q\tilde{X} + (1 - q)\tilde{X} - V_p - (I - I_0) \). The difference between the two payoffs is \( f(\hat{c}_1)\Delta X - \hat{c}_1 - \frac{\rho}{2} - k_1 \), which is positive by assumption. Therefore the entrepreneur is worse off by deviating. If a state \( n \) firm deviates by continuing to use VC financing, because there is no asymmetric information between the entrepreneur and the VC, by the VC’s binding IR, the entrepreneur gets

\[
q\tilde{X} + (1 - q)\tilde{X} - R - V_p - (I - I_0). \tag{A-3}
\]
If a state $n$ firm uses angel financing instead, by Proposition 2, the entrepreneur gets

$$q\bar{X} + (1-q)\bar{X} - V_p - (I - I_0),$$

(A-4)

so that the entrepreneur is better off by not deviating.

### A.3. Proof of Proposition 3

We prove this proposition using the optimal contract characterized in Proposition 4. The out-of-equilibrium belief is that a firm deviating from equilibrium is of type $n$. The state $n$ firm will not have the incentive to mimic the state $p$ by obtaining VC financing by condition (22), which is satisfied by the contract design as specified in Proposition 4. A state $p$ firm will not benefit from obtaining angel financing because condition (25) is satisfied. To see why, note that, by using the VC and the angel’s IRs to simplify condition (25), we get:

$$f(c^*_1)\Delta X - k_1 - c^*_1 - \frac{\delta}{2} \geq 0,$$

which is true by assumption. If a state $p$ deviates by offering an out-of-equilibrium contract to the VC, the VC would believe he is of state $n$ and the VC would not exert effort. As a result, the firm is worse off by deviating. If a state $n$ firm offers an out-of-equilibrium contract to a VC, it reveals its type. Because a state $n$ firm cannot benefit from VC financing and the cost of capital of VC financing is greater, the firm is better off using angel financing.

### A.4. Proof of Proposition 4

As in the proof of Proposition 1, it is easy to see that the entrepreneur’s objective is to maximize $f(c^*_1)\Delta X - c^*_1 - k_1 - \frac{\delta}{2}$.

First, at the optimum, $c^*_1 < c^b$. This is because of the entrepreneur’s IC, condition (28). Condition (28) can be simplified as:

$$f(c^*_1)(\Delta X - (d^*_{1} \bar{X} - b^P_{1} \bar{X})) \geq k_1.$$  

(A-5)

It is clear that $d^*_{1} \bar{X} - b^P_{1} \bar{X} < \Delta X$, since otherwise condition (A-5) will not be satisfied.

Second, at the optimum, the $n$-state firm’s incentive compatibility constraint to not mimic a $p$-state firm (recall that the firm designs the financing contract to maximize firm value net of cash flows to the VC) implies condition (A-5), the entrepreneur’s IC to exert effort. Therefore, the entrepreneur’s IC is not binding. The $n$-state firm’s incentive compatibility constraint is condition (31), which is equivalent to:

$$\frac{R}{2} + c^*_1 \geq P^P_{1} f(c^*_1(P^P_{1})).$$

(A-6)

By the assumption $f(c^*_1)\Delta X - k_1 - c^*_1 - \frac{\delta}{2} \geq 0$, so that condition (A-6) implies condition (A-5). This result implies that condition (A-5) is not binding at the optimum. Thus $\hat{c}_1 \geq c^*_1$ follows because if this is not true, then (i) $c^*_1$ satisfies all the constraints in the problem in Section 3.1; (ii) the two problems have the same objective function $f(c^*_1)\Delta X - c^*_1$; and (iii) $\hat{c}_1 < c^*_1 < c^b$. Therefore, $c^*_1$ would be a better solution than $\hat{c}_1$ for the problem in Section 3.1, which violates the optimality of $\hat{c}_1$. Further, if $f(c^*_1)\Delta X - k_1 - c^*_1 - \frac{\delta}{2} > 0$, then $\hat{c}_1 > c^*_1$ because $\hat{c}_1$ will violate condition (A-6) for sure.

