The Pricing of Initial Public Offerings: A Dynamic Model with Information Production

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ABSTRACT
This paper presents an information-theoretic model of IPO pricing in which insiders sell stock in both the IPO and the secondary market, have private information about their firm’s prospects, and outsiders may engage in costly information production about the firm. High-value firms, knowing they are going to pool with low-value firms, induce outsiders to engage in information production by underpricing, which compensates outsiders for the cost of producing information. The information is reflected in the secondary market price of equity, giving a higher expected stock price for high-value firms.

The underpricing of initial public offerings (IPOs) is one of the most extensively documented anomalies in financial economics (see, e.g., McDonald and Fisher (1972), Logue (1973), Ibbotson (1975), Ritter (1984), and Ritter (1991)). Ibbotson, Sindelar, and Ritter (1988) report an average initial return of 16.4% for IPOs made during 1960 to 1987, computed from the offer price to the closing price on the first day of trading in the secondary market. An explanation of the underpricing puzzle popular with investment bankers and other practitioners is that underpricing generates publicity about the firm making the IPO and induces investors to learn more about that firm. They contend that this leads to a runup in the secondary market share price, and consequently, is in the best interests of firms going public.

The argument has not been subjected to serious examination in the academic literature. In this paper, we model the above hypothesis and develop a scenario in which underpricing is generated by the desire of firm insiders to induce information production about their firm. In our model, insiders have private information about the quality of their firm’s projects; outsiders may

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285
acquire information at a cost to reduce this information asymmetry. The firm (or its insiders) sells stock in an IPO and again in a second offering, made after trading begins in the secondary market. Insiders of high-value firms are motivated to maximize outsider information production so that this information will be reflected in the secondary market price of their firm's equity, increasing its expected value. However, since information production is costly, only a lower IPO share price will induce more outsiders to produce information. The equilibrium initial offer price, which may involve underpricing under certain conditions, emerges from this tradeoff.

Much of the existing literature has focused on dissipative signaling by insiders (see, e.g., Ross (1977), Leland and Pyle (1977)) or information production (certification) by financial intermediaries (see, e.g., Campbell and Kracaw (1980)) as ways of dealing with information asymmetry in the equity market. Our model demonstrates that costly information production by outside investors may be of equal importance in minimizing the impact of private information in IPOs. In order to focus on information production by outsiders, we have chosen a model structure here that does not have separating equilibria, ruling out credible signaling by insiders; in addition, we do not model certification by financial intermediaries. However, in practice, insiders may have access to more than one of these mechanisms for dealing with information asymmetry; they may then compare the costs involved in using each of these alternatives and choose that one (or a combination) which maximizes their net payoff.

Our model generates several implications consistent with the recent empirical evidence on IPOs. First, IPOs which are oversubscribed to a greater degree are associated with more underpricing, as documented by Beatty and Ritter (1986). Second, the extent of underpricing is greater for firms with projects that are costlier to evaluate, a prediction supported by Muscarella and Vetsuypens (1987) and Ritter (1991). Third, it is often in the issuers' interest to price equity in the IPO below the highest price at which they can sell, since this results in larger combined proceeds from the initial and second offerings. Consistent with this is the fact that issuers and investment bankers view underpriced, oversubscribed IPOs as successful. Supporting evidence is also provided by Muscarella and Vetsuypens (1989), who document significant underpricing even in the IPOs of investment banks. We will discuss these and other implications further in Section IV.

The rest of the paper is organized as follows. In Section I, we discuss how our paper relates to other work. In Section II we describe the model, and characterize the equilibrium in the new issues market in Section III. In Section IV we describe the empirical implications of the model, relating it to the existing evidence. We conclude in Section V. The proofs of all propositions are in the Appendix.

I. Relation to Other Work

Rock (1986) develops a model where uninformed investors face a bias in IPO share allocation due to the presence of a group of informed outsiders. Firms
are forced to underprice in order to compensate uninformed investors for this adverse selection, since they would otherwise receive below-average returns and withdraw from the new issues market. Thus underpricing is a cost imposed on the issuing firm by the informed outsiders. In contrast, information production by outsiders is endogenous to our model, and underpricing arises here because high-value firm insiders benefit from inducing a greater extent of information production.

