A Theory of Corporate Boards and Forced CEO Turnover

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ABSTRACT

We develop a theory of corporate boards and their role in forcing CEO turnover. We consider a firm with an incumbent CEO of uncertain management ability and a board consisting of a number of directors whose role is to evaluate the CEO and fire her if a better replacement can be found. Each board member receives an independent private signal about the CEO’s ability, after which board members vote on firing the CEO (or not). If the CEO is fired, the board hires a new CEO from the pool of candidates available. The true ability of the firm’s CEO is revealed in the long run; the firm’s long-run share price is determined by this ability. Each board member owns some equity in the firm, and thus prefers to fire a CEO of poor ability. However, if a board member votes to fire the incumbent CEO but the number of other board members also voting to fire her is not enough to successfully oust her, the CEO can impose significant costs of dissent on him. In this setting, we show that the board faces a coordination problem, leading it to retain an incompetent CEO even when a majority of board members receive private signals indicating that she is of poor quality. We solve for the optimal board size, and show that it depends on various board and firm characteristics: one size does not fit all firms. We develop extensions to our basic model to analyze the optimal composition of the board and the effect of board members observing imprecise public signals in addition to their private signals on board decision-making. Finally, we develop a dynamic extension to our basic model to analyze why many boards do not fire CEOs even when they preside over a significant, publicly observable, reduction in shareholder wealth over a long period of time. We use this dynamic model to distinguish between the characteristics of such boards from those that fire CEOs proactively, before such shareholder wealth reductions take place.
1 Introduction

As one CEO said, “I often say my directors can come in and vote one of two ways — either ‘yes’ or ‘I resign’.”

Barbara Lyne, “The Executive Life,”
New York Times, January 2, 1992

The seeming inability of corporate boards to monitor the performance of firm management proactively, and thus prevent corporate crises, has become the topic of debate among both practitioners and academics over the last few years in the wake of various corporate scandals. Citing several case studies where boards allowed CEOs to destroy shareholder value over a number of years before pressuring the CEOs to resign, Monks and Minow (1995) ask: “Why does it take boards so long to respond to deep-seated competitive problems? And, if one of the leading responsibilities of directors is to evaluate the performance of the CEO, why do boards wait too long for proof of managerial incompetence before making a move?” A number of related questions also arise in the above context. First, are there situations where boards do not fire CEOs, even when a majority of board members are individually convinced that the CEO is of poor quality? In particular, what are the characteristics of boards that fire poor quality CEOs proactively versus waiting until considerable destruction in shareholder has occurred? Second, what is the relationship between board size and the effectiveness of board decision-making in terms of monitoring the CEO? Third, what is the relationship between board composition (the proportion of outside directors) and the effectiveness of board decision-making and forced CEO turnover? Finally, how will the various policy proposals for corporate governance and board reform that have been suggested by academics, practitioners, and policy-makers affect the ability of the board to appropriately monitor the CEO and fire him or her as necessary?

While there has been little research on the dynamics of board’s CEO firing decisions, there has been a large empirical literature in finance addressing some of the other questions we have raised above. However, the above evidence has been mixed. For example, while some papers argue that smaller boards of directors are more effective and therefore maximize shareholder value (e.g., Yermack (1996)), others challenge this notion, documenting that, depending on various firm characteristics, either large or small board sizes may be

An exception is Ertugrul and Krishnan (2009), who empirically study the characteristics of boards that fire CEOs proactively, before the firm sustains a significant loss in shareholder value.
appropriate from the point of view of enhancing shareholder value (see, e.g., Coles et al. (2008)). Similarly, the evidence is also inconclusive on the question of whether a larger proportion of inside directors enhances shareholder value: see, e.g., Linck et al. (2008), Boone et al. (2007), Coles et al. (2008). While theoretical models of corporate boards (see, e.g., Hermalin and Weisbach (1998), Harris and Raviv (2008), Adams and Ferreira (2007)) have helped to resolve some of the above ambiguities by providing guidance for empirical research, a number of interesting questions remain unanswered. The objective of this paper is to provide answers to such questions by developing a theoretical analysis of corporate boards and their role in forcing CEO turnover based on trade-offs that have not been studied before in the literature.

We consider a firm with an incumbent CEO, whose quality is uncertain, and a board consisting of a number of directors whose role is to evaluate the CEO and decide whether to fire her or to retain her. Each board member receives an independent private signal about the CEO’s quality, after which board members vote regarding whether or not to fire the CEO. While we consider other voting rules, the bulk of our analysis is carried out under the assumption that a majority of votes is required to fire the CEO. If the CEO is fired, the board hires a new CEO from the pool of candidates available. The quality of the firm’s CEO is revealed in the long run; the firm’s long-run share price is determined by the quality of its CEO. We assume that each board member owns some equity in the firm, so that they have an incentive to fire a bad CEO and hire a good one (or retain a good CEO), based on their assessment of the CEO’s quality. However, if a board member votes to fire the incumbent CEO but fails to oust him, the CEO can impose significant costs of dissent on him (by not re-nominating that director to the board or otherwise imposing costs on him even if he continues on the board).2

We first study the benchmark equilibrium where the board members’ dissent-costs are zero. We show that, in the absence of dissent-costs, each board member votes informatively (i.e., they vote to fire the CEO if they get a bad signal and vote to retain her if they get a good signal). Further, in the absence of dissent-costs and if the board’s voting rule is optimal, we show that the board’s firing decision efficiently aggregates all board members’ private signals, in the sense that the board fires the CEO only if, in the opinion of a social

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2Mace (1971) discusses anecdotal evidence of CEOs exercising authority in selecting candidates for the board, in effect hand-picking nominees. Similarly, Lorsch and MacIver (1994) report survey evidence indicating that CEOs wield major influence in selecting new board members. Tejada (1997) presents a news account of an outside director of a prominent company being denied nomination for reelection after criticizing management.
planner who has access to all board members’ signals, the incumbent CEO is worse (in terms of expected quality) than a potential replacement; the board retains the CEO is the reverse is true. We also show that, under reasonable conditions, a simple majority voting rule is optimal in the above sense.

We then analyze our basic model with dissent-costs. We show that, once such dissent-costs are significant, coordination problems arise between board members, so that the board may become suboptimally passive. In particular, the board may choose to retain the CEO even when a majority of board members are privately convinced that the CEO is of poor quality. This is because, even when each board member has a bad signal about the CEO’s quality, his probability assessment that enough other board members will vote against the CEO (and thus will be able to successfully fire the CEO and avoid incurring dissent-costs) is not large enough, so that the board member is better off voting to retain the CEO. In this case, we show that whether each board member votes informatively or not depends on how well they are incentivized: if each board member’s equity holdings in the firm is large enough, so that his dissent-cost to equity loss ratio is low, then he votes informatively; he votes to retain the CEO even if he receives a bad signal about the CEO’s ability if this ratio is high.

We then analyze the effect of board size on the quality of the board’s decision-making. On the one hand, since each board member’s signal about the CEO’s quality is independent, a larger number of board members increases the amount of information that is potentially available (collectively) to the board. On the other hand, a larger board worsens the coordination problems across board members, since in a larger board, a larger number of votes is required to fire the CEO. We show that the optimal board size emerges from the above trade-off. Further, this optimal board size depends on the precision of each board member’s signal and his dissent-cost to equity loss ratio. Our results contradict the hypothesis of Lipton and Lorsch (1992) and Jensen (1993) that larger boards are always less effective, and sheds light on the findings of the recent empirical literature, which documents that while smaller board sizes are value maximizing for simpler firms, larger board sizes seem to be optimal for more complex firms (with greater information requirements for evaluating the CEO’s performance): see, e.g., Coles et al. (2008).

In section 4, we extend our basic model to analyze issues of optimal board composition. Here we distinguish between insider board members and independent outsiders: while outsiders are similar to the board members in our basic model (who receive independent private signals about the CEO’s quality, and who cannot credibly communicate their signals
to each other prior to voting), insiders’ signals are correlated with each other, and they can credibly communicate their private signals to one another. The advantage of insider dominated boards is that they do not suffer from coordination problems; the disadvantage is that, since insiders’ signals are correlated, the board is effectively making a decision based on much less information compared to outsider dominated boards. By the same token, the advantage of outsider dominated boards is that, since each outsider has an independent signal, such boards have a much greater amount of information (collectively) available to them; the disadvantage of outsider dominated boards is that, since outsider directors (similar to directors in our basic model) suffer from coordination problems, they may not be able to use this information efficiently in many situations.

The optimal composition of boards emerges from the above trade-off between having a larger number of outsiders versus insiders: while in some situations outsider dominated boards are optimal, in other situations a board consisting of a combination of insiders and outsiders is optimal; insider dominated boards are optimal in yet other situations. We show that the optimal composition of the board depends on the dissent-cost to equity loss ratio of board members, and the informational requirements of evaluating the CEO’s performance in managing the firm. Our results thus contradict the conventional wisdom that outsider dominated boards are always value-maximizing. Further, they explain the recent empirical evidence documenting that, while a larger number of outsiders is value-maximizing for complex firms with greater informational requirements in evaluating and monitoring the CEO (see Coles et al. (2008)), this is not necessarily the case for simpler firms and those where firm-specific knowledge (likely to be available to insiders) is more important (see Linck et al. (2008) and Coles et al. (2008)). It also explains the evidence in Boone et al. (2007), who document that the proportion of independent outsiders on corporate boards is negatively related to measures of the CEO’s influence over the board (which increase dissent-costs) but positively related to the equity ownership of outside directors (which increase their equity loss).

In section 5, we extend the basic model in a different direction: we study the effect of a public signal on the coordination problem among directors in the basic model. One

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3Given that insiders, by definition, are connected to the firm and the incumbent CEO, it seems to be natural to assume that insiders’ signals about the CEO’s quality will be correlated. For the same reason, it is likely that it is easier for insiders (given their common background and potential familiarity with each other) to credibly communicate their signals to each other compared to the case for outsiders (who may come from diverse backgrounds, and may be relatively unfamiliar with each other; outsiders may have competing interests as well, making credible communication difficult).
might at first think that publicly observable information regarding the CEO’s quality (for example, the accounting performance of the firm under the CEO’s management) is a natural coordination mechanism. In order to examine this question, we assume in this section that, in addition to their own private signals, all directors observe a noisy public signal of the CEO’s quality, before voting.\footnote{Our assumption here is that the public signal is imprecise, so that it does not make the board members’ private signals redundant. Of course, if the public signal is entirely precise, all board members will ignore their private signals and will vote to fire the CEO after they observe a negative public signal (for example).} In this setting, we show that the effect of the signal on board decision-making is asymmetric. On the one hand, if is indeed the case that a negative public signal improves board decision-making, in the sense that board members vote informatively for ranges of the dissent-cost to equity loss ratio where, in the absence of such a signal, they would have voted to retain the CEO (ignoring their private signals). On the other hand (and more interestingly), a positive public signal worsens board decision-making, in the sense that board members vote to retain the CEO in the presence of such a signal for values of the dissent-cost to equity loss ratio for which they would have chosen to vote informatively otherwise. The intuition here is that, when a board member receives a negative private signal but a positive public signal, he may be prevented from voting against the CEO, since, given the positive public signal, he assesses a greater probability that many other board members vote to retain the CEO, thus making it impossible to oust her. In summary, we show that having noisy public measures of CEO performance available to the board will not always improve board decision-making, since such signals can sometimes make the board members’ coordination problem worse rather than better.

Finally, in section 6 we develop a dynamic version of our basic model to address the question we raised at the beginning of this paper, namely, why many corporate boards wait too long in the face of shareholder value destruction before firing the CEOs involved, and the difference between the characteristics of such boards and those that fire CEOs proactively before significant destruction in shareholder value takes place. In this dynamic model, we introduce a second round of voting on the incumbent CEO (if she was not fired as a result of the first round of voting), with a highly informative public signal (such as significant destruction in shareholder value or not) between the two rounds of voting. We also assume that the equity loss of the board is significantly higher if a low quality CEO is fired only after board members receive the public signal (consistent with the higher loss in the value of their equity holdings that board members would sustain compared to the case where they replace the low quality CEO earlier). In this setting, we show that the
coordination problem faced by board members may explain the board’s reluctance to act to remove CEOs until considerable value destruction has taken place. The dynamic trade-off faced by board members is the following. On the one hand, as time goes by, the firm’s share price goes further and further down, thus increasing the board members’ equity loss. On the other hand, as more and more evidence of the CEO’s incompetence arrives (publicly) over time, each board members’ assessment of a sufficient number of other members also voting against the CEO to oust her increases, thus reducing their own probability of incurring a dissent-cost. The timing of the CEO’s dismissal emerges from this dynamic trade-off between expected equity loss and expected dissent-cost. We show that boards that are better incentivized (in terms of dissent-cost to equity loss ratio) will fire CEOs earlier, before significant shareholder value destruction has taken place. The predictions of our dynamic model explain the empirical findings of Ertugrul and Krishnan (2009), who document that boards that dismiss CEOs early (i.e., in the absence of significant negative prior stock returns) are those that are characterized by higher equity ownership by board members, higher board equity compensation as a fraction of total compensation, higher institutional ownership, and better firm corporate governance characteristics.

There have been several theoretical models of corporate boards in the existing literature, driven by trade-offs different from the ones we analyze here. The seminal theoretical analysis in this literature was by Hermalin and Weisbach (1998). In their model, board structure is the outcome of negotiation between the CEO and outside directors. CEOs who generate surplus for their firms (for whom good substitutes are unavailable) wield considerable influence over their outside directors and use this influence to capture some of the surplus they generate by placing insiders in open board positions. Adams and Ferreira (2007) develop a model in which the CEO’s preferred projects, which yield him control benefits, differ from those of shareholders. The CEO faces a trade-off in disclosing information to the board: if he reveals his information, he receives better advice from it, but may lose control benefits, since an informed board will also monitor him more intensively. They thus show that management-friendly boards, which can pre-commit not to use the information provided by the CEO to monitor him, may be value-maximizing for the firm.

Harris and Raviv (2008) develop a model of a corporate board consisting of both insiders and outsiders and which can profitably use the information held by both insiders and outsiders to make optimal project choices. Insiders have private information relevant to this choice, but have private benefits that lead their incentives to be misaligned with those of shareholders; outsiders, whose interests are perfectly aligned with those of shareholders,
are initially uninformed but can engage in producing information relevant to project choice at a cost. Control of the board entitles the controlling party (insiders or outsiders) either to make decisions themselves or to delegate decisions to the other party. In this setting, they characterize the conditions under which there will be insider versus outsider control of the board, when each party will delegate decision-making to the other party, the extent of communication between the two parties, and the number of outside directors. Raheja (2005) also models a board consisting of both insiders and outsiders in charge of project selection, where insiders have decision-relevant private information, have private benefits that distort their incentives, and where outsiders can engage in costly production of information relevant to project choice. Outsiders’ information costs are reduced if insiders reveal their private information. She argues that since insiders are the source of future CEOs in her model, outsiders can use their CEO succession votes to motivate insiders to reveal their private information. She solves for the combination of insiders and outsiders on the board that leads to optimal project selection.