Third, the $p$-state firm’s IC is satisfied as long as $f(c^*_1)\Delta X - k_1 - c^*_1 - \frac{\delta}{2} \geq 0$, which is true by assumption. Therefore, at the optimum, condition (A-6) is binding. Otherwise, the entrepreneur can increase $P^P_{1}$ to $P^P_{1}^*$ by increasing $d^*_{1}$ and fixing $b^P_{1}$ to induce the VC to exert more effort $c^*_1, c^*_1 > c^*_1$. At the same time he can increase $P^P_{1}$ such that the VC’s IR is just satisfied. The new contract improves the value of his objective. Solving condition (A-6) as an equality and choosing the maximal solution gives us the optimal $P^{P*}_{1}$. Thus we have shown (a). The optimal $b^P_{1}$ is solved by setting the VC’s IR condition (30) as an equality and then substituting $d^*_{1} \bar{X} = b^P_{1} \bar{X} + P^P_{1}$ and the binding condition (A-6).
For limited liability to be not binding, we need $b^*_B \leq 1$ and $b^*_n \geq 0$. $b^*_B \leq 1$ is equivalent to $P^*_1 + V_p - P^*_n q \leq X$. Together with the budget constraint, we get $I - I_0 \leq P^*_B \leq X - V_p + P^*_n q$. (c) follows because the condition implies that the inequalities become equalities. $b^*_n \geq 0$ is equivalent to $P^*_n \geq P^*_n q - V_p$, which is never binding because the entrepreneur can always raise $P^*_n$. Part (d) follows because $d^*_n = b^*_n = 1$ if and only if $P^*_B X \leq \Delta X$, but $P^*_n X = P^*_n + V_p - P^*_n q \leq I - I_0 + V_p - P^*_n q$. (e) is already shown. Thus we have proven part (i). The proof of part (ii) is similar to the proof of part (ii) in Proposition 1.

### A.5. Proof of Proposition 5

We prove this proposition using the optimal contract characterized in Proposition 6. The out-of-equilibrium belief is that a firm deviating from equilibrium is of type B firm. The type B firm will not have the incentive to mimic a type G firm by obtaining VC financing because condition (A-8) is satisfied (using the VC and the angel’s IRs to simplify condition (A-7)). The type G firm has no incentive to deviate by offering an out-of-equilibrium contract to the VC either, because the VC would believe he is of type B firm and thus would not exert any effort to create value for the firm. Thus, the type G firm adopts VC financing using the equilibrium contract, and the type B firm adopts angel financing.

### A.6. Proof of Proposition 6

The incentive compatibility conditions of the type B firm, which ensure that the type B entrepreneur is better off using angel financing rather than VC financing, is given by condition (A-7).

$$\lambda [W_p(c^*_1) - V^B_p(c^*_1) - (I - I_0) - c^*_1 - \frac{R}{2} - k_1] + (1 - \lambda)[W_n - V^B_n - (I - I_0)] + I^B_0 - I_0 \geq$$

$$\lambda [W_p(\hat{c}_1) - V^G_p(\hat{c}_1) - (I - I_0) - \hat{c}_1 - \frac{R}{2} - k_1] + (1 - \lambda)[W_n - V^G_n - (I - I_0)] + I^G_0 - I_0.$$  

(A-7)

The left-hand side of condition (A-7) gives the payoff to the type B firm from not mimicking the type G firm, thereby revealing its type and obtaining only angel financing. The right-hand side gives the payoff to the type B firm from mimicking the type G firm, thereby revealing its type and obtaining only angel financing. In condition (A-7), $W_p(c^*_1) = (q + f(c^*_1))\bar{X} + (1 - q - f(c^*_1))X$ and $W_n = q\bar{X} + (1 - q)X$; these are the firm’s time 1 expected values in equilibrium if it is in state $p$ and state $n$, respectively, provided that it is financed by an angel at time 0. Remember that the firm makes a take-it-or-leave-it offer to the time 0 financier when it switches financing sources, the value of the time 0 financier’s security at time 1 is determined by his reservation value of these securities. Therefore, $V^B_p = (q + f(c^*_1))a_0^B X + (1 - q - f(c^*_1))b_0^B X$, and $V^B_n = a_0^B q\bar{X} + b_0^B (1 - q)X$; these are the time 1 values of the angel’s contract $(a_0^B, b_0^B)$ when the state of the firm is $p$ and $n$, respectively. Denote by $W_p(\hat{c}_1)$ and $W_n$ the time 1 value in state $p$ and in state $n$ respectively if the type B firm mimics the type G by using VC financing at time 0. Because there is no asymmetric information at time 1 if the firm is financed by a VC at time 0, in equilibrium the VC will exert effort $\hat{c}_1$, and the entrepreneur will exert effort $k_1$ if the firm is in state $p$. Therefore, $W_p(\hat{c}_1) = (q + f(\hat{c}_1))\bar{X} + (1 - q - f(\hat{c}_1))X$ and $W_n = q\bar{X} + (1 - q)X$. Similarly, denote by $V^G_p(\hat{c}_1)$ and $V^G_n$ the time
Thus, greater than that obtained by mimicking the type B firm (and obtaining angel financing). The third term is the profit from selling overvalued securities to the VC, $VG_1$ values of the VC’s contract $(aG, bG)$ when the state of the firm is $p$ and $n$, respectively. $V_p^G(\hat{c}_1) \equiv (q + f(\hat{c}_1))aG_0X + (1 - q - f(\hat{c}_1))bG_0X$, and $V_n^G \equiv q aG_0X + (1 - q)bG_0X$.