In Allen and Faulhaber (1989), Welch (1989), and Grinblatt and Hwang (1989), underpricing, along with the fraction of the firm retained by insiders, is a signal to convey firm insiders’ private information to outsiders. Allen and Faulhaber (1989) and Welch (1989) use the idea of repeat sales, with an assumption that additional information about firm value becomes available exogenously between the sales of equity, to drive the signaling equilibrium with underpricing. While the assumption of insiders with private information is common to signaling models and our model, the tradeoff that drives the pricing of new issues is different in our setting: here the equilibrium is driven by high-value firm insiders inducing costly information production by outsiders.

Benveniste and Spindt (1989) take an auction-design approach to model the pricing of new issues, arguing that underpricing is generated by the underwriters’ desire to extract from “regular IPO investors” information useful in setting the IPO offer price. Other theories of underpricing include the lawsuit avoidance theories of Ibbotson (1975) and Tinic (1988), the IPO “fads” theory of Shiller (1990), and the “cascades” theory of Welch (1992). While such influences may also play a role in the pricing of new issues, we will focus here on the hypothesis that underpricing is generated by insiders inducing costly information production about their firm, a hypothesis not examined in the papers cited.

II. The Model

The model has three dates. At date 0, insiders of a private firm, holding a total of $N$ shares, sell a portion of this equity at a price of $F$ per share in an IPO, to raise an amount $S$ of new capital. Trading commences in the secondary market at date 1, when insiders sell the remaining $N - S/F$ shares they hold to outsiders. At date 2, all cash flows are realized and distributed to shareholders. The expected value of these cash flows is denoted by $V$. The number of bidders in the IPO becomes public knowledge between date 0 and date 1. The model uses the following assumptions:

**Assumption 1:** The equity market is characterized by asymmetric information. Firms may be of two types: “high value” ($V = V_H$) or “low value” ($V = V_L$), $V_H > V_L > 0$. While firm insiders know their firm’s type, outside investors

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1 Entrepreneurs and venture capitalists, adept at startups, may not be as capable of managing mature firms and may therefore divest their holdings in the firm, transferring control to professional managers. Allen and Faulhaber (1989) make a similar assumption.
observe only the unconditional probability $\alpha$ of a firm being of the high-value type.

**Assumption 2:** All agents are risk neutral. The risk-free rate is zero.

**Assumption 3:** At any point in time, there is only one firm making an IPO. The alternative to investing in the IPO is to invest in the riskless asset.

**Assumption 4:** The IPO is made at a “fixed” price; i.e., once the offer price is announced, issuers are not allowed to adjust the offer price in accordance with demand. Each investor can bid for only one share in the IPO.\(^2\)

**Assumption 5:** The IPO is made as a firm commitment offering, so that the offering takes place even if all shares are not sold, any unsold shares being taken up by the investment banker at the equilibrium offer price $F$.\(^3\)

**Assumption 6:** Before bidding in the IPO, outsiders can conduct an “evaluation” of the firm at a cost $C$, which gives them additional (noisy) information about the firm.\(^4\) This reduces the informational disadvantage of outsiders with respect to firm insiders. The outcome of this evaluation is either “good” ($e = G$) or “bad” ($e = B$).

$$\text{Prob}(e = G|V = V_H) = \beta, \quad \text{Prob}(e = G|V = V_L) = \theta, \quad 1 > \beta > 0.5 > \theta > 0.$$  \hfill (1)

$\beta$ and $\theta$ together represent the precision of the information available to investors; a higher $\beta$ or a lower $\theta$ increases the precision. We will impose the following parametric restriction on the cost and precision of this information:

$$\frac{\alpha(1 - \beta)V_H + (1 - \alpha)(1 - \theta)V_L + C/K}{\alpha(1 - \beta) + (1 - \alpha)(1 - \theta)} < \alpha V_H + (1 - \alpha)V_L,$$  \hfill (2)

where $K$ is a lower bound for the probability of a bidder obtaining an allocation of shares in the IPO. Outsiders’ evaluations are stochastically independent of each other. No outsider can credibly sell or otherwise transfer information directly to others.