While some of the above models also address two of the central issues regarding corporate boards that we study here, namely, board size and optimal board composition, the above models address these questions based on trade-offs quite different from that in our model. Further, none of these models study issues related to the effect of public signals on board decision-making, and the dynamics of CEO firing decision that we study here. Our paper is also related to the large empirical literature on corporate boards and the relation between boards and firm characteristics, which we discuss in section 7 in the context of the empirical and policy implications of our model: see Hermalin and Weisbach (2003) for an excellent review of the literature on corporate boards.

The rest of the paper is organized as follows. Section 2 describes the set-up of our basic model. Section 3 characterizes its equilibrium and analyzes issues related to optimal board

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5Warther (1998) develops a simple model of a two-member board in which the CEO can eject hostile board members, which can be viewed as the imposition of a dissent-cost on board members who disagree with the CEO. He shows that this results in board members being reluctant to oppose management and votes being often unanimous in favor of management. However, given that he exogenously assumes a two-member board, he is unable to study any of the issues of coordination among board members that we analyze here; neither does he analyze optimal board size, board composition, the effect of public signals, or the dynamics of CEO turnover. Almazan and Suarez (2003) show that passive (weak) boards may be optimal in a setting where severance pay and weak boards are substitutes for costly incentive compensation paid to the CEO. Gillette et al. (2003) show both theoretically and experimentally that when agency problems are especially severe, having uninformed outsiders in control of a board can prevent inefficient outcomes. Hirshleifer and Thakor (1994) model the interaction between internal (corporate boards) and external (acquisitions) governance mechanisms in the maintenance of managerial quality.
size. Section 4 extends our basic model to analyze optimal board composition. Section 5 extends our basic model to study the effect of an imprecise public signal on the effectiveness of board decision-making. Section 6 presents a dynamic extension of our basic model to study the timing of forced CEO turnover. Section 7 discusses the empirical and policy implications of our model. Section 8 concludes. The proofs of all propositions are confined to the appendix.

2 The model

Our basic model consists of four dates. It starts at time 0 with an incumbent CEO in place. The CEO can be of either good or bad quality (denoted G for good and B for bad). The board consists of \( n \) directors and does not include the CEO (we will use the terms board member and director interchangeably throughout the paper). At time 1 each of \( n \) board members obtains a signal, \( s_i, i = 1, \ldots, n \), about the CEO’s quality. At time 2, based on the signals they received at time 1, board members vote whether to keep the current CEO or to replace her. If the board fires the CEO, it must hire a replacement from the pool of available candidates. The quality of this pool is indexed by \( \gamma \): \( \gamma \) is the ex-ante probability of hiring a high-quality replacement. At time 3 the true quality of the then-incumbent CEO is revealed (either of the CEO existing at time 0 if she was not fired or the new CEO hired at time 2). The game ends and all payoffs are distributed. The timeline of the model is depicted in figure 1. All agents are risk-neutral.

The signal that each board member receives can be either good, denoted \( g \), or bad, denoted \( b \). The signals are assumed to be independent across directors and informative. In particular, \( p(s_i = g|G) = p(s_i = b|B) = \alpha > \frac{1}{2} \). Thus, \( \alpha \) gives the precision of each board member’s signal about the CEO’s quality. We assume that all board members have signals of identical precision. The prior probability assessment of a board member that the incumbent CEO is good is \( \mu \).

2.1 The board members’ objective

The objective of each board member in making his voting decision is to retain the CEO if she is of good quality and fire her if she is of bad quality (and hire a good quality candidate).

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6This assumption is introduced for ease of exposition and is in innocuous if one assumes that the CEO always votes to retain herself. Adding the CEO to the board in this case will simply decrease the effective board size to \( n - 1 \).
replacement if the incumbent CEO is fired by the board) but only to the extent that this affects his own costs and benefits, as described below.\textsuperscript{7} We assume without loss of generality that the maximum payoff a director can receive is 0.\textsuperscript{8} If it is revealed at time 3 that the CEO chosen by the board is of bad quality, each board member suffers a cost, $q, q > 0$. $q$ is the loss in value to directors from making a bad decision that arises from the reduction in the value of the firm’s equity held by individual board members (due to the mismanagement of the firm by a low quality CEO). We will refer to $q$ from now on in the the directors’ “equity loss.” Under this interpretation, one can think of the equity loss as being proportional to the difference in the long-run value of the firm’s equity under a good quality CEO versus a bad quality CEO. Thus, one way to increase a board member’s equity loss would be to increase the equity fraction in his compensation. $q$ can also be thought of as reputational damage or a direct cost imposed by shareholders on the members of a board which retains a bad CEO.

In addition to the above cost of choosing a low quality CEO, each board member may suffer a cost of dissent. In particular, if a director votes to fire the CEO but fails to receive enough support from fellow board members, he suffers a cost, $c, c > 0$. This “dissent-cost” can be thought of as potential punishment imposed by the CEO (such as the

\textsuperscript{7}If good quality CEOs can generate higher long-run cash flows for the firm than bad quality CEOs, this translates into the assumption that the board wishes to maximize the firm’s long-run value net of personal costs incurred.

\textsuperscript{8}This is a normalization. If we assumed a fixed amount here (e.g., representing the board member’s compensation that is independent of firm value), our results would go through qualitatively unchanged.
CEO’s reluctance to nominate the director in the next election cycle) or as the reputational
damage from being perceived as a “troubemaker” by other directors on this board or by
other boards who refuse to grant this director a seat. What matters to individual directors
is the magnitude of \( c \) relative to \( q \). We will refer to \( \frac{c}{q} \) as the directors’ dissent-cost to equity
loss ratio.

Director \( i \)’s vote is denoted by \( v_i \). We say that \( v_i = 0 \) if director \( i \) votes to keep the
CEO and \( v_i = 1 \) if director \( i \) votes to fire the CEO. Given the above, director \( i \)’s objective
when making his voting decision, denoted by \( \pi_i \), is given by:

\[
\pi_i(v_i; s_i, v_{-i}, k, n, c, q, \alpha, \gamma, \mu) = - \left\{ I\left( \sum_{j=1}^{n} v_j \geq k \right) q(1 - \gamma) + I\left( \sum_{j=1}^{n} v_j < k \right) qp(B|s_i) \right\} \]

\[ - I\left( \sum_{j=1}^{n} v_j < k \right) cv_i, \]

where \( I \) denotes an indicator function in each term (it takes the value of 1 if the relation
in parenthesis is satisfied and 0 otherwise), \( v_{-i} \) is the voting configuration of all directors
except \( i \), and \( p(B|s_i) \) denotes the probability that the current CEO is of low quality given
director \( i \)’s signal. The terms in curly brackets in (1) represent the cost of choosing a bad
CEO. The first term is the expected cost of choosing a bad CEO when the current CEO is
fired: the corresponding indicator function, \( I\left( \sum_{j=1}^{n} v_j \geq k \right) \), takes on the value of 1 when
there are enough votes to oust the CEO (\( \sum_{j=1}^{n} v_j \geq k \)). In that case, the probability that
the replacement is of low quality is given by \( 1 - \gamma \), and hence the expected equity loss is
\( -q(1 - \gamma) \). The second term in curly brackets is the expected equity loss from retaining
the incumbent: the corresponding indicator function takes on the value of 1 when there are
not enough votes to oust the CEO (\( \sum_{j=1}^{n} v_j < k \)). In that case, the probability that
the board member will suffer equity loss is given by \( p(B|s_i) \) so that the expected equity
loss is \( -qp(B|s_i) \). The last term in (1), outside the curly brackets, represents the expected
dissent-cost. A board member incurs it when two conditions are satisfied: (i) he votes
to fire the CEO (\( v_i = 1 \)), and (ii) there are not enough votes to oust the incumbent
(\( \sum_{j=1}^{n} v_j < k \), so that the corresponding indicator function takes on the value of 1). For
example, if board member \( i \) votes to fire the CEO but fails to attain enough support from
other board members, his expected payoff will be \( -qp(B|s_i) - c \).
2.2 The board members’ voting rule

Each director decides how to vote based on his updated assessment (based on his prior probability \( \mu \) and his private signal) of the quality of the CEO, his conjecture about the voting behavior of other directors (since this affects his chance of being on the winning side if he chooses to vote to fire the CEO, for example), and his equity loss and dissent-cost, which together determine the value of his objective (1). Using Bayes’ rule, each board member’s updated probability assessment of the CEO being of bad quality conditional on his private signal, denoted \( p(B|s_i) \), is given by:

\[
p(B|g) \equiv p(B|s_i = g) = \frac{(1 - \mu)(1 - \alpha)}{(1 - \mu)(1 - \alpha) + \mu \alpha}, \quad (2)
\]

\[
p(B|b) \equiv p(B|s_i = b) = \frac{(1 - \mu)\alpha}{(1 - \mu)\alpha + \mu(1 - \alpha)}. \quad (3)
\]

The assumption of informative signals guarantees that

\[
p(B|b) > 1 - \mu > p(B|g). \tag{4}
\]

**Definition 1.** Voting rule, \( k \), is the minimum number of votes required to fire the CEO.

**Definition 2.** Voting is sincere if \( v_i = 1 \) iff \( p(B|s_i) > 1 - \gamma \).

**Definition 3.** Voting is informative if \( v_i = 1 \) iff \( s_i = b \).

Sincere voting implies that each board member will vote to fire the CEO if after observing the private signal, this director’s assessment of the CEO’s quality is lower than that of the potential replacement. Informative voting implies that the director reveals his private information: i.e., the board member will vote in the direction of his private signal about the CEO’s quality. We will assume throughout that \( p(B|b) > 1 - \gamma > p(B|g) \); otherwise, sincere voting can never be informative. Consider, for instance, the case when \( p(B|b) > p(B|g) > 1 - \gamma \). Then, even after observing private signal \( g \), a director’s private assessment of the incumbent CEO’s quality is lower than of a potential replacement. With this assumption, on the other hand, informative voting is always sincere.

The board makes decisions according to a voting rule defined above. Assume that there exists a benevolent social planner who can observe all board members’ private signals. Assuming that the number of directors who observe a bad private signal is \( r \), the social
planner’s updated probability of the incumbent’s quality is given by

\[ p(B|r) = \frac{(1 - \mu)\alpha^r(1 - \alpha)^{n-r}}{(1 - \mu)\alpha^r(1 - \alpha)^{n-r} + \mu(1 - \alpha)^r\alpha^{n-r}}. \]  \hspace{1cm} (5)

The social planner would therefore fire the incumbent if and only if

\[ \frac{(1 - \mu)\alpha^r(1 - \alpha)^{n-r}}{(1 - \mu)\alpha^r(1 - \alpha)^{n-r} + \mu(1 - \alpha)^r\alpha^{n-r}} > 1 - \gamma. \]  \hspace{1cm} (6)

Let \( r^* \) be the smallest integer that makes (6) hold. We then define the optimal voting rule as the one that sets \( k = r^* \). In other words, under the optimal voting rule, the CEO is fired only when she is considered worse than outside replacement, given all directors’ cumulative private information. Notice that \( r^* \) generally depends on the parameters of the situation. However, in an important special case that may be applicable to real life boards, simple majority is the optimal voting rule.

**Proposition 1. (Optimal voting rule).** Assume that \( \mu = \gamma \) and that \( n \) is odd. Then, the optimal voting rule is given by \( k = \frac{n+1}{2} \) (simple majority).

It seems feasible that in real-life board considerations the quality of potential replacement is often comparable with board members’ prior about the underlying quality of the current CEO.

### 3 Equilibrium of the basic model

In this section we characterize the equilibrium of our basic model. The equilibrium concept we use is symmetric pure strategy Efficient Bayesian Nash equilibrium. An equilibrium consists of: (i) a vote by each director (either to fire or to keep the current CEO); (ii) each director’s conjecture about the signals of the other directors. Each director’s vote maximizes his expected payoff given his beliefs about the distribution of signals among the other board members and their voting strategies.

\(^9\)Our definition of an optimal voting rule parallels the existing voting literature: see, e.g., Austen-Smith and Banks (1996).
3.1 The benchmark equilibrium with no dissent-costs

Before we solve for the equilibrium of our basic model and its enriched versions (in later sections), we consider the benchmark case in which each director’s dissent-cost, $c$, is zero, i.e., the case when the only cost imposed on board members is the equity loss. The following proposition describes the equilibrium in this case.

**Proposition 2.** (Benchmark Equilibrium).

(i) The benchmark case described above has a unique equilibrium, in which each board member votes informatively. In particular, board member $i$ votes to keep the CEO iff $s_i = g$ and to fire the CEO iff $s_i = b$.

(ii) A board that adopts an optimal voting rule efficiently aggregates all board members’ information in the absence of dissent-costs, in the sense that if the board decides to retain the CEO, her expected quality is better than that of a potential replacement; if the board decides to fire the CEO, her expected quality is worse than that of a potential replacement.

This equilibrium achieves the efficient outcome preferred by a benevolent social planner. Since all directors vote informatively and the voting rule is chosen optimally, the outcome of this vote is equivalent to aggregating all directors’ private information in making the final decision of the board regarding whether to fire the incumbent CEO or to retain her.

Another important observation concerns the optimal board size. If the board effectively aggregates private information, it is clearly optimal to have as many independent signals as possible. This logic is made precise in the following proposition.

**Proposition 3.** (Optimal board size without dissent-costs). The quality of the social planner’s decision is increasing in the number of board members, $n$.

Proposition 3 says that when board members are willing to reveal their private valuation it is better to have as many of them as possible, since this increases the amount of information collectively available to the board (or the social planner, in the benchmark case).

3.2 Equilibrium of the basic model with dissent-costs

We now characterize the equilibrium of our model in the presence of dissent-costs, which leads to imperfect coordination among board members.
Proposition 4. (Equilibrium of the basic model). Index board members by \(i\) and let \(e_i = 1\) if the \(i^{th}\) board member observes a bad signal \((s_i = b)\); \(e_i = 0\) if the signal is good \((s_i = g)\). Then the game described above has a unique equilibrium, so that:

(i) If the dissent-cost to equity loss ratio \(\frac{\xi}{q} \leq \Upsilon(k, n, \alpha, \gamma, \mu | b)\), each director votes informatively, in particular, director \(i\) votes to retain the CEO iff \(s_i = g\) and to fire the CEO iff \(s_i = b\);

(ii) If \(\frac{\xi}{q} > \Upsilon(k, n, \alpha, \gamma, \mu | b)\), each director votes to retain the CEO regardless of his private signal, where

\[
\Upsilon(k, n, \alpha, \gamma, \mu | b) = \frac{p\left(\sum_{j=1}^{n} e_j \geq k | b\right)\left(p(B | b) - (1 - \gamma)\right)}{p\left(\sum_{j=1}^{n} e_j < k | b\right)}
\]

and all conditional probabilities represent beliefs of director \(i\) given his signal.