The type G firm’s incentive compatibility condition (A-8) ensures that its payoff from not mimicking the type B firm and truly revealing its type (and obtaining VC financing) is greater than that obtained by mimicking the type B firm (and obtaining angel financing). Thus,

$$(\lambda + f(c_p^G)) [W_p(\hat{c}_1) - V_p^G(\hat{c}_1) - (I - I_0) - \hat{c}_1 - R \frac{2}{2} - k_1] + (1 - \lambda - f(c_p^G)) [W_n - V_n^G - (I - I_0)] + f_o^G - I_0 - k_0$$

(A-8)

Thus the entrepreneur’s contract design problem is summarized by:

$$\max_{aG^0, bG^0, cG} (\lambda + f(c_0)) [W_p(\hat{c}_1) - V_p^G(\hat{c}_1) - (I - I_0) - \hat{c}_1 - R \frac{2}{2} - k_1] + (1 - (\lambda + f(c_0))) [W_n - V_n^G - (I - I_0)] + f_o^G - I_0 - k_0$$

s.t. $c_0 \in \arg\max_{c_0} \{\lambda G V_p^G + (1 - \lambda G) V_n^G - c_0\},$ (A-9)

$$(\lambda + f(c_0)) [W_p(\hat{c}_1) - V_p^G(\hat{c}_1) - (I - I_0) - \hat{c}_1 - R \frac{2}{2} - k_1] + (1 - (\lambda + f(c_0))) [W_n - V_n^G - (I - I_0)] + f_o^G - I_0 - k_0$$

(A-10)

$$(\lambda + f(c_0))^G [W_p(\hat{c}_1) - V_p^G(\hat{c}_1) - (I - I_0) - \hat{c}_1 - R \frac{2}{2} - k_1] + (1 - (\lambda + f(c_0))) [W_n - V_n^G - (I - I_0)] + f_o^G - I_0 - k_0$$

(A-11)

Thus (A-10) is the VC’s incentive compatibility condition, ensuring the VC puts forth the optimal level of effort. Condition (A-11) is the entrepreneur’s incentive compatibility condition, ensuring that he exerts high effort. Condition (A-12) is the VC’s individual rationality condition, ensuring that the VC gets adequate compensation for the investment he provides for the firm and his effort cost, as well as his opportunity cost. Let’s look at the type B’s incentive compatibility condition, condition (A-7). Simplifying it by using the VC’s IR condition (A-12) and the angel’s IR, $V_n^B = f^B_0$, we get:

$$\lambda [f(\hat{c}_1) \Delta X - \hat{c}_1] - (f(c_0^*) \Delta X - c_0^*) - R \frac{2}{2} - c_0 + f(c_0) f(\hat{c}_1)(aG_0X - bG_0X) \leq 0.$$ (A-15)

The first term is the value created by having VC financing earlier if the firm is in state $p$, which is the benefit of a more efficient financing at time 1, $\delta \equiv (f(\hat{c}_1) \Delta X - \hat{c}_1) - (f(c_0^*) \Delta X - c_0^*)$. The second term is the extra financing cost by choosing VC financing. The third term is the profit from selling overvalued securities to the VC, $V_p^G G_n^G = f(\hat{c}_1)(aG_0X - bG_0X)$. To prove (i) (a), (b), (c), and (d), we need to first show that...
the entrepreneur’s IC to exert high effort—that is, condition (A-11) is not binding. Simplifying condition (A-11), we get

\[ f(c_0)(f(c_1)\Delta X - \tilde{c}_1 - \frac{R}{2} - k) - k_0 \geq f(c_0)(f(c_1)(d_0^{GR}X - b_0^{GR}X)). \]  