\(^2\) This assumption serves to simplify our exposition of the investors’ information production condition. Rock (1986) uses a similar assumption to bound demand from each informed investor.

\(^3\) In a firm commitment offering, the underwriter bears all the risk involved in selling the issue and may lose money on some new issues if the offering is undersubscribed. The fee charged to all issuers will therefore be large enough to cover any such losses. We will assume that the underwriter is at the same informational level as other investors, and cannot ex ante distinguish between the two types of firms in setting his fee as long as the offer price set by them is the same in equilibrium. Consequently, we will not explicitly model the role of the underwriter, and we ignore the underwriting fee in our analysis. While the assumption of a firm commitment offering makes the exposition of our model considerably simpler, the basic intuition goes through irrespective of contract choice.

\(^4\) This information production cost can be thought of as the cost of gathering, evaluating, and assimilating available information about the firm’s projects.
A. The Outsiders' Information Production Condition

When both types of firms set the same IPO share price (which we will show to be the case in equilibrium), the offer price does not convey any information about firm type. Outsiders, therefore, make one of the following three choices based on their prior valuation of the firm: engage in uninformed bidding, produce information about the firm and then decide whether or not to bid, or ignore the IPO and invest in the riskless asset.

The outsiders' prior valuation of a share is $\alpha V_H + (1 - \alpha) V_L$. Therefore, for $F > \alpha V_H + (1 - \alpha) V_L$, there is no uninformed bidding, since the expected payoff from uninformed bidding is negative. However, for an offer price such that $V_L < F \leq \alpha V_H + (1 - \alpha) V_L$, the outsiders' choice between informed and uninformed bidding will depend on the cost and precision of the information available to them. We can show that there exists a certain cutoff value of the offer price, $F$, above which the expected payoff from informed bidding is strictly greater than that from uninformed bidding (so that only informed investors bid for shares). We can also show that $F$ is lower if the information available to outsiders is cheaper or more precise; further, given the parametric restriction (2), $F < \alpha V_H + (1 - \alpha) V_L$.

We now analyze the case in which the IPO share price is in the range $F \leq F \leq V_H$, and as a consequence, informed bidding dominates uninformed bidding. We will demonstrate later that the equilibrium offer price is always in this range, so that only informed bidding exists in equilibrium. Each outsider then chooses either to produce information or to ignore the IPO and invest in the riskless asset. An outsider will produce information if (and only if) his expected (net) payoff from information production is nonnegative; i.e., the expected benefit from producing information outweighs the cost.

In computing his expected payoff from information production, each outsider works backward (in the spirit of dynamic programming) from his date 2 payoff, assuming optimal behavior at each date. Since informed bidding dominates uninformed bidding in this range of the offer price, information has value in the outsiders' decision to bid or not bid for shares. This implies that all information producers who obtain good evaluations bid in the IPO, while those with bad evaluations invest in the riskless asset. Incorporating

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5 If the two types of firms set different offer prices in equilibrium, outsiders can infer firm type by observing the initial offer price, and no outsider will engage in costly information production.

6 $F$ is always higher than $V_L$, since for any offer price $F \leq V_L$, all outsiders will prefer uninformed bidding, knowing they can never lose from bidding across the board in all IPOs.

7 To illustrate this numerically, set $V_H = 80$, $V_L = 60$, $\alpha = 0.5$, $\beta = 0.75$, $\theta = 0.01$, $C = 0.01$, and $K = 0.01$. A simple computation shows that informed bidding dominates uninformed bidding for any offer price above 60.32; i.e., $F = 60.32 < \alpha V_H + (1 - \alpha) V_L = 70$.