The intuition for Proposition 4 is as follows. The benefit from keeping the current CEO for a director with a good signal is the chance to avoid the equity loss \(q\). Since directors who vote to keep the current CEO do not suffer the cost of dissent if the CEO is fired, they have nothing to lose by voting truthfully. It follows that board members who receive bad signals of the current CEO’s quality determine the outcome of the vote. If they vote to fire the CEO, they reduce the probability of suffering the equity loss \(q\). However, if they vote truthfully and fail to attain enough support they will suffer \(c\), the cost of dissent, regardless of whether the CEO is good or bad. Hence, if a director observes a signal \(b\), he faces the trade-off between equity loss and the dissent-cost. When \(c\) is high relative to \(q\), board members will choose to disregard their signals and vote to keep the current CEO. Hence, it is the ratio \(\frac{\xi}{q}\) that will determine whether board members vote informatively or not. The threshold value of \(\frac{\xi}{q}\) above which truth telling is impossible is given by

\[
\Upsilon(k, n, \alpha, \gamma, \mu | b) = \frac{p\left(\sum_{j=1}^{n} e_j \geq k | b\right)\left(p(B | b) - (1 - \gamma)\right)}{p\left(\sum_{j=1}^{n} e_j < k | b\right)}.
\]

Notice that (7) makes intuitive sense. \(\Upsilon(k, n, \alpha, \gamma, \mu | b)\) is positively related to the probability that enough directors observe bad signals, given that director \(i\) observes \(b\). It is also positively related to the probability that the current CEO is of low quality given that director \(i\) observes \(b\): the higher this probability, the more strongly directors with a bad signal believe that the incumbent’s quality is low. It is also positively related to the quality of outside replacement, \(\gamma\) (negatively related to \((1 - \gamma)\): a high expected quality of replacement makes it more worthwhile to fire the incumbent. On the other
hand, \( \Upsilon(k, n, \alpha, \gamma, \mu|b) \) is inversely related to the conjectured number of positive signals observed by board members (given director \( i \)'s own signal is \( b \)): if director \( i \) believes that only a small number of other board members obtained a bad signal, he understands that it is unlikely they will have enough votes to oust the current CEO, so that he will vote to retain the CEO even after observing a bad signal in order to avoid the dissent-cost \( c \).

It is natural to consider the ratio \( \frac{c}{q} \) to measure the board’s incentives.

**Definition 4.** Consider two boards, \( A \) and \( B \), and denote the corresponding dissent-costs and equity loss by \( c_A, q_A, c_B, \) and \( q_B \), respectively. We say that board \( A \) is better incentivized than board \( B \) if \( \frac{c_A}{q_A} < \frac{c_B}{q_B} \).

It is immediate from Proposition 4 that better incentivized boards are more likely to sustain informative voting (in the sense defined above). Our analysis emphasizes two sources of influence that a director faces: the CEO (and potentially other directors) via dissent-costs \( c \), and the shareholders and directors’ own shareholdings through the equity loss \( q \). Providing better incentives involves changes to either or both of those parameters.

**Proposition 5.** (Suboptimally passive boards). In the presence of dissent-costs, boards are likely to keep the incumbent CEO more often than is socially optimal.

In particular, if dissent-costs are sufficiently high relative to the equity loss the board will not fire the CEO even if the majority of directors observe a bad signal of CEO’s quality. This may explain the criticism of current boards in the financial press: in general, they are less likely to fire bad CEOs than is desirable. As long as some boards suffer from coordination problems, the average turnover decision in the economy will be suboptimal.

**Proposition 6.** (Large vs. small boards).

(i) When simple majority is the optimal voting rule and \( n \) is odd:

(a) if \( \mu > \alpha z(\alpha, n) \), then adding directors to the board decreases the threshold value of \( \frac{c}{q} \) above which informative voting is unsustainable,

(b) if \( \mu < \alpha z(\alpha, n) \), then adding directors to the board increases the threshold value of \( \frac{c}{q} \) above which informative voting is unsustainable,

(c) if \( \mu = \alpha z(\alpha, n) \), then adding directors to the board doesn’t change the threshold value of \( \frac{c}{q} \) above which informative voting is unsustainable, where

\[
z(\alpha, n) = \frac{2^{2n(1-\alpha)} - (n+1)}{2^{2n(1-\alpha)} - (n+1) + 2(1-\alpha)} < 1.
\]
(ii) In case of a different voting rule such that as the number of directors goes from \( n \) to \( n+1 \), the minimum number of votes required to fire the CEO changes from \( k \) to \( k+1 \), the threshold value of \( c/q \) above which informative voting is unsustainable decreases as the board size grows.

Proposition 6 implies, in particular, that when \( \alpha \leq \mu \), then larger board are less likely to sustain informative equilibria. The trade-off driving the relationship between board size and informative voting is as follows. On the one hand, a larger board brings more information into the decision making process. On the other hand, bigger boards increase the difficulty of coordinating across board members. Whether a larger board fosters or inhibits informative voting depends on whether the first effect dominates the second, or vice versa. If simple majority is the optimal voting rule (part (i) of Proposition 6), two cases need to be considered. When the signal available to an individual director is not very precise (\( \mu > \alpha z(\alpha,n) \)), then the second effect dominates the first, and informative voting becomes less likely when the board becomes larger. On the other hand, if the signal is very precise (\( \mu < \alpha z(\alpha,n) \)), then the first effect dominates and as the board’s size grows, so does the likelihood of informative voting.

The intuition behind part (ii) of the above proposition is best seen from the extreme case of a unanimous voting rule. If unanimity is the optimal voting rule, then larger boards are unequivocally less likely to provide efficient outcomes. In essence, in a larger board a director with a negative private signal needs to obtain support from a greater number of other board members in order to oust the CEO. Obtaining this additional support is inhibited by the presence of dissent-costs, which makes it less likely that directors will vote truthfully.

**Proposition 7.** *(Optimal board size).*

(i) If \( \alpha \leq \mu \) and majority is the optimal voting rule, then there exists an optimal board size, \( n^* \).

(ii) If unanimity is the optimal voting rule, then there exists an optimal board size, \( n^* \).

The trade-off driving the preceding proposition is the same as in Proposition 6. A larger board collectively has access to more information but size increases the difficulty of coordination among directors. Compare Proposition 3 to Proposition 6. The former states that larger boards obtain a more precise cumulative signal, conditional on having
all directors reveal their private valuation. The latter, however, provides conditions under which large boards inhibit informative voting. In general, boards must be large enough to obtain a reasonably precise signal but small enough to enable informative voting. More directors bring additional information into decision making. However, this will occur only if voting is informative. If they are unwilling to reveal their information, additional board members only exacerbate the coordination problem among board members.

**Proposition 8.** *(Comparative statics).*

(i) The threshold value of $\frac{c}{q}$ above which informative voting is unsustainable is decreasing in $\mu$.

(ii) If $\mu = \frac{1}{2}$, then the threshold value of $\frac{c}{q}$ above which informative voting is unsustainable is increasing in $\alpha$.

(iii) The threshold value of $\frac{c}{q}$ above which informative voting is unsustainable is increasing in $\gamma$.

Proposition 8 is quite intuitive. If the assessment of the incumbent’s quality is high, it is unlikely that there will be enough negative votes to oust her, even conditional on observing a bad private signal. When signals are more precise, directors are more confident about the quality of their information and are more willing to vote informatively. A high expected quality of a potential replacement makes it more worthwhile to fire the incumbent CEO since conditional on firing, the board is more likely to obtain a good CEO.

4 Insiders versus outsiders

One of the recurring topics in the literature on corporate boards is the relationship between insiders and outsiders on the board. In this section we explore the role that these types of directors play in the CEO turnover decision. As before, we assume that there are no public signals and that each director observes a private signal of the incumbent CEO’s quality.

We introduce two types of board members: insiders and outsiders. We assume that there are $h$ insiders and $n - h$ outsiders on the board. We define insiders as board members employed by the firm. Since insiders work in the same environment they are likely to have the ability to predict what signals other insiders observe. To keep the model tractable, we assume that this ability is perfect: i.e., insiders can see each other’s signals. At the
same time, the similarity of information received by insiders means that their signals are correlated. For simplicity of modeling, we make the extreme assumption that the correlation across insiders’ signals is perfect: i.e., all insiders receive either a good signal or a bad signal.\footnote{Our results would be unchanged if, instead of assuming perfect correlation across insiders’ signals, we were to assume that a fixed proportion of insiders, \( \rho \), \( 0 < \rho < 1 \), receive the same signal: If the CEO is of good quality, then with probability \( \alpha \), \( \rho h \) insiders observe a good signal, with \( (1 - \rho)h \) receiving a bad signal. Thus, \( \rho \) would represent the degree to which insiders as a group get similar signals. Similarly, if the CEO is bad, then with probability \( \alpha \), \( \rho h \) insiders observe a bad signal, with \( (1 - \rho)h \) receiving a good signal.} We continue to assume, however, the same precision of each insider’s signal as in the basic model. Thus, \( p(s_i = g | G) = p(s_i = b | B) = \alpha > \frac{1}{2} \). Outsiders have the same characteristics as board members in our basic model.

In summary, we introduce two differences between insiders and outsiders in this section. First, on the positive side, insiders do not suffer from coordination problems, unlike outsiders (who suffer from the inability to coordinate their voting behavior that we discussed for all directors throughout previous sections). On the negative side, however, since insiders’ signals are correlated, each insider does not bring as much information to the board’s decision as an outsider, who receives a signal that is independent (of other outsiders as well as insiders). All other characteristics (in terms of dissent-cost, equity loss and so on) are the same for insiders and outsiders, and various other assumptions in this section remain the same as in the basic model.

Let \( f \) denote insiders’ aggregate signal. In particular, we let \( f = b \) if all insiders observe a bad signal and \( f = g \) if insiders receive a good signal. Notice that the inference that insiders make is equivalent to having one director observe a signal based on their cumulative information. In particular,

\[
p(B | f = g) = \frac{(1 - \mu)(1 - \alpha)}{(1 - \mu)(1 - \alpha) + \mu \alpha} = p(B | g), \tag{8}
\]

\[
p(B | f = b) = \frac{(1 - \mu)\alpha}{(1 - \mu)\alpha + \mu(1 - \alpha)} = p(B | b). \tag{9}
\]

### 4.1 Equilibria

Before we discuss equilibria we note that due to their perfect coordination insiders will vote as a block. Since they observe each other’s signal, they aggregate this information to update their private valuation. In fact, after observing every other insider’s signal, any inside director has the same information as any other inside director. Hence, they cannot
vote differently in a symmetric equilibrium.

4.1.1 Equilibrium when the board is insider dominated

First we consider the case when insiders dominate the board \((h \geq k)\). Since insiders always vote as a block and their block is large enough to determine the outcome, the way outsiders vote is irrelevant. Hence, it is optimal for outsiders to always vote to keep the incumbent CEO. This way outsiders always avoid the cost of dissent. To see this clearly, consider the case when an outsider observes a bad signal of CEO’s quality. If he votes to fire the incumbent while insiders don’t, this outsider will suffer the cost of dissent. If, on the other hand, insiders vote to fire the CEO, his vote makes no difference: the CEO will leave even if this outsider votes to keep her. Thus, the payoff from voting to keep the CEO is always at least as high as the payoff from voting to fire the CEO even for outsiders with a bad signal. The following proposition characterizes the equilibrium in this case.

Proposition 9. (Equilibrium when insiders dominate the board). If insiders dominate the board \((h \geq k)\), they vote to fire the CEO if and only if \(f = b\). Outsiders always vote to retain the CEO, regardless of their private signals.

Proposition 9 implies that when insiders dominate the board, they completely ignore outsiders’ information and either retain the CEO or fire her in a “palace coup.” The fact that insiders dominate the board and vote as a block means that they can always avoid the cost of dissent. If they vote to fire the CEO, she is let go and insiders do not suffer any dissent-cost. However, a board dominated by insiders may act on very little information, since insiders’ signals are correlated with each other, and the board ignores any information held by outsiders.

4.1.2 Equilibrium when the board is outsider dominated

We now consider boards where insiders do not have enough power to determine the outcome of the game \((h < k)\). Insiders still have access to every insider’s information and therefore they still vote as a block. However, they need to assess the probability of outsiders observing a particular signal and voting according to that signal. Outsiders, in their turn, have to assess the probability that insiders observe a bad signal and side with outsiders in ousting the CEO. Otherwise, they need to assess the probability that they can have enough votes to fire the CEO without insiders’ support. Recall that, since outsiders receive independent
Proposition 10. (Equilibrium when outsiders dominate the board). Index outside board members by $i$ and let $e_i = 1$ if the $i^{th}$ board member observes a bad signal ($s_i = b$); $e_i = 0$ if the signal is good ($s_i = g$). Then the game described above has a unique equilibrium, so that:

(i) If the dissent-cost to equity loss ratio $\frac{c}{q} \leq \Theta(k, n, q, \alpha, \gamma, \mu, h)$, each outside director votes informatively, in particular, director $i$ votes to retain the CEO iff $s_i = g$ and to fire the CEO iff $s_i = b$, and all insiders vote to fire the CEO iff $f = b$ and to retain her if $f = g$;

(ii) If $\frac{c}{q} > \Theta(k, n, q, \alpha, \gamma, \mu, h)$, each director votes to retain the CEO regardless of his private signal, where

$$
\Theta(\cdot, h) = \frac{p(f=b|b)p\left(\sum_{j=1}^{n-h} e_j \geq k-h\right|b) + p(f=g|b)p\left(\sum_{j=1}^{n-h} e_j \geq k\right|b)}{1-p(f=b|b)p\left(\sum_{j=1}^{n-h} e_j \geq k-h\right|b) - p(f=g|b)p\left(\sum_{j=1}^{n-h} e_j \geq k\right|b)} \left(\rho(B|b) - (1 - \gamma)\right).
$$

All probabilities above represent beliefs of director $i$ given his signal (in the case of insiders it’s their cumulative signal based on $f$).

Proposition 10 is in line with the basic intuition of our model: outsiders vote informatively only if they expect that there will be a sufficient number of directors who will join them. Notice that in this case it is outsiders’ assessment that matters. If outsiders are unwilling to vote informatively, insiders will never vote to fire the CEO since they do not have a large enough block.

4.2 Optimal board composition

The more interesting and perhaps relevant question is whether there is an optimal composition of the board. We characterize optimal board composition in the following proposition.