(A-16)

Because \( f(c_0)(f(c_1)\Delta X - \tilde{c}_1 - \frac{R}{2} - k) - k_0 \geq 0 \) for \( c_0 = c_0^* \) by assumption, we can conclude that at the optimum, condition (A-15) implies condition (A-16). That is, condition (A-11) is not binding at the optimum.

This result characterizes the optimal \( P^{GR}_0 = c_0^{GR}X - b_0^{GR}X \) at time 0; that is, \( P^{GR}_0 \) is the maximum solution to the binding condition (A-15). The optimal \( c_0^{GR*} \) is determined by

\[ f'(c_0^{GR*}) = \frac{1}{f(c_1)(d_0^{GR}X - b_0^{GR}X)}. \]

And then we can get \( a_0^{GR*} \) and \( b_0^{GR*} \) using the VC’s IR condition (A-12) and the binding condition (A-15). Thus we have proved (a) and (b).

For limited liability to be not binding, we need \( b_0^{GR*} \leq 1 \) and \( b_0^{GR*} \geq 0 \). \( b_0^{GR*} \leq 1 \) is equivalent to \( \frac{\Delta X}{X} \leq \frac{P^{GR}_0}{P^{GR}_0 +!} + \lambda \Delta X - P^{GR}_0 \frac{\lambda f(c_1) + q}{X} \leq X \). Using the budget constraint, we get \( L_0 \leq P^{GR}_0 \leq X + P^{GR}_0 \frac{\lambda f(c_1) + q}{X} - \lambda \delta \). (c) follows because it implies that the inequalities become equalities. \( b_0^{GR*} \geq 0 \) is equivalent to \( f^{GR}_0 \geq f^{GR}_0 \frac{\lambda f(c_1) + q}{X} - \lambda \delta \), which is never binding because the entrepreneur can always increase the entrepreneur’s IC to exert high effort—that is, condition (A-11) is not binding. Part (d) follows because \( a_0^{GR*} < b_0^{GR*} \) if and only if \( \frac{P^{GR}_0}{b_0^{GR*}X} < \Delta X \), but \( \frac{P^{GR}_0}{b_0^{GR*}} = \frac{P^{GR}_0}{P^{GR}_0 +!} + \lambda \delta - P^{GR}_0 \frac{\lambda f(c_1) + q}{X} \leq L_0 + \lambda \delta - P^{GR}_0 \frac{\lambda f(c_1) + q}{X} \).

To prove (i) (e), we need to show that \( c_0^{GR*} \leq \tilde{c}_1 \).

The second inequality is proven; we need to show only the first. Suppose \( c_0^{GR*} \geq \tilde{c}_1 \). By the first-order condition of the VC at time 0, \( f'(c_0^{GR*}) = \frac{1}{f(c_1)(d_0^{GR}X - b_0^{GR}X)} \), we can find \( a'_1 \) and \( b'_1 \) such that \( f'(c_0^{GR*}) = \frac{1}{(a'_1X - b'_1X)} \). We then have

\[ f(c_0^{GR*})f(c_1)(a_0^{GR*}X - b_0^{GR*}X) = (a'_1X - b'_1X)f(c_0^{GR*}). \]

Since \( c_0^{GR*} \geq \tilde{c}_1 \), an n-state firm’s IC condition (A-6) is violated (weakly). That is, \( (a'_1X - b'_1X)f(c_0^{GR*}) - (\frac{\Delta X}{X}) \geq 0 \). Using the above two conditions to simplify the left-hand side of condition (A-15), we have \( \lambda \delta - \frac{\Delta X}{X} \geq 0 \). That is, type B entrepreneur’s IC constraint (A-15) cannot hold, which is a contradiction. Thus we have (e).

The proof of part (ii) is similar to the proof of part (ii) in Proposition 1.

To prove (iii), we need also to show that the entrepreneur’s IR and limited liability constraints are satisfied at time 0 and time 1.