8 In Rock (1986), most investors ("uninformed investors") are assumed to be unable to produce information (perhaps due to a prohibitively high cost), while a group of outsiders have privileged access to information ("informed investors"). This generates simultaneous bidding by uninformed and informed investors in that model. In contrast, all outsiders have the same cost of information production in this model, and, depending on the cost and precision of the information, all choose either informed or uninformed bidding.
this optimal bidding behavior, the following condition must be satisfied for
outsiders to engage in information production:

\[
\text{Prob}(e = G) \text{Prob}(V = V_H | e = G) b(V_H - F) +
\text{Prob}(e = G) \text{Prob}(V = V_L | e = G) b'(V_L - F) - C \geq 0. \quad (3)
\]

Here \(\text{Prob}(e = G)\) is the outsiders’ prior probability assessment of getting a
good evaluation; \(\text{Prob}(V = V_H | e = G)\) and \(\text{Prob}(V = V_L | e = G)\) are respec-
tively the probabilities, conditional on a good evaluation, of the firm being of
high or low value; \(b\) and \(b'\) are the bidders’ assessment of the probabilities of
obtaining an allocation of shares in a high-value and a low-value firm,
respectively. The probabilities \(b\) and \(b'\) are less than 1 whenever there is a
chance that the IPO is oversubscribed (and the equity is consequently ra-
tioned).

Using (1), we obtain:

\[
\text{Prob}(e = G) = \beta \text{Prob}(V = V_H) + \theta \text{Prob}(V = V_L) = \beta\alpha + \theta(1 - \alpha). \quad (4)
\]

We compute, using Bayes’ rule:

\[
\text{Prob}(V = V_H | e = G) = \frac{\beta\alpha}{\beta\alpha + \theta(1 - \alpha)},
\]

\[
\text{Prob}(V = V_L | e = G) = \frac{\theta(1 - \alpha)}{\beta\alpha + \theta(1 - \alpha)}. \quad (5)
\]

Using (4) and (5) we can transform the information production condition (3)
to obtain (6):

\[
\beta\alpha b(V_H - F) + \theta(1 - \alpha)b'(V_L - F) - C \geq 0. \quad (6)
\]

The expected payoff from producing information, given by the left-hand
side of (6), depends on the allocation probabilities \(b\) and \(b'\), which depend on
the number of information producers, \(n\). As a prelude to analyzing how the
expected payoff varies with \(n\), we will now study how \(b\) and \(b'\) change with
\(n\).

Of the \(n\) information producers, a certain number obtain good evaluations
and hence bid in the IPO. For \(n \leq S/F\), the number of bidders is always less
than the number of shares offered for sale, implying that \(b = b' = 1\). As \(n\)
exceeds \(S/F\), these probabilities \(b\) and \(b'\) fall below 1, since the larger the
number of information producers, the larger the number who can be expected
to obtain good evaluations, and consequently, bid for shares. Thus, \(b\) is a
function of \(S/F, n, \) and \(\beta; b'\) is a function of \(S/F, n, \) and \(\theta\). Since \(\beta > \theta\), the
proportion of information producers who bid for shares can always be ex-
pected to be higher for a high-value firm than for a low-value firm. Conse-
quently, for \(n \geq S/F\), the probability of receiving an allocation of shares is
always less for a high-value firm than for a low-value firm. To incorporate this intuition, we assume:

**Assumption 7:** \( b = b(n; F; S; \beta); b' = b'(n; F; S; \theta). \)

\( b = b' = 1, \) for \( n \leq S/F; \) \( b < b' < 1, \) for \( n > S/F. \)

We assume that \( b \) and \( b' \) are continuous in their arguments, with continuous partial derivatives. Denote the partial derivatives of \( b \) with respect to \( n, \)
\( F \) and \( S \) by \( b_n, b_F, \) and \( b_S, \) respectively; let \( b'_n, b'_F, \) and \( b'_S \) denote the corresponding partial derivatives of \( b'. \) We will make the following intuitive assumptions about these derivatives:

**Assumption 8:** For \( n \geq S/F, \)

\[
egin{align*}
  b_n < 0, & \quad b'_n < 0, & \quad \left| \frac{b_n}{b} \right| \geq \left| \frac{b'_n}{b'} \right|; & \quad b_F < 0, & \quad b'_F < 0, & \quad \left| \frac{b_F}{b} \right| \geq \left| \frac{b'_F}{b'} \right|; \\
  b_S > 0, & \quad b'_S > 0, & \quad \frac{b_S}{b} \geq \frac{b'_S}{b'}.
\end{align*}
\]

Using Assumption 7 and Assumption 8 in (6), we can show that for \( n > S/F, \) the expected payoff from information production decreases as the number of information producers increases, until, at a certain value of \( n, \) it falls to zero. \(^{10}\) The information production condition (6) will then hold as an equality.