Proposition 11. (Optimal board composition).

(i) If the dissent-cost to equity loss ratio $\frac{c}{q} \leq \Upsilon(k, n, \alpha, \gamma, \mu|b)$, then the board should include at most one insider.

(ii) If $\Upsilon(k, n, \alpha, \gamma, \mu|b) < \frac{c}{q} \leq \Theta(k, n, q, \alpha, \gamma, \mu, h)$, then the board should include both insiders and outsiders.
(iii) If \( \frac{c}{q} > \Theta(k, n, q, \alpha, \gamma, \mu, h) \), then only insiders should be on the board, where

\[
\Upsilon(k, n, \alpha, \gamma, \mu | b) = \frac{p \left( \sum_{j=1}^{n} e_j \geq k | b \right) \left( p(B | b) - (1 - \gamma) \right)}{p \left( \sum_{j=1}^{n} e_j < k | b \right)} \text{ and }
\]

\[
\Theta(\cdot, h) = \frac{p(f = b | b)p \left( \sum_{j=1}^{n-h} e_j \geq k-h | b \right) + p(f = g | b)p \left( \sum_{j=1}^{n-h} e_j \geq k | b \right)}{1 - p(f = b | b)p \left( \sum_{j=1}^{n-h} e_j \geq k-h | b \right) - p(f = g | b)p \left( \sum_{j=1}^{n-h} e_j \geq k | b \right)} \left( p(B | b) - (1 - \gamma) \right).
\]

All probabilities above represent beliefs of director i given his signal (in the case of insiders it’s their cumulative signal based on f).

The intuition behind Proposition 11 is as follows. Since insiders’ signals are correlated, having them on the board provides little informational advantage. In fact, their cumulative inference is the same as the inference made by a single director with an independent signal. Hence, to improve decision-making it is optimal to have as many outsiders on the board as possible as long as they are willing to reveal their private information. When the dissent-cost to equity loss ratio is low, as in (i) above, outsiders would be willing to vote informatively even if the board included only outsiders. In this case it is optimal to have at most one insider on the board (since when there’s only one insider, he is equivalent to an outsider), so that the informational benefits of a greater number of independent signals provided by outsiders are realized.

Insiders, on the other hand, are able alleviate the coordination problem that outside director face because inside directors observe each other’s signal. When the cost of dissent is moderate but high enough that a board consisting exclusively of outsiders will not vote informatively (as in (ii)), having insiders can improve decision-making, so that the board should consist of both insiders and outsiders. In this case, having insiders reduces the effective board size and thus mitigates the coordination problem. Once the effective board size is smaller due to the presence of insiders, outsiders will vote informatively. Notice that we explicitly emphasize the dependence of \( \Theta \) on \( h \), the number of insiders. It is only for those values of \( h \) for which the dissent-cost to equity loss ratio does not go beyond this threshold that informative voting by outsiders is feasible.\(^{11}\)

Finally, if the dissent-cost to equity loss ratio is very high, as in (iii), so that outside directors are never willing to vote informatively, only insiders should be on the board. This ensures that the board will avoid the cost of dissent and will be able to fire a bad CEO with a positive probability. Outsiders will always vote to keep the CEO, so there is

---

\(^{11}\) In general, there can be several values of \( h \) that would satisfy this requirement. On the other hand, if

\[
\Theta(k, n, q, \alpha, \gamma, \mu, h) < \frac{p \left( \sum_{j=1}^{n} e_j \geq k | b \right) \left( p(B | b) - (1 - \gamma) \right)}{p \left( \sum_{j=1}^{n} e_j < k | b \right)}
\]

for all \( h = 1, \ldots, n \), then \( \Theta(k, n, q, \alpha, \gamma, \mu, h) \) is irrelevant.
no informational advantage in adding them to the board in this situation. However, the
disadvantage of such an insider dominated board is that, while having only insiders avoids
the coordination problem, the board will have access to significantly less information in
this case.

5 Public signals

So far we have been assuming that directors were only able to obtain private information
about the CEO’s quality. We have shown that in this case lack of coordination may lead to
inefficient decision-making. In this section we investigate what implications adding a public
signal brings to our model. We argue that publicly observable information is a natural
coordination mechanism. Simple intuition would suggest that having such a mechanism
should improve decision-making. Somewhat surprisingly, however, we show that it is not
always the case.

We assume that, in addition to their own private signals, all directors observe a public
signal of CEO’s quality. The public signal can be either \( H \) (high) or \( L \) (low). As in our basic
model, we assume that directors are homogeneous (there’s no distinction between insiders
and outsiders). The informativeness of the public signal is determined by parameter \( \phi \):\(^{12}\)

\[
\begin{align*}
p(H|G) &= p(L|B) = \phi; \\
p(L|G) &= p(H|B) = 1 - \phi; \\
\phi &> \frac{1}{2}.
\end{align*}
\]

We also assume that public and private signals are conditionally independent. That is,
\( p(s_{\text{private}}, S_{\text{public}}|Q) = p(s_{\text{private}}|Q)p(S_{\text{public}}|Q) \) for \( s_{\text{private}} \in \{g,b\}, S_{\text{public}} \in \{H,L\}, \) and
\( Q \in \{G,B\} \).

5.1 Equilibria

Due to the efficiency requirement, if the public signal is very precise compared to private
signals, directors should disregard their private information and fire the CEO upon ob-
serving public signal \( L \) but retain her upon observing public signal \( H \). This happens when
\( p(B|b,H) < 1 - \gamma < p(B|g,L) \). In order to keep matters interesting, we assume that the

\(^{12}\)For simplicity of modeling, we use only one parameter to describe the public signal’s precision.
precision of the public signal does not allow directors to disregard their private signals. This is a reasonable assumption, since otherwise the board of directors brings no value into the turnover decision and it would be optimal to simply dismiss the CEO after observing negative public information (such as a fall in the stock price or in operating performance).

To capture the above ideas, in this section only we impose the following restriction:

$$p(B|g, H) < p(B|g, L) < 1 - \gamma < p(B|b, H) < p(B|b, L).$$ (11)

**Proposition 12.** (Equilibrium with a public signal). Index board members by $i$ and let $e_i = 1$ if the $i^{th}$ board member observes a bad private signal ($s_i = b$); $e_i = 0$ if the private signal is good ($s_i = g$). Then, in the presence of a public signal, the game described above has a unique equilibrium, so that:

(i) After observing public signal $H$:

(a) If the dissent-cost to equity loss ratio $\frac{c}{q} \leq \Upsilon(k, n, q, \alpha, \gamma, \mu|b, H)$, each director votes informatively, in particular, director $i$ votes to retain the CEO iff $s_i = g$ and to fire the CEO iff $s_i = b$;

(b) If $\frac{c}{q} > \Upsilon(k, n, q, \alpha, \gamma, \mu|b, H)$, each director votes to retain the CEO regardless of his private signal, where

$$\Upsilon(k, n, q, \alpha, \gamma, \mu|b, H) = \frac{p(\sum_{j=1}^{n} e_j \geq k | b, H) \left( p(B|b, H) - (1 - \gamma) \right)}{p(\sum_{j=1}^{n} e_j < k | b, H)}.$$

(ii) After observing public signal $L$:

(a) If the dissent-cost to equity loss ratio $\frac{c}{q} \leq \Upsilon(k, n, q, \alpha, \gamma, \mu|b, L)$, each director votes informatively, in particular, director $i$ votes to retain the CEO iff $s_i = g$ and to fire the CEO iff $s_i = b$;

(b) If $\frac{c}{q} > \Upsilon(k, n, q, \alpha, \gamma, \mu|b, L)$, each director votes to retain the CEO regardless of his private signal, where

$$\Upsilon(k, n, q, \alpha, \gamma, \mu|b, L) = \frac{p(\sum_{j=1}^{n} e_j \geq k | b, L) \left( p(B|b, L) - (1 - \gamma) \right)}{p(\sum_{j=1}^{n} e_j < k | b, L)}.$$

Notice that unlike in the basic model there are two thresholds in this case, depending on the type of the public signal that board members observe. This is because, after observing a

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13 This assumption ensures that the private signal continues to be informative to board members even in the presence of a public signal (i.e., the public signal is not too precise).
public signal each director reassesses the validity of his own private information, conditional on the public information. The quality of decision-making by the board depends on the relation between $\frac{c}{q}$ and these conditional thresholds.

**Proposition 13.** (Quality of decision making with a public signal).

(i) If $\frac{c}{q} \leq \Upsilon(k,n,q,\alpha,\gamma,\mu|b,H)$, directors vote informatively regardless of the public signal, i.e., the public signal is irrelevant.

(ii) If $\Upsilon(k,n,q,\alpha,\gamma,\mu|b,L) < \frac{c}{q} \leq \Upsilon(k,n,\alpha,\gamma,\mu|b)$, directors vote informatively only if there is no public signal or if the public signal is L, i.e., the public signal worsens the board's decision-making on average.

(iii) If $\Upsilon(k,n,\alpha,\gamma,\mu|b) < \frac{c}{q} \leq \Upsilon(k,n,q,\alpha,\gamma,\mu|b,L)$, directors vote informatively only when there is a public signal and it is L, i.e., the public signal improves the board's decision-making on average.

(iv) If $\Upsilon(k,n,q,\alpha,\gamma,\mu|b,L) < \frac{c}{q}$, directors never reveal their private signals and always vote to keep the CEO, i.e., the public signal is irrelevant again, where

$$
\Upsilon(k,n,q,\alpha,\gamma,\mu|x) \equiv \frac{p\left(\sum_{j=1}^{n} e_j \geq k-1|x \right)\left(p(B|x)-(1-\gamma)\right)}{p\left(\sum_{j=1}^{n} e_j < k-1|x \right)}.
$$

Part (i) of Proposition 13 states that when the dissent-cost to equity loss ratio is low, the public signal is irrelevant. Since $\frac{c}{q}$ is low, the directors would have revealed their private information even in our basic model. On the other hand, when this ratio goes up but remains below $\Upsilon(k,n,\alpha,\gamma,\mu|b)$ of our basic model, as in part (ii) of the above proposition, the presence of a public signal may worsen decision-making. This happens because conditional on observing a low public signal or no public signal at all, directors vote informatively. Observing a high public signal, however, may prevent board members with bad private signals from voicing their opinion, since they may assess that many other directors may vote for retaining the CEO, given the high public signal. A favorable public signal may thus make it less likely that there will be enough directors to overturn the incumbent CEO.

Part (iii) of Proposition 13 implies that when the dissent-cost to equity loss ratio is higher than $\Upsilon(k,n,\alpha,\gamma,\mu|b)$ of our basic model but is below $\Upsilon(k,n,q,\alpha,\gamma,\mu|b,H)$, the presence of a public signal is beneficial. This happens because in this case the only way for directors to vote informatively is to observe a low public signal. Directors with bad private
signals would have sided with the CEO in our basic model. Upon learning negative public information about her quality, however, there is a higher probability that enough other directors will also want to oust her. This higher chance of success facilitates informative voting. When $\frac{c}{q}$ is very high, as in part (iv) of Proposition 13, even a low public signal will not make board members reveal their private information. In this case, the public signal is irrelevant.

6 The dynamic model: Proactive boards and early firing decisions

We now turn to the dynamic aspect of the board’s decision-making. In particular, we ask the following question: which boards are likely to fire their CEO sooner rather than later? The financial press has been long asking the question why some boards seem to keep their CEOs until the damage is so obvious that the stock price is in a free fall. This subject, however, has seen little attention in the academic literature, with Ertugrul and Krishnan (2009) being the sole empirical paper on the topic.

In this section we modify our initial game to allow two rounds of voting: one after directors observe their private signals and another after some public information about the current CEO is revealed. We also assume the board is homogeneous (there’s no difference between insiders and outsiders). To make the notion of early versus late firing decision operational, we say that the CEO is fired early if the board made this decision prior to the revelation of a negative public signal. The CEO is said to be fired late if the decision came only after a negative public signal. The negative public signal we have in mind is a drop in share price (loss of shareholder value) under the current CEO.

The timeline of the extended model is as follows. At time 1 each director observes a signal about the CEO’s ability. At time 2 the board votes whether to fire the CEO. If the CEO is retained, at date 3 the public signal of her quality is revealed. If a new CEO is hired, at time 3 directors observe private signals about her ability, and there will be no public signal at time 3 if a new CEO is hired at time 2. At time 4 the second round of voting takes place and at time 5 the true quality of the then-incumbent CEO is revealed. Unlike before, we assume that the public signal is highly informative, that is, $p(B|g, H) < p(B|b, H) < 1 - \gamma < p(B|g, L) < p(B|b, L)$. As we discussed in the earlier section on public signals, in this case directors should disregard their private signals
upon observing public information. Apart from modeling simplicity, we introduce this assumption because it represents the trade-off we intend to study in this section. One may view this public signal as a result of a series of revelations, which are compressed into a single publicly observed sufficient statistic (\(H\) for high or \(L\) for low).

We also impose a penalty for keeping a bad CEO for too long. In particular, if it is revealed at time 5 that the CEO is of poor quality but the board failed to fire her before public information was revealed, the equity loss of each board member becomes \(2q\) instead of \(q\). This assumption reflects the idea that some damage is being done to the firm during the bad CEO’s tenure, so that the board members’ equity loss is greater. Without such an increase it would always be optimal to wait until the public signal is revealed, since this would allow board members to always avoid the dissent-cost without suffering any equity loss. If at time 2 a new CEO was hired, there is no increase in cost since the CEO is only in office for one period. For simplicity, we also assume that \(\mu = \gamma\), i.e., the board members’ probability assessment of the incumbent CEO being of type \(G\) is the same as the probability assessment of a CEO from the outside pool being of type \(G\).

**Proposition 14.** (Firing in the dynamic game). In the two-round voting game described above, there exists a threshold, \(\Psi(\gamma, \alpha, \phi, k, n)\), such that if \(\frac{\phi}{q} > \Psi(\gamma, \alpha, \phi, k, n)\), the board never fires the incumbent CEO in the first round, but waits for the second round of voting.
If \( \frac{\xi}{q} \leq \Psi(\gamma, \alpha, \phi, k, n) \), the board will fire the incumbent CEO in the first round with a positive probability.