First we look at the type G firm. After some algebra we can show that if \( I \leq X - (\tilde{c}_1 + \frac{R}{2} + c_0^{GR*} + (1 - \lambda - f(c_0^{GR*})(f(c_1)P^{GR}_0 - P^{GR}_0 \frac{\lambda f(c_1) + q}{X} + f(c_1))) \) and \( X + (1 - q)X - I - (\frac{\Delta X}{X}) \geq 0 \), the entrepreneur’s time 1 IRs (which are implied by limited liability and his IC), limited liability, and budget constraints can be satisfied by choosing \( P^{GR}_0 = L_0 \) and \( P^{GR}_0 = P^{GR}_1 = I - L_0 \). At time 0, the entrepreneur’s payoff is at least \( qX + (1 - q)X - I \), which is greater than zero if the above two conditions hold. The limited liability and budget constraints can be satisfied if \( L_0 \leq X + P^{GR}_0 \frac{\lambda f(c_1) + q}{X} - \lambda \delta \), as is given in part (i) (b).

Next, we look at the type B firm. By an argument similar to that used for the type G firm, the type P firm’s limited liability and budget constraints can be satisfied at time 1 if \( I - L_0 \leq X - (\frac{\Delta X}{X}) + (1 - \lambda) f(c_1)P^{GR}_0 \). The entrepreneur’s IR in state \( n \) is satisfied if \( qX + (1 - q)X - I - L_0 \geq (\frac{\Delta X}{X}) + (1 - \lambda) f(c_1)P^{GR}_0 \), which can be satisfied if \( qX + (1 - q)X - I \geq 0 \). In state \( p \), this is implied by limited liability and his IC condition, assuming that he exerts high effort. Finally, at time 0, the entrepreneur’s IR is satisfied, since his payoff is at least \( qX + (1 - q)X - I \). Limited liability and budget constraints at time 0 can be satisfied if \( L_0 \leq X + \lambda f(c_1)P^{GR}_0 \).
To summarize, if
\[
I \leq \min \{ X - \left[ \frac{R}{2} + \hat{c}_1 + R + c_0^{G*} + (1 - \lambda - f(\hat{c}_0^{G*}))f(\hat{c}_1)P_0^{G*} - P_1^{G*}(q + f(\hat{c}_1)) \right],
\]
\[
qX + (1 - q)X - \left( \frac{R}{2} + c_0^{G*} - (\lambda + (f(\hat{c}_0^{G*}))P_0^{G*}) \right), X - (1 - \lambda)f(\hat{c}_1)P_0^{B*} + P_1^{G*} q
\]
the equilibrium holds. Thus (iii) is proved.

A.7. Proof of Proposition 7
The optimal contract design problem of the type G firm is the following:
\[
\max_{a_0, b_0, P_0} (\lambda + f(c_0))[W_p(\hat{c}_1) - V_p(\hat{c}_1) - (I - I_0) - \hat{c}_1 - \frac{R}{2} - k_1]
\]
\[
+ (1 - \lambda - f(c_0))[W_a - V_a - (I - I_0)] + P_0^* - I_0 - k_0
\]
\[
s.t. c_0 \in \arg\max_{c_0} (\lambda + \theta f(c_0))V_p + (1 - (\lambda + \theta f(c_0)))V_n - c_0.
\]
\[
(\lambda + f(c_0))[W_p(\hat{c}_1) - V_p(\hat{c}_1) - (I - I_0) - \hat{c}_1 - \frac{R}{2} - k_1]
\]
\[
+ (1 - \lambda - f(c_0))[W_a - V_a - (I - I_0)] + P_0^* - I_0 - k_0
\]
\[
\geq \lambda [W_p(\hat{c}_1) - V_p(\hat{c}_1) - (I - I_0) - \hat{c}_1 - \frac{R}{2} - k_1]
\]
\[
+ (1 - \lambda)[W_a - V_a - (I - I_0)] + P_0^* - I_0.
\]
\[
(\lambda + \theta f(c_0))V_p + (1 - (\lambda + \theta f(c_0)))V_n \geq P_0^* + \frac{R}{2} + c_0.
\]
\[
0 \leq a_0 \leq 1, 0 \leq b_0 \leq 1,
\]
\[
P_0^* \geq I_0 \geq I_0.
\]
Here condition (A-19) is the VC’s incentive compatibility condition, ensuring the VC puts forth the optimal level of effort. Condition (A-20) is the entrepreneur’s incentive compatibility condition, ensuring that he exerts high effort. Condition (A-21) is the VC’s individual rationality condition, ensuring that the VC gets adequate compensation for the investment he provides for the firm and his effort cost, as well as his opportunity cost. Since condition (A-21) is binding (otherwise the entrepreneur can increase $I_0^*$), the entrepreneur’s objective can be simplified to:
\[
f(c_0)(f(\hat{c}_1)\Delta X - \hat{c}_1 - \frac{R}{2} - k_1) - c_0 - \frac{R}{2} - k_0 - (1 - \theta) f(c_0)(V_p - V_n).
\]