**Lemma 1:** For \( F \in (F, \bar{F}), \) where \( \bar{F} \equiv [\beta \alpha V_H + \theta(1 - \alpha) V_L - C]/[\beta \alpha + \theta(1 - \alpha)], \) the information production condition (6), holding as an equality, defines a decreasing function between \( n, \) the number of information producers corresponding to zero expected payoff from information production, and the offer price \( F. \) This function is of the form. \(^{11}\)

\[
n = n(F; S, C, \alpha, \beta, \theta, V_H, V_L); \quad \frac{\partial n}{\partial F} < 0, \quad \frac{\partial n}{\partial S} > 0, \quad \frac{\partial n}{\partial C} < 0, \quad \frac{\partial n}{\partial \alpha} > 0.
\]

(7)

**Proof:** This follows directly from the Assumptions 7 and 8, and the implicit function theorem.

\(^9\) We will not be concerned here with the specific forms of the functions \( b \) and \( b', \) which can allow for different allocation rules for shares in the IPO; we merely need to impose these intuitive restrictions on the values of \( b \) and \( b'. \)

\(^{10}\) To see this intuitively, notice from (6) that the expected payoff from information production is the sum of three terms: The first (and only positive) term, \( \alpha \beta b(V_H - F), \) becomes smaller with \( n, \) since \( b \) decreases with \( n. \) The second (negative) term, \( \theta(1 - \alpha)b'(V_L - F), \) also becomes numerically smaller with \( n, \) since \( b' \) is decreasing in \( n. \) However, because the first term becomes smaller at a faster rate than the second, and because of the third (negative) term, \( -C, \) which is independent of \( n, \) the sum of the three terms becomes smaller with \( n, \) and falls to zero at a certain value of \( n. \)
Thus, for each value of the offer price in the interval \((F, \bar{F})\), there is a certain value of \(n\) such that each information producer obtains zero expected payoff. If the number of information producers is smaller than this, all information producers obtain a positive expected payoff, and there is an incentive for additional investors to produce information; if it is larger, all information producers obtain a negative expected payoff. We will assume that, in equilibrium, all information producers obtain zero expected payoff, and consequently, will be concerned only with the number of information producers defined by (7); from now on, \(n\) refers to this number.\(^{12}\)

\(\bar{F}\), the upper bound of the domain of the function (7), corresponds to that value of \(F\) at which \(b\) and \(b'\) equal 1, which occurs when \(n\) equals \(S/F\). This is the offer price above which no investor produces information or bids for shares (it is easy to verify that the information production condition is never satisfied for \(F > \bar{F}\)). Further, we can show that \(\bar{F} \geq \alpha V_H + (1 - \alpha) V_L\), given the parametric restriction (2).\(^{13}\) The outsiders' bidding behavior for different ranges of the offer price is depicted in Figure 1.

B. The Secondary Market Price of Equity

Before the IPO, outsiders have heterogeneous information about the firm. Of the \(n\) investors producing information, a certain number, \(\delta\), obtain good evaluations, and the remainder, bad evaluations. All outsiders with good evaluations bid in the IPO, while those with bad evaluations ignore the new issue and invest in the riskless asset.

Before trading begins in the secondary market, however, the number of bidders in the IPO becomes public knowledge, which transmits information across outsiders. For any \(F\), all outsiders know \(n\); further, the number of bidders in the IPO is the same as \(\delta\). Therefore, the information sets of

\(^{11}\) For the purpose of establishing the comparative statics of the model (Proposition 3), we will make the additional assumption that the second-order derivatives of this function, \(\partial^2 n/\partial S \partial F\), \(\partial^2 n/\partial a \partial F\), and \(\partial^2 n/\partial F^2\), are small compared to its first-order derivatives. Since these depend on the \(b\) and \(b'\) functions and their first- and second-order derivatives, this is essentially an additional restriction on the \(b\) and \(b'\) functions.