The trade-off faced by board members is as follows. As in the basic model, they face equity loss if it is revealed that they chose a bad CEO and the dissent-cost if they attempt to fire the incumbent but fail. However, in this dynamic setting they have the additional choice of whether to fire the CEO in the first round by voting according to their private signals, or to wait until some public information is observed. Since we assume that the public signal is more precise than directors’ private signals, waiting is beneficial. There is some residual uncertainty, however, so that even after observing a positive public signal the CEO’s quality may be revealed as bad at time 5. In that case, the board members’ equity loss will be significantly greater than it would have been had the board fired the CEO in the first round of voting (at time 2). If the dissent-cost to equity loss ratio is high, board members would rather risk this greater equity loss than face the higher probability of immediate punishment in the form of dissent-costs. When this ratio is low, directors will vote according to their private information in the first round since the potential of increased equity loss outweighs the benefits of avoiding the cost of dissent. To summarize, as time goes by and more evidence of poor firm performance becomes available to board members, their equity loss becomes greater; however, the coordination problem faced by board members is also reduced over time, since, once each board member becomes aware that others are also receiving more and more such evidence of CEO’s incompetence (over time), their probability assessment of a sufficient number of board members voting against the CEO goes up (and correspondingly, their probability assessment of having to incur the dissent-cost goes down). Proposition 14 implies that better incentivized boards (smaller \( \frac{\xi}{q} \)) are more likely to fire the CEO earlier, before the damage is done to the firm.

7 Empirical and policy implications

Our model has several testable and policy implications. We highlight a few of these below.

(i) Passive boards, suboptimal firing decisions, and the effects of shareholder activism: Our model develops a new rationale for why many boards are suboptimally passive (i.e., they choose to retain the existing CEO for too long). We showed that when the dissent-cost to equity loss ratio is large (for example, when the CEO has the ability to dictate the composition of the board without any uncertainty), coordination problems between board
members prevent the board from firing the CEO even when collectively, the board has enough information to know that it is optimal for shareholders to fire the existing CEO and hire a new one from outside. Further, our model implies that if the board’s assessment of the quality of the existing CEO relative to the pool of outside CEOs is low, then the board is more likely to fire the existing CEO and hire a new CEO from the outside.

The above has several testable predictions. First, more entrenched CEOs with greater ability to influence the payoff of board members are less likely to be fired. Second, there will be a positive relationship between a CEO being fired and the pool of outside candidates for the CEO’s job and a negative relationship between industry-adjusted firm performance and the CEO being fired. Third, firms in industries where board members are able to evaluate CEOs more precisely (i.e., where board members receive more precise signals of CEO’s quality) are more likely to make better CEO firing decisions. Thus, boards of firms in more homogeneous industries are likely to make better firing decisions than those in more heterogeneous industries (where it is harder to benchmark the CEO’s performance relative to other firms). Further, boards in which members have, on average, larger stock ownership (larger $q$ in our model) and are institutional or otherwise activist shareholders (lower $c$) are more likely to make better decisions in terms of firing the CEO.

Evidence supporting the first prediction above is provided by Allen (1981), who documents that CEOs having greater power over directors have longer tenures. Strong evidence supporting the second and third predictions above is provided by Parrino (1997). First, he documents a strong negative relationship between industry-adjusted firm performance and the likelihood that an outsider is appointed CEO. Second, he documents that the likelihood of forced CEO turnover and outside succession are both greater in homogeneous industries (that consist of similar firms) than in heterogeneous industries.

(ii) Optimal board size: The optimal board size in our model is determined by the trade-off between the information requirements of the board in evaluating the CEO, which favors a larger board (the larger the board is, the greater is the amount of information available to the board as a whole), and the greater coordination problems that arise with larger boards. Thus, our model predicts that, in order to maximize firm value, firms will have the smallest board size consistent with the information requirements of the board in evaluating and monitoring the performance of the CEO. Thus, our model predicts that larger, multi-divisional, and more complex firms (in the sense that the board requires more information in evaluating the CEO’s performance) will have larger boards than simple firms. In the latter case, our model predicts that the board members will (optimally) be
better incentivized (smaller $\frac{c}{q}$) either through larger holdings in the firm's equity (larger equity loss $q$) or through appointing board members (like activist shareholders) who suffer from smaller dissent-costs. Several empirical papers provide evidence consistent with the first prediction above of our model. Yermack (1996) shows that, controlling for firm size and other firm characteristics, there is an inverse relationship between market value (Tobin’s Q) and the size of the board of directors. He also shows that measures of operating efficiency and profitability are negatively related over time to board size within firms, and that smaller boards are more likely to dismiss CEOs following periods of poor performance. Related evidence is provided by Kini et al. (1995), who document that board size shrinks after successful tender offers of underperforming firms. Evidence supporting the second prediction of our model above is provided by Mikkelsen et al. (1997) and Boone et al. (2007), who document that firms start out with smaller boards at IPO, with board size increasing significantly after the firm has become seasoned. The latter paper also documents that board size is related to measures of the scope and complexity of the firm’s operations such as firm size, firm age, and the number of business segments the firm operates in. Finally, Coles et al. (2008) document that complex firms, which have greater advising requirements than simple firms, have larger boards with more outside directors. They also document that the relation between a firm’s market value (Tobin’s Q) and board size is U-shaped: Tobin’s Q increases (decreases) in board size for complex (simple) firms.

(iii) Optimal board composition: Conventional wisdom among practitioners seems to be that a greater level of board independence (a larger fraction of outside directors) unambiguously increases firm performance. For example, TIAA-CREF, one of the largest pension funds in the world, has stated that it will invest only in companies that have a majority of outside directors on its board; similarly, CALPERS, another large pension fund, recommends that the CEO should be the only inside director on a firm’s board. The empirical evidence, however, has been mixed: For example, Coles et al. (2008) present evidence challenging the notion that restrictions on board size and insider representation necessarily enhance firm value (see also Bhagat and Black (2001)).

Our model contributes significantly to the above debate about the optimal proportion of insiders versus outsiders on corporate boards. Our analysis suggests that there may be an optimal mix of insiders versus outsiders on corporate boards: the advantage of having insiders arises from reducing coordination problems, while the advantage of having outsiders arises from the additional information they bring to the board (at the cost of greater coordination problems). Thus, our analysis suggests that the fraction of insiders
to outsiders (and whether the board should be insider or outsider dominated) depends on the dissent-cost to equity loss ratio. Consequently, our model predicts that a greater proportion of independent directors (outsiders) is not necessarily value improving in all situations: rather, it suggests that firms where the CEO has greater influence, generating higher dissent-costs for board members (as measured by the CEO’s share ownership and job tenure), will be associated with a smaller fraction of independent outsiders. Evidence supporting this prediction is provided by Boone et al. (2007), who document such a relationship. On the other hand, our model predicts that more complex firms (with greater informational requirements for the board in evaluating the firm’s and the CEO’s performance) should have a greater proportion of outsiders; further, in such boards, the outside board members should have greater equity holdings, so as to reduce the dissent-cost to equity loss ratio. Evidence supporting the former prediction is provided by Coles et al. (2008), who document that more complex firms, with greater advising requirements than simpler firms, have larger boards with more outside directors. Evidence supporting the latter prediction that the proportion of independent outside directors will be greater in firms with lower dissent-cost to equity loss ratio is provided by Boone et al. (2007), who document that the proportion of independent outsiders on a firm’s board is positively related to the equity ownership of outside directors. Finally, our model predicts that a board with a greater proportion of (appropriately incentivized) outsiders is more likely to replace a poorly performing CEO. Evidence supporting this prediction is supported by Weisbach (1988) and Borokhovich et al. (1996).

(iv) The ambiguous effect of low-precision public signals on board performance: Our model demonstrates that, contrary to common intuition, low-precision public signals may in fact worsen the board’s decision-making instead of improving it (relative to the case of no public signal). Thus, short-term improvements in accounting performance by a firm may induce a board member to vote to retain the current CEO, even though he is privately convinced that the firm is pursuing a wrong long-term strategy under the current CEO. This arises from the fact that such a public signal worsens coordination problems among board members by increasing each member’s fear that other members may vote to retain the CEO, thus increasing his likelihood of having to incur dissent-costs if he votes against the CEO.

(v) The dynamics of forced CEO turnover and its relation to board characteristics: Both practitioners and academics have bemoaned the tendency of boards to allow CEOs to pursue wrong-headed policies for a number of years, and fire them only after millions (if not
billions) of dollars of shareholder value has been destroyed. For example, Monks and Minow (1995) document a number of cases where CEOs were able to destroy shareholder value for a number of years, and raise the following questions: “Why does it take boards so long to respond to deep-seated competitive problems? And, if one of the leading responsibilities of directors is to evaluate the performance of the CEO, why do boards wait too long for proof of managerial incompetence before making a move?” Jensen (1993) makes a similar point and calls this a failure of corporate internal control systems, and comments: “They seldom respond in the absence of a crisis.”

The dynamic version of our model provides some answers to the above questions raised by Monks and Minow (1995), regarding why, even when board members are individually aware of the wrong-headedness of the CEO’s policies, they do not immediately vote to fire the CEO. Our dynamic analysis indicates that the timing of a board’s firing of a CEO who is destroying shareholder value is driven by a trade-off between the equity loss incurred by board members versus the probability of incurring a dissent-cost. On the one hand, as time goes by and more evidence of poor firm performance becomes available to board members, their equity loss becomes greater; on the other hand, the coordination problem faced by board members is also reduced over time, since, once each board member becomes aware that others are also receiving more and more such evidence (over time), their probability assessment of having to incur a dissent-cost also goes down. Thus, our dynamic analysis has two predictions. First, CEOs who are entrenched and have greater influence over the board (e.g., as measured by longer tenure) are likely to be allowed to pursue policies that reduce share price for a longer time period before being asked to resign by the board. Second, boards that are better incentivized (smaller dissent-cost to equity loss ratio) are more likely to fire CEOs proactively, before considerable destruction of shareholder value has taken place. Evidence consistent with the latter prediction of our model is provided by Ertugrul and Krishnan (2009), who document that boards that dismiss CEOs early (i.e.,

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14 Monks and Minow (1995) give the case studies of the CEOs of General Motors, Westinghouse, American Express, IBM, Eastman Kodak, Scott Paper, and Borden, who were pressured to resign in the face of their companies’ long-term underperformance. They comment that, while the above moves by their companies’ boards were heralded in the media as breakthroughs in boardroom activism, in all these instances the board took the necessary drastic action years too late. Jensen (1993) points out that “The [...] GM board revolt [...] which resulted in the firing of CEO Robert Stempel exemplifies the failure, not the success, of GM’s governance system. General Motors, one of the world’s high-cost producers in a market with substantial excess capacity, avoided making major changes in its strategy for over a decade. The revolt came too late: the board acted to remove the CEO only in 1992, after the company had reported losses of $6.5 billion in 1990 and 1991.”
in the absence of significant negative prior stock returns) are those that are characterized by higher equity ownership by board members, higher board equity compensation as a fraction of total compensation, higher institutional ownership, and better firm corporate governance characteristics.\textsuperscript{15}

\textit{(vi) Policy implications for corporate governance and board reform:} Our model suggests several ways in which corporate governance, especially corporate boards, can be reformed to improve firm performance. Our analysis suggests that, broadly speaking, reforms that reduce the dissent-costs of board members or increase their equity loss would significantly increase the likelihood of boards firing incompetent CEOs. One proposal that would reduce dissent-costs would be to grant security of tenure (i.e., guarantee their presence on the board for some length of time) for current board members, thus reducing their dissent-costs in the event they vote against the current CEO but end up being unable to oust him or her. Another reform proposal that would reduce dissent-costs arising from our analysis would make it easier for dissidents to acquire a seat on the board: due to the cost of proxy fights, few dissidents bother to make board challenges.\textsuperscript{16} One such proposal currently being considered by the SEC would let owners of at least 1\% of a company with an equity value of $700$ million or more (3\% for smaller companies) include information about their board nominees in corporate proxy materials. Another such proposal would reimburse successful dissident board candidates for all their campaign expenses (and partially reimburse candidates who obtained at least 40\% of the votes cast).\textsuperscript{17} Another such proposal that would reduce dissent-costs would be to reduce the involvement of CEOs in the selection of board members and have boards choose directors through nominating committees composed only of independent members of the board (see, e.g., The Working Group on Corporate Governance (1991)).\textsuperscript{18}

\textsuperscript{15}Ertugrul and Krishnan (2009) document that firms that dismiss CEOs early do not experience poor operating performance after the CEO dismissal relative to a control sample, suggesting that such dismissals are not cases of value-reducing mistakes by the board.

\textsuperscript{16}It can cost hundreds of thousands of dollars or more for an outside contender to run against a director. For example, a campaign for a board seat may involve repeated mailings to every investor; directors endorsed by the firm management, on the other hand, can make use of the company coffers to finance their candidacies. Thus, RiskMetrics, a proxy-advisory firm, points out that there were just 75 shareholder contests in 2009 (as of October of that year); challengers won one seat each in 58 of those fights.

\textsuperscript{17}Such a proposal was adopted by HealthSouth Corporation in October 2009. For details of this and other proposals being considered by the SEC, see Joann S. Lublin, \textit{Reimbursements aim for a fairer proxy fight.} The Wall Street Journal, page A22, October 27, 2009.

\textsuperscript{18}Shivdasani and Yermack (1999) document that when the CEO is involved in the selection of a new director, firms appoint fewer independent outside directors and more “grey” outsiders. They define a CEO as “involved” in such selection (i) if the board has a separate nominating committee and the CEO serves as
A second set of board reform proposals suggested by our analysis would increase the equity loss of board members. One such proposal would be to increase the proportion of board members’ compensation that depends directly on the firm’s long-run share value. Another proposal (perhaps harder to implement) would require board members to invest a significant amount of their own wealth in the firm’s equity.

8 Conclusion

In this paper, we have developed a theory of corporate boards and their role in forcing CEO turnover. We considered a firm with an incumbent CEO of uncertain management ability and a board consisting of a number of directors whose role is to evaluate the CEO and fire her if a better replacement can be found. In our setting, each board member receives an independent private signal about the CEOs ability, after which board members vote on firing the CEO (or not). If the CEO is fired, the board hires a new CEO from the pool of candidates available. The true ability of the firms CEO is revealed in the long run; the firms long-run share price is determined by this ability. Each board member owns some equity in the firm, and thus prefers to fire a CEO of poor ability. However, if a board member votes to fire the incumbent CEO but the number of other board members also voting to fire her is not enough to successfully oust her, the CEO can impose significant costs of dissent on him. In this setting, we show that the board faces a coordination problem, leading it to retain an incompetent CEO even when a majority of board members private signals indicate that she is of poor quality. We solved for the optimal board size, and show that this depends on various board and firm characteristics: one size does not fit all firms. We developed extensions to our basic model to analyze the optimal composition of the board between insiders and outsiders and the effect of board members observing imprecise public signals (such as the firms short-term operating performance) in addition to their private signals on the effectiveness of board decision-making. Finally, we developed a dynamic extension to our basic model to analyze why many boards do not fire CEOs even when they preside over a significant, publicly observable, reduction in shareholder wealth over a long period of time, and used this dynamic model to distinguish between the characteristics of such boards from those that fire CEOs proactively before such wealth reductions take place.

a member or (ii) if such a committee does not exist and directors are selected by the entire board including the CEO.
References


Carlos Tejada. Longtime Tandy director quits board, says he was punished for faulting CEO. *The Wall Street Journal*, January 16, 1997.