The first four terms in condition (A-24) are the value created by the VC and the entrepreneur’s effort. The last term is the extra financing cost the type G firm has to pay the VC because of asymmetric information. The VC’s first-order condition is
\[
f'(c_0) = \frac{1}{\theta f(\hat{c}_1)P_0}.
\]
There are two cases. First, let condition (A-20) be binding. Then $f(c_0)(f(\hat{c}_1)\Delta X - \hat{c}_1 - \frac{R}{2} - k_1) - k_0 = f(c_0)f(\hat{c}_1)P_0$ and $f'(c_0) = \frac{1}{\theta f(\hat{c}_1)P_0}$ determine the optimal $P_0^*$ and $c_0^*$. In this case, $(a_0^*, b_0^*)$ are solved by using condition (A-21). Second, let condition (A-20) be not binding. In this case, the optimal $P_0^*$ and $c_0^*$ is determined by taking first-order condition of condition (A-24), $f'(c_0)(f(\hat{c}_1)\Delta X - \hat{c}_1 - \frac{R}{2} - k_1) - (1 - \theta) f'(c_0)f(\hat{c}_1)P_0 = 0$, and $f'(c_0) = \frac{1}{\theta f(\hat{c}_1)P_0}$. In this
case, $a_0$ and $b_0$ are solved using condition (A-21). $c_0 < \tilde{c}_1$ follows by comparing conditions (A-20) and (13), as in the proof of Proposition 6 (i) (e).

Next, we check the entrepreneur’s incentive to deviate. The out-of-equilibrium belief is that a deviating firm is of type B firm. If a type G firm deviates and offers another contract to the VC, the VC would exert zero effort. By our assumption (35), in this case the type G firm is worse off. The type G firm entrepreneur clearly has no incentive to seek angel financing either, by condition (35). The type B entrepreneur would not seek angel financing since $\lambda \delta > \frac{R}{2}$. Comparing the type B firm’s payoff if it offers an out-of-equilibrium contract to the VC and thereby reveals his type to his equilibrium payoff, we find that type B firm has no incentive to deviate if and only if $\lambda \delta - \frac{R}{2} \leq \frac{R}{2} + f(c_0^*)\theta f(\tilde{c}_1)P_0^* - c_0^*$. By the VC’s IC constraint (A-19), we get $f(c_0^*)\theta f(\tilde{c}_1)P_0^* - c_0^* \geq 0$. Therefore, the type B firm will not deviate. Finally, similar to the proof of Proposition 6, it can be shown that the entrepreneur’s IR, limited liability, and budget constraints at time 0 and time 1 are satisfied in equilibrium. The condition corresponding to condition (A-17) is:

$$I \leq \min\{\bar{X} - \left[\frac{R}{2} + \tilde{c}_1 + \frac{R}{2} + c_0^* + (1 - \lambda - \theta f(c_0^*))f(\tilde{c}_1)P_0^* - P_1^*(q + f(\tilde{c}_1))\right],$$

$$q\bar{X} + (1 - q)\bar{X} - \left[\frac{R}{2} + c_0^* - (\lambda + \theta f(c_0^*))P_0^*\right].$$  \hspace{1cm} (A-25)