A Appendix: proofs of propositions

Proof of Proposition 1

When $\gamma = \mu$, (6) can be rewritten as

$$
\frac{(1 - \mu)\alpha^r(1 - \alpha)^{n-r}}{(1 - \mu)\alpha^r(1 - \alpha)^{n-r} + \mu(1 - \alpha)^r} > 1 - \mu. \quad (A.1)
$$

We have:

$$
\alpha^r(1 - \alpha)^{n-r} > (1 - \mu)\alpha^r(1 - \alpha)^{n-r} + \mu(1 - \alpha)^r \quad \Leftrightarrow \\
\alpha^r(1 - \alpha)^{n-r} > (1 - \alpha)^r \quad \Leftrightarrow \\
(2r - n)(\ln \alpha - \ln(1 - \alpha)) > 0 \quad \Leftrightarrow \\
2r > n \text{ (since } \alpha > \frac{1}{2}) \ .
$$

We assumed that $n$ is odd; $r$ is an integer, hence, the lowest $r$ that satisfies (A.1) is $r^* = \frac{n+1}{2}$.

Proof of Proposition 2

If there were no dissent-costs, directors’ objective would no longer be given by (1), but rather by

$$
\pi_i^{\text{benchmark}}(v; s_i, k, n, q, \alpha, \gamma, \mu) = - I\left(\sum_{j=1}^{n} v_j \geq k\right) q(1 - \gamma) \\
- I\left(\sum_{j=1}^{n} v_j < k\right) qp(B|s_i).
$$

We say that a director is pivotal if his vote determines the outcome of the vote. First consider the case when the director is not pivotal. Since there are no costs of dissent, his vote does not alter his payoff. He can as well vote informatively. Now consider the case when the director is pivotal. It only happens when exactly $k - 1$ other directors vote to fire the CEO. Since $k - 1 < k$, if this director’s signal is $g$, then his inference based on aggregate information available is

$$
p(B|g^{\text{pivotal}}) = \frac{(1 - \mu)\alpha^{k-1}(1 - \alpha)^{n-k+1}}{(1 - \mu)\alpha^{k-1}(1 - \alpha)^{n-k+1} + \mu(1 - \alpha)^{k-1}\alpha^{n-k+1}} < 1 - \gamma. \quad (A.4)
$$

If, on the other hand, this director’s signal is $b$, then his inference is

$$
p(B|b^{\text{pivotal}}) = \frac{(1 - \mu)\alpha^{k}(1 - \alpha)^{n-k}}{(1 - \mu)\alpha^{k}(1 - \alpha)^{n-k} + \mu(1 - \alpha)^{k}\alpha^{n-k}} > 1 - \gamma. \quad (A.5)
$$
It follows that
\[ \pi_i^{\text{benchmark}}(v_i = 0; g \overset{\text{pivotal}}{\cdot}) > \pi_i^{\text{benchmark}}(v_i = 1; g \overset{\text{pivotal}}{\cdot}) \] (A.6)
since
\[ -qp(B|g \overset{\text{pivotal}}{\cdot}) > -q(1 - \gamma). \] (A.7)

Analogously,
\[ \pi_i^{\text{benchmark}}(v_i = 0; b \overset{\text{pivotal}}{\cdot}) < \pi_i^{\text{benchmark}}(v_i = 1; b \overset{\text{pivotal}}{\cdot}) \] (A.8)
since
\[ -qp(B|b \overset{\text{pivotal}}{\cdot}) < -q(1 - \gamma). \] (A.9)

Hence, the pivotal board member is better off voting informatively. Thus, informative voting is an equilibrium. This equilibrium is efficient since its outcome coincides with the social planner’s choice.

**Proof of Proposition 3**

Consider the case when the CEO’s true quality is good. In this case, the expected number of good signals observed by directors is \( n(1 - \alpha) \) for any given \( n \) (because the mean of a Binomial distribution is given by \( n(1 - \alpha) \)). Assuming that \( n(1 - \alpha) \) is an integer, we can rewrite (5) to obtain the expected updated probability of CEO’s quality conditional on having \( n \) independent signals:

\[
E(p(B|n)|G) = \frac{(1 - \mu)\alpha^{n(1-\alpha)}(1 - \alpha)^{n\alpha}}{(1 - \mu)\alpha^{n(1-\alpha)}(1 - \alpha)^{n\alpha} + \mu(1 - \alpha)^n(1-\alpha)\alpha^{n\alpha}}. \] (A.10)

The smaller this probability, the better is the social planner’s inference. Notice that, since \( \alpha > \frac{1}{2} \),

\[
\frac{\partial E(p(B|n)|G)}{\partial n} = \frac{\mu(1 - \mu)\alpha^{n(1-\alpha)}(1 - 2\alpha)(\ln \alpha - \ln (1 - \alpha))}{(1 - \mu)\alpha^{n(1-\alpha)}(1 - \alpha)^{n\alpha} + \mu(1 - \alpha)^n(1-\alpha)\alpha^{n\alpha})^2} < 0. \] (A.11)

The case when the CEO’s true ability is low is analogous.

**Proof of Proposition 4**

There are four possible symmetric pure voting strategies given director \( i \)’s signal:

1. always vote to fire the CEO, \( \{v_i = 1|s_i = g; v_i = 1|s_i = b\} \);
2. always vote to keep the CEO, \( \{v_i = 0|s_i = g; v_i = 0|s_i = b\} \);
3. vote according to the signal, \( \{v_i = 0|s_i = g; v_i = 1|s_i = b\} \);
4. vote in the opposite direction to the signal, \( \{ v_i = 1 | s_i = g; v_i = 0 | s_i = b \} \).

In this case it won’t suffice to consider only pivotal board members since even when a director isn’t pivotal, his vote influences his payoff via the dissent-cost. It is easier to think of equilibria in terms of board members’ signals rather than their votes. We therefore let \( e_i = 1 \) if \( s_i = b \) and \( e_i = 0 \) if \( s_i = g \). We can now rewrite (1) to obtain directors’ expected payoffs in each of the candidate equilibria above, conditional on their private signals. The payoffs to a director with \( s_i = g \) from strategies 1, 2, 3, and 4, respectively, are given by

\[
- q(1 - \gamma), \quad (A.12)
\]

\[
- qp(B|g), \quad (A.13)
\]

\[
- qp(B|g)p(\sum_{j=1}^{n} e_j < k|g) - q(1 - \gamma)p(\sum_{j=1}^{n} e_j \geq k|g), \quad (A.14)
\]

\[
- (c + qp(B|g))p(\sum_{j=1}^{n} (1 - e_j) < k|g) - q(1 - \gamma)p(\sum_{j=1}^{n} (1 - e_j) \geq k|g). \quad (A.15)
\]

The payoffs to a director with \( s_i = b \) from strategies 1, 2, 3, and 4, respectively are given by

\[
- q(1 - \gamma), \quad (A.16)
\]

\[
- qp(B|b), \quad (A.17)
\]

\[
- (c + qp(B|b))p(\sum_{j=1}^{n} e_j < k|b) - q(1 - \gamma)p(\sum_{j=1}^{n} e_j \geq k|b), \quad (A.18)
\]

\[
- qp(B|b)p(\sum_{j=1}^{n} (1 - e_j) < k|b) - q(1 - \gamma)p(\sum_{j=1}^{n} (1 - e_j) \geq k|b). \quad (A.19)
\]

We assumed that \( p(B|g) < 1 - \gamma < p(B|b) \). It follows that \(-qp(B|g) > -q(1 - \gamma) > -qp(B|b)\). Now, \((A.13) > (A.12) \) but \((A.16) > (A.17)\): directors who observe signal \( g \) prefer to retain the CEO while directors who observe \( b \) want to replace her.

It is easy to show that strategy 4 cannot survive in equilibrium. We prove by contradiction. Assume that voting in the opposite direction to one’s signal and believing that everyone else does this is an equilibrium. Consider directors who observe \( g \). They believe that all directors with signal \( b \) vote to keep the CEO. Hence, if directors with good signals vote in accordance with their equilibrium strategy, their payoff is given by \((A.15)\). If, however, they vote to keep the current CEO while the other type does not change its strategy, their payoff is given by \((A.13)\). Since \((A.13) > (A.15)\), directors who observe \( g \) always deviate. Contradiction.

Among the three remaining strategies, strategy 1 can also be eliminated. Again consider board members with signal \( g \). If directors with this signal deviate from strategy 1, the lowest payoff they can attain is \((A.14)\), which is greater than \((A.12)\). Hence, they always deviate.
Notice now that only two possible equilibrium strategies remain: strategy 2 and strategy 3. In both of these, directors who observe $g$ vote in accordance with their signal. Thus, it is directors with a bad signal of CEO’s quality who determine the equilibrium. They face the trade-off between (A.17) and (A.18) and vote according to their signal (informatively) if and only if (A.18) $\geq$ (A.17). Hence, a truth-telling equilibrium exists if and only if

$$-qp(B|b) \leq -(c + qp(B|b))p(\sum_{j=1}^{n} e_j < k|b) - q(1 - \gamma)p(\sum_{j=1}^{n} e_j \geq k|b). \quad (A.20)$$

Otherwise, directors who observe $b$ vote to retain the CEO. Algebraic manipulations transform (A.20) into

$$c \leq \frac{p(\sum_{j=1}^{n} e_j \geq k|b)(p(B|b) - (1 - \gamma))}{p(\sum_{j=1}^{n} e_j < k|b)}. \quad (A.21)$$

**Proof of Proposition 5**

Compare equilibria in Proposition 2 and Proposition 4. It is immediately clear that they coincide except when $\frac{c}{q} > \frac{p(\sum_{j=1}^{n} e_j \geq k|b)(p(B|b) - (1 - \gamma))}{p(\sum_{j=1}^{n} e_j < k|b)}$. In this case, had there been no dissent-cost, the CEO would have been fired with a positive probability (as long as the number of directors’ negative signals equals or exceeds $k$). In the presence of dissent-costs the board will never fire the CEO. It was shown above that the equilibrium in Proposition 2 produces the same outcome as the social planner. Hence, any deviation from this equilibrium is suboptimal.

**Proof of Proposition 6**

Part (i). Let $\omega(n,k) \equiv p(\sum_{j=1}^{n} e_j \geq k|b)$. It suffices to show that $\omega(n+2,k+1) < \omega(n,k)$ when $\mu > \alpha \frac{2n\alpha - (n+1)}{2n\alpha - (n+1) + 2(1 - \alpha)}$, and $\omega(n+2,k+1) = \omega(n,k)$ when $\mu = \alpha \frac{2n\alpha - (n+1)}{2n\alpha - (n+1) + 2(1 - \alpha)}$. Verbally, when $n$ increases by 2 (to keep it odd) and $k$ rises by 1 (to keep the majority rule), the probability that enough directors observe a bad signal decreases if $\mu > \alpha \frac{2n\alpha - (n+1)}{2n\alpha - (n+1) + 2(1 - \alpha)}$.

Let $\lambda \equiv p(e_j = 1|b) = \frac{\mu - 2\mu\alpha + \alpha^2}{\mu - 2\mu\alpha + \alpha}$. Then,

$$\omega(n,k) = p(\sum_{j=1}^{n} e_j \geq k|b)$$

$$= p(\sum_{j=1}^{n-1} e_j \geq k - 1|b)$$

$$= \sum_{m=k-1}^{n-1} \binom{n-1}{m} \lambda^m (1 - \lambda)^{n-1-m}. \quad (A.22)$$
The second line in (A.22) follows from the fact that once a board member observes \( b \), he only needs to consider the probability that \( k - 1 \) other directors obtained a bad signal. There are three possible combinations of the signals observed by the added directors:

1. their signal is split (one observes a bad signal and the other observes a good signal), with probability \( 2\lambda(1 - \lambda) \);
2. both of them observe a bad signal, with probability \( \lambda^2 \);
3. both of them observe a good signal, with probability \( (1 - \lambda)^2 \).

Notice that

\[
\omega(n+2,k+1) = 2\lambda(1 - \lambda)\omega(n,k) + \lambda^2\omega(n,k-1) + (1 - \lambda)\omega(n,k+1).
\]  

(A.23)

(A.23) follows by the following logic. The left-hand side is the probability that at least \( k + 1 \) directors observe a bad signal. If the signal of the new board members is split, which happens with probability \( 2\lambda(1 - \lambda) \), then to have at least \( k + 1 \) bad signals it must be the case that there are at least \( k \) bad signals among the remaining \( n \) directors, which happens with probability \( \omega(n,k) \). If both of the added board members observe bad signals, \( k - 1 \) negative signals among the previous \( n \) directors suffices to bring the total to \( k + 1 \). If both of the added board members observe good signals, there must already be \( k + 1 \) bad signals.

Further observe that by (A.22), the following probabilities can be rewritten:

\[
\omega(n,k - 1) = \omega(n,k) + \binom{n-1}{k-1}\lambda^{k-1}(1 - \lambda)^{n-k},
\]

\[
\omega(n,k + 1) = \omega(n,k) - \binom{n-1}{k}\lambda^k(1 - \lambda)^{n-k-1}.
\]  

(A.24)

Combining (A.23) with (A.24) yields

\[
\omega(n+2,k+1) - \omega(n,k) = \\
\lambda^2\frac{n-1}{k-1}\lambda^{k-1}(1 - \lambda)^{n-k} - \lambda^2\binom{n-1}{k}\lambda^k(1 - \lambda)^{n-k-1} = \\
\frac{(n-1)!}{(k-1)!(n-k-1)!}\lambda^k(1 - \lambda)^{n-k}\left(\frac{\lambda}{n-k} - \frac{1 - \lambda}{k}\right).
\]  

(A.25)

The sign of (A.25) is determined by \( \frac{\lambda}{n-k} - \frac{1 - \lambda}{k} \). Noting that \( k = \frac{n-1}{2} \), this can be rewritten as
\[
\frac{\lambda}{n-k} - \frac{1-\lambda}{k} = \frac{2\lambda}{n+1} - \frac{2-2\lambda}{n-1}. \tag{A.26}
\]

Hence,

\[
\omega(n+2,k+1) < \omega(n,k) \iff \frac{\lambda}{n+1} < \frac{1-\lambda}{n-1} \iff \lambda < \frac{n+1}{2n} \tag{A.27}
\]

In deriving (A.27) we used the fact that \(2n\alpha - (n+1) + 2(1-\alpha) > 0\) because \(\alpha > \frac{1}{2} > \frac{n-1}{2n-1}\). The rest of the proposition follows.

**Part (ii).** The proof is analogous to Part (i). We need to show that \(\omega(n+1,k+1) < \omega(n,k)\). First notice that

\[
\omega(n+1,k+1) = \lambda \omega(n,k) + (1-\lambda)\omega(n,k+1). \tag{A.28}
\]

Now observe that

\[
\omega(n,k+1) = \omega(n,k) - \binom{n-1}{k} \lambda^k (1-\lambda)^{n-k-1}. \tag{A.29}
\]

Hence,

\[
\omega(n+1,k+1) - \omega(n,k) = - \binom{n-1}{k} \lambda^k (1-\lambda)^{n-k-1} < 0. \tag{A.30}
\]

**Proof of Proposition 7**

**Part (i).** Notice that if \(\alpha \leq \mu\), then \(\mu > \alpha z(\alpha,n)\) for any \(n\) since \(z(\alpha,n) < 1\). It follows from Proposition 6 that as \(n\) grows, informative equilibria become less likely, or, equivalently, the threshold above which informative voting is unsustainable falls. Given \(\xi\), let \(n^*\) be the maximum \(n\) so that (A.21) holds. We will show that \(n^*\) dominates any \(n > n^*\) and any \(n < n^*\). Consider any \(n > n^*\). The board will always keep the incumbent, regardless of directors’ signals, while at \(n^*\) a bad CEO is fired with a positive probability. Hence \(n^*\) dominates any \(n > n^*\). Consider any \(n < n^*\). The board votes informatively. Hence, the outcome of the vote coincides with the social
planner's choice. By Proposition 3, \( n^* \) produces more efficient outcomes. Hence, \( n^* \) dominates any \( n < n^* \).

**Part (ii).** If unanimity is the optimal voting rule then it follows from Proposition 6 that as \( n \) grows, informative equilibria become less likely, or, equivalently, the threshold above which informative voting is unsustainable falls. Given \( \frac{c}{q} \), let \( n^* \) be the maximum \( n \) so that (A.21) holds. We will show that \( n^* \) dominates any \( n > n^* \) and any \( n < n^* \). Consider any \( n > n^* \). The board will always keep the incumbent, regardless of directors' signals, while at \( n^* \) a bad CEO is fired with a positive probability. Hence \( n^* \) dominates any \( n > n^* \). Consider any \( n < n^* \). The board votes informatively. Hence, the outcome of the vote coincides with the social planner's choice. By Proposition 3, \( n^* \) produces more efficient outcomes. Hence, \( n^* \) dominates any \( n < n^* \).

**Proof of Proposition 8**

**Part (i).** We want to show that \( \Upsilon(k, n, \alpha, \gamma, \mu|b) = \frac{p(\sum_{j=1}^{n} e_j \geq k|b)}{p(\sum_{j=1}^{n} e_j < k|b)} \) decreases as \( \mu \) rises. It is easy to see that

\[
\frac{\partial p(B|b)}{\partial \mu} = \frac{\partial}{\partial \alpha} \frac{(1-\mu)\alpha}{(1-\mu)\alpha + \mu(1-\alpha)}
= -\alpha(\alpha + \mu - 2\alpha\mu) - (\alpha - \alpha\mu)(1 - 2\alpha)
= \frac{\alpha(\alpha - 1)}{(\mu - 2\alpha\mu + \alpha)^2} < 0, \text{ since } \alpha < 1.
\]

(A.31)

Hence, the difference \( p(B|b) - (1 - \gamma) \) is decreasing in \( \mu \). It remains to show that \( p(\sum_{j=1}^{n} e_j \geq k|b) \) is also decreasing in \( \mu \).

As before, let \( \lambda \equiv p(e_j = 1|b) = \frac{\mu - 2\mu\alpha + \alpha^2}{\mu - 2\mu\alpha + \alpha} \). Then,

\[
\frac{\partial \lambda}{\partial \mu} = \frac{(1 - 2\alpha)(\mu - 2\mu\alpha + \alpha) - (\mu - 2\mu\alpha + \alpha^2)(1 - 2\alpha)}{(\mu - 2\mu\alpha + \alpha)^2}
= \frac{(1 - 2\alpha)(1 - \alpha)}{(\mu - 2\mu\alpha + \alpha)^2} < 0, \text{ since } \frac{1}{2} < \alpha < 1.
\]

(A.32)

To complete the proof notice that \( p(\sum_{j=1}^{n} e_j \geq k|b) \) is 1 minus the cumulative distribution function of a Binomial distribution with parameter \( \lambda \). It is therefore increasing in \( \lambda \).

**Part (ii).** We want to show that when \( \mu = \frac{1}{2} \), \( \Upsilon(k, n, \alpha, \gamma, \mu|b) \) increases as \( \alpha \) rises. It is easy
to see that

\[
\frac{\partial p(B|b)}{\partial \alpha} = \frac{\partial}{\partial \alpha} \left( \frac{(1-\mu)\alpha}{(1-\mu)\alpha + \mu(1-\alpha)} \right) = \frac{(1-\mu)((1-\mu)\alpha + \mu(1-\alpha)) - (1-\mu)\alpha(1-2\mu)}{(1-\mu)\alpha + \mu(1-\alpha))^2} = \frac{(1-\mu)\mu}{(1-\mu)\alpha + \mu(1-\alpha))^2} > 0, \text{ since } \mu < 1.
\] (A.33)

Hence, the difference \( p(B|b) - (1-\gamma) \) is increasing in \( \alpha \). It remains to show that \( p(\sum_{j=1}^{n} e_j \geq k|b) \) is also increasing in \( \alpha \) when \( \mu = \frac{1}{2} \). Notice that when \( \mu = \frac{1}{2} \), \( \lambda = \frac{\mu+2\mu\alpha+\alpha^2}{\mu-2\mu\alpha+\alpha} = 1 + 2\alpha^2 - 2\alpha \).

Thus,

\[
\frac{\partial \lambda}{\partial \alpha} = 4\alpha - 2 > 0 \text{ since } \frac{1}{2} < \alpha < 1.
\] (A.34)

To complete the proof notice that \( p(\sum_{j=1}^{n} e_j \geq k|b) \) is 1 minus the cumulative distribution function of a Binomial distribution with parameter \( \lambda \). It is therefore increasing in \( \lambda \).

**Part (iii).** Notice directly that \( \Upsilon(k,n,\alpha,\gamma,\mu|b) = p(\sum_{j=1}^{n} e_j \geq k|b) \) is increasing in \( \gamma \).

**Proof of Proposition 9**

Insiders’ vote determines the outcome. Hence, outsiders are irrelevant to the board’s decision-making and always vote to retain the CEO in order to avoid the cost of dissent.

Since insiders always avoid the cost of dissent by voting as a block, they solve the following problem:

\[
\max_{v_i} \pi^{\text{insider}}_i(v_i; f) = -qp(B|f)(1-v_i) - q(1-\gamma)v_i.
\] (A.35)

Hence, they vote to fire the CEO \( (v_i = 1) \) if and only if \( -qp(B|f) < -q(1-\gamma) \), or, equivalently,

\[
p(B|f) > 1-\gamma.
\] (A.36)

The result follows since \( p(B|f = g) = p(B|g) \), \( p(B|f = b) = p(B|b) \), and \( p(B|b) > 1-\gamma > p(B|g) \) by assumption.

**Proof of Proposition 10**

First consider insiders. They trade off expected costs of dissent versus their expected equity loss. We only need to consider the case when insiders collectively observe a bad signal since if the signal is good they never vote to fire the CEO. Insiders with a bad signal vote informatively if and
only if
\[ cp \left( \sum_{j=1}^{n-h} e_j \leq k - h | f = b \right) < qp \left( \sum_{j=1}^{n-h} e_j \geq k - h | f = b \right) \left( p(B|f = b) - (1 - \gamma) \right). \tag{A.37} \]
(A.37) reflect the fact that there must be at least \( k - h \) votes against the CEO cast by outsiders (whose number is \( n - h \)). (A.37) is equivalent to
\[ \frac{c}{q} \leq \frac{p\left( \sum_{j=1}^{n-h} e_j \geq k - h | f = b \right) (p(B|f = b) - (1 - \gamma))}{p\left( \sum_{j=1}^{n-h} e_j < k - h | f = b \right)}. \tag{A.38} \]
Consider outsiders. Again, it is outsiders with bad private signals who determine the equilibrium since after observing a good signal they always vote to retain the CEO. Outsiders with bad signals must consider two scenarios: when insiders vote to fire the CEO and when insiders vote to keep the CEO. If insiders vote to fire the CEO, there needs to be \( k - h \) outsiders to oust her. If insiders vote to retain the CEO, the number of outsiders necessary to oust her is \( k \). It follows that outsiders vote informatively when
\[ \frac{c}{q} \leq \frac{p(f = b|b)p\left( \sum_{j=1}^{n-h} e_j \geq k - h | f = b \right) + p(f = g|b)p\left( \sum_{j=1}^{n-h} e_j \geq k | b \right)}{1 - p(f = b|b)p\left( \sum_{j=1}^{n-h} e_j \geq k - h | f = b \right) - p(f = g|b)p\left( \sum_{j=1}^{n-h} e_j \geq k | b \right)} \times (p(B|b) - (1 - \gamma)). \tag{A.39} \]
It is easy to see that (A.39) is the binding constraint. Notice that since insiders’ cumulative signal is equivalent to having a single independent signal, \( p(B|f = b) = p(B|b) \), \( p\left( \sum_{j=1}^{n-h} e_j \geq k - h | f = b \right) = p\left( \sum_{j=1}^{n-h} e_j \geq k - h | b \right), \) \( p(f = b|b) = \lambda, \) \( p(f = g|b) = 1 - \lambda, \) where \( \lambda \equiv p(e_j = 1|b) = \frac{\mu - 2 \mu \alpha + \alpha^2}{\mu - 2 \mu \alpha + \alpha} \) as before.
(A.38) can now be rewritten as
\[ \frac{c}{q} \leq \frac{p\left( \sum_{j=1}^{n-h} e_j \geq k - h | b \right)}{1 - p\left( \sum_{j=1}^{n-h} e_j \geq k - h | b \right)} \left( p(B|b) - (1 - \gamma) \right). \tag{A.40} \]
(A.39) can be rewritten as
\[ \frac{c}{q} \leq \frac{\lambda p\left( \sum_{j=1}^{n-h} e_j \geq k - h | b \right) + (1 - \lambda)p\left( \sum_{j=1}^{n-h} e_j \geq k | b \right)}{1 - \lambda p\left( \sum_{j=1}^{n-h} e_j \geq k - h | b \right) - (1 - \lambda)p\left( \sum_{j=1}^{n-h} e_j \geq k | b \right)} \times (p(B|b) - (1 - \gamma)). \tag{A.41} \]
Now notice that

\[ p(\sum_{j=1}^{n-h} e_j \geq k - h \mid b) > \lambda p(\sum_{j=1}^{n-h} e_j \geq k - h \mid b) + (1 - \lambda) p(\sum_{j=1}^{n-h} e_j \geq k \mid b). \]  

(A.42)

(A.42) follows because \( \lambda < 1 \) and \( p(\sum_{j=1}^{n-h} e_j \geq k - h \mid b) > p(\sum_{j=1}^{n-h} e_j \geq k \mid b) \). Hence, whenever \( \frac{q}{\xi} \) satisfies (A.39) it also satisfies (A.38). If \( \frac{q}{\xi} \) satisfies (A.38) but does not satisfy (A.39), then outsiders always vote to retain the CEO. Since insiders don’t have enough votes to overturn that decision, they also vote to retain the CEO.

**Proof of Proposition 11**

Part (i). Notice that when the board consists exclusively of outsiders, board members receive \( n \) independent signals about CEO’s quality. If there is only one insider on the board, there are also \( n \) independent signals about CEO’s quality (a single insider is equivalent to an outsider). In general, if there are \( h \) insiders on the board, \( h \geq 1 \), the number of independent signals obtained by directors is \( n - h + 1 \).

To measure the quality of the board’s decision making we use the likelihood of firing a bad CEO. By Proposition 3, the larger the number of independent signals, the higher this likelihood is. Notice that when \( \frac{q}{\xi} \leq \Upsilon(k, n, \alpha, \gamma, \mu \mid b) \), a board consisting exclusively of outsiders or a board with at most one insider will vote informatively. Such a board reveals the maximum possible number of independent signals about CEO’s ability (\( n \)). Hence, this board composition is optimal when \( \frac{q}{\xi} \leq \Upsilon(k, n, \alpha, \gamma, \mu \mid b) \).

Part (ii). When \( \frac{q}{\xi} > \Upsilon(k, n, \alpha, \gamma, \mu \mid b) \), a board with at most one insider will always vote to retain the CEO. Such a board obtains \( n \) independent signals about CEO’s quality but doesn’t reveal any of them and a bad CEO is fired with zero probability. If \( \frac{q}{\xi} \leq \Theta(k, n, q, \alpha, \gamma, \mu, h) \) for some \( h = 2, \ldots, n - 1 \), then a board with more than one insider will vote informatively by Proposition 10. Hence, it will reveal \( n - h + 1 \) independent signals and will fire a bad CEO with a positive probability.

Part (iii). When \( \frac{q}{\xi} > \Theta(k, n, q, \alpha, \gamma, \mu, h) > \Upsilon(k, n, \alpha, \gamma, \mu \mid b) \) for all \( h = 2, \ldots, n - 1 \), then by proof of Proposition 10 outsiders always vote to retain the CEO. If insiders do not dominate the board, they also vote to retain the CEO (by Proposition 10). If they dominate the board, they vote in accordance with their cumulative inference and a bad CEO is fired with a positive probability. Further, since outside directors always vote to retain the CEO, there is no informational advantage of adding them to the board. Hence, it is optimal to have only inside board members.