### A.8. Proof of Proposition 8

The out-of-equilibrium belief is that the firm is of type B firm. Clearly the type B firm has no incentive to deviate since, if it does, by assumption (36), the marginal benefit of VC financing, $\lambda \delta$, is lower than the marginal cost, $\frac{R}{2}$, and the firm is worse off. If the type G firm deviates by seeking VC financing, the VC would believe that he is of type B and thus not exert any effort. Therefore, the firm is worse off because it has to pay the VC an additional amount, $\frac{R}{2}$. The entrepreneur’s IRs, limited liability, and budget constraints are satisfied by condition (A-17). The conditions in part (ii) ensure that the angel’s contract has enough power (upside) to raise sufficient fund for investment at time 0, but not too much power to prevent efficient contracting at time 1, as stated in part (iii). If the contract is too powerful, the angel would capture too much value created by the VC and the entrepreneur in state $p$ at time 1. As a result, the entrepreneur will not be able to capture any benefit from VC financing and thus have no incentive to seek VC financing and buy out the angel in the first place. In short, a too powerful angel contract at time 0 prevents efficient value creation by the VC at time 1 as described in Section 3.2. If, however, the conditions in part (ii) are satisfied, then efficient value creation by the VC will proceed. The entrepreneur is thus indifferent between various contracts that satisfy these conditions.

### A.9. Proof of Proposition 9

This follows directly from Propositions 5, 6, and 7.

### A.10. Proof of Proposition 10

This follows directly from Propositions 5, 6, 7, and 8.

### Appendix B. Numerical illustration of the dynamic evolution of financing contracts

In this section we give a numerical example illustrating the situations we characterized before. We illustrate only the moderate-scarcity regime of VC financing. Let the model...
parameters be the following: \( \theta = 0.05, \lambda = 0.1, I = 107, q = 0.2, k_0 = 1, k_1 = 55. \frac{X}{k} = 20 \) and \( I_0 = 101. \) Further, the VC’s production function is: \( f(c) = 0.651, b^*_1 = 0.729 \) (note that \( b^*_1 < b^*_1 \) here). This contract can be implemented by convertible preferred or convertible debt; it cannot be implemented using common equity alone, or using equity plus call options (warrants). The firm raises another additional amount of investment, \( I - I_0 = 6, \) from the VC at time 1. The VC swaps the old (time 0) contract for a new contract at time 1. The VC will exert an amount of effort \( \hat{c}_1 = 17.29, \) and the entrepreneur will choose to exert high effort. The power of the contract \( (d^*_cX - b^*_1 X) = 122.246. \)

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36 We assume that \( f(c) = f(c^B) \) for \( c > c^B, \) so that the probability of project success in the model is always well defined—that is, less than one.

37 This is only for concreteness. Since the equilibrium in the moderate-scarcity regime is separating, the type B firm is indifferent to the amount raised as long as it is at least 101 (see Proposition 6 (ii)).
Case-2: The firm is in state $n$.

The VC will leave the firm at time 1. The firm therefore has to raise new financing from an angel. The angel will have to pay 104.694 to the firm. The firm will use 98.694 to buy out the VC and use the remainder to invest. This contract can be implemented using a variety of securities, including common equity $(a_1^{ps} = b_1^{ps} = 0.968)$, convertible preferred, or convertible debt, as long as the contract’s value is 104.694. The entrepreneur will exert low effort.

**Time 1 financing if the time 0 financing is done by an angel:**

Case-1: The firm is in state $p$.

By Propositions 3 and 4, we know that the firm will switch to VC financing and buy out the angel. The contract given to the VC is: $a_1^{ps} = 0.484, b_1^{ps} = 0.999$ (here, $a_1^{ps} < b_1^{ps}$). The power of the contract is 45.299; notice that the power of the contract is less here than for a previously VC-financed firm. This contract can be implemented by convertible preferred, but this time, the face value is $0.999 \times 100 = 99.99$. The firm raises from the VC 108.946. The firm pays the angel 102.946 to buy out his contract and invests an amount $I - I_0 = 6$. The VC will exert effort $c_1 = 5$, and the entrepreneur will also choose to exert high effort at a cost of 55.

Case-2: The firm is in state $n$.

The firm will continue to use angel financing in this case. The firm raises an additional amount of investment $I - I_0 = 6$ from the angel. The contract given to the angel can take a variety of forms, including common equity $(a_1^{ps} = b_1^{ps} = 0.763)$, as long as its value is 106.784. The entrepreneur will exert low effort in equilibrium.

Finally, in this equilibrium, the angel finances 95% of the earlier-stage firms, while VCs finance the remaining 5%. Further, among the later-stage firms financed by the VC, 74.8% are previously angel-financed firms financed by VCs for the first time, while 25.2% are second-round financings of firms previously financed by the same VC.

References


Venture Capitalists Versus Angels