**Proof of Proposition 12**

Conditional on observing public signal \( H \), (11) becomes \( p(B \mid g, H) < 1 - \gamma < p(B \mid b, H) \). This condition is equivalent to the condition that \( p(B \mid g) < 1 - \gamma < p(B \mid b) \) in the basic model, with
\( p(B|g, H) \) replacing \( p(B|g) \) and \( p(B|b, H) \) replacing \( p(B|b) \). Therefore, after observing public signal \( H \), the game is equivalent to our basic game with \( p(B|g, H) \) replacing \( p(B|g) \) and \( p(B|b, H) \) replacing \( p(B|b) \). Hence, by Proposition 4 the board either always keeps the CEO or each director votes according to his private valuation. The latter outcome is possible if and only if

\[
\frac{c}{q} \leq \frac{p(\sum_{j=1}^{n} e_j \geq k - 1|b, H) (p(B|b, H) - (1 - \gamma))}{p(\sum_{j=1}^{n} e_j < k - 1|b, H)}. \tag{A.43}
\]

Analogously, conditional on observing \( L \), directors will vote truthfully and reveal their private valuations if and only if

\[
\frac{c}{q} \leq \frac{p(\sum_{j=1}^{n} e_j \geq k - 1|b, L) (p(B|b, L) - (1 - \gamma))}{p(\sum_{j=1}^{n} e_j < k - 1|b, L)}. \tag{A.44}
\]

**Proof of Proposition 13**

First we calculate updated probabilities conditional on observing a particular public signal.

\[
p(B|g, H) = \frac{p(B)p(g, H|B)}{p(B)p(g, H|B) + p(G)p(g, H|G)}
= \frac{p(B)p(g|B)p(H|B)}{p(B)p(g|B)p(H|B) + p(G)p(g|G)p(H|G)}
= \frac{(1 - \mu)(1 - \alpha)(1 - \phi) + \mu \alpha \phi}{(1 - \mu)(1 - \alpha)(1 - \phi) + \mu \alpha \phi}, \tag{A.45}
\]

where the second line follows from the assumption of conditional independence between public and private signals. It is easy to see that \( p(B|g, H) < p(B|g) \):

\[
\frac{(1 - \mu)(1 - \alpha)(1 - \phi)}{(1 - \mu)(1 - \alpha)(1 - \phi) + \mu \alpha \phi} < \frac{(1 - \mu)(1 - \alpha)}{(1 - \mu)(1 - \alpha) + \mu \alpha} \iff \phi > 1 - \phi \iff \phi > \frac{1}{2}. \tag{A.46}
\]

Similarly,

\[
p(B|g, L) = \frac{(1 - \mu)(1 - \alpha)\phi}{(1 - \mu)(1 - \alpha)\phi + \mu \alpha (1 - \phi)} > p(B|g), \tag{A.47}
\]

11
because each director knows that if a replacement is hired, she will stay in the second round and will be revealed at time 5. Going back to time 2, the game is now equivalent to the initial game simply keep the replacement they hired after the first round. The true quality of this replacement proposition 4 says that the board doesn’t fire the CEO in the second round (at time 4): directors is equivalent to the initial one-period game (with \( \gamma \) round (at time 2). In this case, the board hired a replacement and in the second round the game never fired in the first round. We prove by contradiction. Assume the CEO was fired in the first rest of this proposition follows from the equilibrium described in Proposition 12 and the relation of \( \Upsilon(\cdot) \) and \( \Upsilon(\cdot, \mu) \) that we have just derived.

**Proposition 14**

We solve the problem within the rational expectations framework by starting with the outcome of the second round. It is easy to see that if \( \frac{B}{2} > \frac{p(\sum_{j=1}^{n} e_j \geq k-1|b) \left(p(B|b) - (1 - \gamma)\right)}{p(\sum_{j=1}^{n} e_j < k-1|b)} \) then the CEO is never fired in the first round. We prove by contradiction. Assume the CEO was fired in the first round (at time 2). In this case, the board hired a replacement and in the second round the game is equivalent to the initial one-period game (with \( \gamma \) replacing \( \mu \)). Given the above restriction on \( \frac{B}{2} \), proposition 4 says that the board doesn’t fire the CEO in the second round (at time 4): directors simply keep the replacement they hired after the first round. The true quality of this replacement will be revealed at time 5. Going back to time 2, the game is now equivalent to the initial game because each director knows that if a replacement is hired, she will stay in the second round and

\[
p(B|b, H) = \frac{(1 - \mu)\alpha(1 - \phi)}{(1 - \mu)\alpha(1 - \phi) + \mu(1 - \alpha)\phi} < p(B|b), \quad (A.48)
\]

\[
p(B|b, L) = \frac{(1 - \mu)\alpha\phi}{(1 - \mu)\alpha\phi + \mu(1 - \alpha)(1 - \phi)} > p(B|b). \quad (A.49)
\]

The probability of director \( j \) observing private signal \( b \), given director \( i \)'s signal, can be rewritten as

\[
p(s_j = b|s_i) = (1 - \alpha) + (2\alpha - 1)p(B|s_i). \quad (A.50)
\]

(A.50) is increasing in \( p(B|s_i) \). We showed above that \( p(B|b, H) < p(B|b) < p(B|b, L) \). Hence, \( p(s_j = b|b, H) < p(s_j = b|b) < p(s_j = b|b, L) \). It follows from the properties of the Binomial distribution that \( p(\sum_{j=1}^{n} e_j \geq k - 1|b, H) \) \( p(\sum_{j=1}^{n} e_j \geq k - 1|b) \) \( p(\sum_{j=1}^{n} e_j \geq k - 1|b, L) \) and \( p(\sum_{j=1}^{n} e_j < k|b, H) \) \( p(\sum_{j=1}^{n} e_j < k|b) \) \( p(\sum_{j=1}^{n} e_j < k|b, L) \). Therefore,

\[
\frac{p(\sum_{j=1}^{n} e_j \geq k - 1|b, H) \left(p(B|b, H) - (1 - \gamma)\right)}{p(\sum_{j=1}^{n} e_j < k - 1|b, H)} < \frac{p(\sum_{j=1}^{n} e_j \geq k - 1|b) \left(p(B|b) - (1 - \gamma)\right)}{p(\sum_{j=1}^{n} e_j < k - 1|b)} < \frac{p(\sum_{j=1}^{n} e_j \geq k - 1|b, L) \left(p(B|b, L) - (1 - \gamma)\right)}{p(\sum_{j=1}^{n} e_j < k - 1|b, L)}. \quad (A.51)
\]

(A.51) is equivalent to \( \Upsilon(k, n, q, \alpha, \gamma, \mu|b, H) < \Upsilon(k, n, \alpha, \gamma, \mu|b) < \Upsilon(k, n, q, \alpha, \gamma, \mu|b, L) \). The rest of this proposition follows from the equilibrium described in Proposition 12 and the relation between \( \Upsilon(k, n, q, \alpha, \gamma, \mu|b, H) \), \( \Upsilon(k, n, \alpha, \gamma, \mu|b) \) and \( \Upsilon(k, n, q, \alpha, \gamma, \mu|b, L) \) that we have just derived.
therefore they will learn her true quality at date 5. Given the restriction on \( \frac{c}{q} \), however, they can never fire the CEO in the initial game. Contradiction.

We assume below that \( \frac{c}{q} < \frac{p(\sum_{j=1}^{n} e_j \geq k-1|B) \{p(B|b)-(1-\gamma)\}}{p(\sum_{j=1}^{n} e_j < k-1|b)} \).

If the CEO was retained in the first round, then the outcome of the second round is completely determined by the public signal observed at time 3. Also, directors in this case are able avoid the cost of dissent since they vote unanimously on the public signal. Given the outcome of the second round, we can now calculate the expected payoff from having the CEO retained in the first round, conditional on director \( i \)'s signal:

\[
\pi^{\text{round 1}}(\text{retained}|s_i) = -q(1-\gamma)p(G|s_i)p(L|G) - 2q(2-\phi)p(B|s_i)p(H|B)
\]

\[
= q(1-\gamma)p(B|s_i)p(L|B) + q(1-\gamma)p(B|s_i)(1-\phi) - 2q(2-\phi)p(B|s_i)(1-\phi)
\]

\[
= q(1-\gamma)(1-p(B|s_i))p(B|s_i)(1-\phi) - 2q(2-\phi)p(B|s_i)(1-\phi)
\]

\[
= q(1-\gamma)(1-\phi)p(B|s_i) - q(1-\gamma)p(B|s_i)\phi
\]

\[
= q(1-\gamma)(1-\phi) - p(B|s_i)q(2 - 2\phi - (1-\gamma)(1-\phi) + (1-\gamma)\phi)
\]

\[
= q(1-\gamma)(1-\phi) - p(B|s_i)q(2 - 2\phi + (1-\gamma)(2\phi - 1))
\]

\[
= q(1-\gamma)(1-\phi) - p(B|s_i)q(1 + (1-2\phi)(1 - 1 + \gamma))
\]

\[
= q(1-\gamma)(1-\phi) - q(1-\gamma)p(B|s_i)q(1 - \gamma(2\phi - 1))
\]

\[
= q(1-\gamma)(1-\phi) - q(B|s_i) + qp(B|s_i)\gamma(2\phi - 1)
\]

\[
\equiv qA(\gamma, \phi, \alpha, n, k|s_i).
\]

Notice that (A.52) is maximized when \( \phi = 1 \), which is quite intuitive: if the public signal is very precise, waiting until this information is revealed is highly valuable. Also notice that \( qA(\gamma, \phi, \alpha, n, k|g) > qA(\gamma, \phi, \alpha, n, k|b) \) since \( 1 - \gamma(2\phi - 1) > 0 \): the value of waiting is greatest for directors who already think the CEO is likely to be good.

We now need to calculate the expected payoff from having the CEO fired in the first round. Since \( \frac{c}{q} < \frac{p(\sum_{j=1}^{n} e_j \geq k-1|B) \{p(B|b)-(1-\gamma)\}}{p(\sum_{j=1}^{n} e_j < k-1|b)} \), directors in the second round will vote truthfully. Hence, the CEO will be fired with the probability that \( k \) or more directors observe a bad signal and will be
retained otherwise. Let $\bar{V}$ denote the number of directors who observe a bad signal in the second round. Then, the payoff from having the CEO fired in the first round is given by

$$\pi^{\text{round1}}(\text{fired}|s_i) = -\gamma p(b|G)cp(\bar{V} < k|G) - \gamma p(b|G)q(1-\gamma)p(\bar{V} \geq k|G)$$

$$- \gamma p(g|G)q(1-\gamma)p(\bar{V} \geq k|G)$$

$$- (1-\gamma)p(b|B)(c + q)p(\bar{V} < k|B)$$

$$- (1-\gamma)p(b|B)q(1-\gamma)p(\bar{V} \geq k|B)$$

$$- (1-\gamma)p(g|B)qp(\bar{V} < k|B)$$

$$- (1-\gamma)p(g|B)q(1-\gamma)p(\bar{V} \geq k|B)$$

$$= -\gamma (1-\alpha)cp(\bar{V} < k|G) - \gamma (1-\alpha)q(1-\gamma)p(\bar{V} \geq k|G)$$

$$- \gamma \alpha q(1-\gamma)p(\bar{V} \geq k|G)$$

$$- (1-\gamma)\alpha(c + q)p(\bar{V} < k|B)$$

$$- (1-\gamma)\alpha q(1-\gamma)p(\bar{V} \geq k|B)$$

$$- (1-\gamma)(1-\alpha)qp(\bar{V} < k|B)$$

$$- (1-\gamma)(1-\alpha)q(1-\gamma)p(\bar{V} \geq k|B)$$

$$= -c\gamma (1-\alpha)p(\bar{V} < k|G) - c(1-\gamma)\alpha p(\bar{V} < k|B)$$

$$- q\gamma (1-\alpha)(1-\gamma)p(\bar{V} \geq k|G)$$

$$- q\gamma \alpha (1-\gamma)p(\bar{V} \geq k|G)$$

$$- q(1-\gamma)\alpha p(\bar{V} < k|B)$$

$$- q(1-\gamma)\alpha q(1-\gamma)p(\bar{V} \geq k|B)$$

$$= c\beta(\gamma, \alpha, n, k) + q\zeta(\gamma, \alpha, n, k).$$

There is no obvious way to simplify (A.53). For our purposes, however, it suffices to notice that there is no deterministic relationship between $A(\gamma, \phi, \alpha, n, k|s_i)$, $B(\gamma, \alpha, n, k)$ and $C(\gamma, \alpha, n, k)$ and that neither $B(\gamma, \alpha, n, k)$ nor $C(\gamma, \alpha, n, k)$ depends on $\phi$ or $s_i$ (which must be true since no signal about the replacement’s quality can be observed before the replacement is hired).

In equilibrium directors choose the voting strategy with the highest payoff. However, if $qA(\gamma, \phi, \alpha, n, k|g) > qA(\gamma, \phi, \alpha, n, k|b) > cB(\gamma, \alpha, n, k) + qC(\gamma, \alpha, n, k)$, it is always optimal to keep the current CEO regardless of the signal. If, on the other hand, $cB(\gamma, \alpha, n, k) + qC(\gamma, \alpha, n, k) > qA(\gamma, \phi, \alpha, n, k|g) > qA(\gamma, \phi, \alpha, n, k|b)$, then it is always optimal to fire the incumbent.

We therefore focus on the interesting case when $qA(\gamma, \phi, \alpha, n, k|g) > cB(\gamma, \alpha, n, k) + qC(\gamma, \alpha, n, k) > qA(\gamma, \phi, \alpha, n, k|b)$. Directors with signal $g$ always vote to keep the current CEO, it is the directors with signal $b$ who determine the equilibrium. They choose to reveal their private valuation if and
only if
\[qA(\gamma, \phi, \alpha, n, k|b) \leq - (c - qA(\gamma, \phi, \alpha, n, k|b)) p\left(\sum_{j=1}^{n} e_j < k - 1|b\right)\]
\[+ cB(\gamma, \alpha, n, k)p\left(\sum_{j=1}^{n} e_j \geq k - 1|b\right) + qC(\gamma, \alpha, n, k)p\left(\sum_{j=1}^{n} e_j \geq k - 1|b\right) \iff\]
\[c\left(p\left(\sum_{j=1}^{n} e_j < k - 1|b\right) - B(\gamma, \alpha, n, k)p\left(\sum_{j=1}^{n} e_j \geq k - 1|b\right)\right) \leq\]
\[( - qA(\gamma, \phi, \alpha, n, k|b) + qC(\gamma, \alpha, n, k)\right) p\left(\sum_{j=1}^{n} e_j \geq k - 1|b\right) \iff\]
\[c \leq \left( - A(\gamma, \phi, \alpha, n, k|b) + C(\gamma, \alpha, n, k)\right) p\left(\sum_{j=1}^{n} e_j \geq k - 1|b\right).
\]

(A.54)

Define
\[\Psi(\gamma, \alpha, \phi, k, n) = \min\{\left( - A(\gamma, \phi, \alpha, n, k|b) + C(\gamma, \alpha, n, k)\right) p\left(\sum_{j=1}^{n} e_j \geq k - 1|b\right),\]
\[
\frac{p\left(\sum_{j=1}^{n} e_j \geq k - 1|b\right) - B(\gamma, \alpha, n, k)p\left(\sum_{j=1}^{n} e_j \geq k - 1|b\right)}{p\left(\sum_{j=1}^{n} e_j < k - 1|b\right)}(p(B|b) - (1 - \gamma))\}.
\]

Then, if \(c > \Psi(\gamma, \alpha, \phi, k, n)\), the CEO is never fired in the first round and if \(c \leq \Psi(\gamma, \alpha, \phi, k, n)\), the CEO is fired in the first round with a positive probability (equal to the probability that \(k\) directors observe a bad signal given the CEO’s true quality).