\(^{12}\) The assumption implicit here is that each investor knows the number of others who are producing information, so that an equilibrium in which only \(n\) investors produce information, and all information producers obtain zero expected payoff, is easily implemented. However, this assumption can be relaxed at the expense of making the model more complex. If each investor does not know the number of others choosing to produce information, each of the \(M\) investors (say) in the IPO market will follow a randomized strategy, producing information with an equilibrium probability \(n/M\) which will be a decreasing function of the IPO offer price) such that the expected payoff from information production is zero. The value of \(n\) given by (7) will then denote the expected number of information producers corresponding to all investors obtaining zero expected payoff from information production. All our results will hold in this case as well, as long as the actual number of information producers becomes known before trading commences in the secondary market. (Milgrom (1981) adopts a somewhat similar approach to model information acquisition in an auction setting.)

\(^{13}\) Using the same parameters as in the numerical illustration in footnote 7, \(\bar{F} = 79.71 > \alpha V_H + (1 - \alpha) V_L = 70\).
outsiders become homogeneous once the number of bidders becomes known, since, for a given $n, \delta$ is a sufficient statistic for the information produced by all outsiders. The equilibrium price of a share of stock will then equal the expectation of the cash flows accruing to each share conditional on the aggregate of the information produced by all outsiders.

**Proposition 1:** In any equilibrium with information production where outsiders rationally learn from prices, the price functional in the secondary market is given by:

$$P(n, \delta) = E[V|n, \delta] = \frac{\beta^\delta(1 - \beta)^{n-\delta} \alpha V_H + \theta^\delta(1 - \theta)^{n-\delta}(1 - \alpha)V_L}{\beta^\delta(1 - \beta)^{n-\delta} \alpha + \theta^\delta(1 - \theta)^{n-\delta}(1 - \alpha)}.$$  \hspace{1cm} (8)

This price functional constitutes a fully revealing rational expectations equilibrium in that it symmetrizes information across all outsiders.

The price in the secondary market incorporates the information produced by all outsiders; given the equilibrium price in the secondary market, each investor in the economy has access to information superior to his own private signal. Consequently, outsiders do not have any incentive to produce information at date 1 because, while the costs of information production remain private, the benefits no longer accrue to individual outsiders.\(^{14}\)

Because of the aggregation of information that occurs at date 1, the information asymmetry between firm insiders and outsiders is reduced.

\(^{14}\) In practice, the price system may be only partially revealing (perhaps due to additional uncertainty in the economy not modeled here). The equilibrium in the secondary market may then be a noisy rational expectations equilibrium. The intuition behind our model holds even in this case, since we merely require that outsiders' incentives to produce information diminish after the start of trading in the equity in the secondary market. See Grossman (1976), Hellwig (1980), and Diamond and Verrecchia (1981) for a discussion of the reduction in investors' incentives to produce information under alternative assumptions about the degree of noise in prices.
However, insiders still have information superior to outsiders: since the number of shares sold by insiders at date 1 is independent of their private information, there is no direct transfer of information from insiders to outsiders. Further, outsiders are indifferent to trading in the equity at date 1, since the stock price accurately reflects all the information available to them.

C. The Insiders’ Objective

At date 0, the objective of insiders is to maximize their expectation of the combined proceeds from the two sales of equity. While insiders know their firm type, $\delta$ is a random variable even to them, since the outsiders’ information is noisy. Consequently, the secondary market price of a share is a random variable even to insiders in an equilibrium with information production. Their actions at date 0 will therefore be based on their expectation of the secondary market price with respect to the probability distribution of $\delta$, conditional on firm type. The probability of obtaining a particular realization of $\delta$ for a given $n$, conditional on the firm types $V_H$ and $V_L$, respectively, are:

$$\text{Prob}(\delta|V = V_H, n) = \left(\frac{n}{\delta}\right)\beta^\delta(1 - \beta)^{n-\delta},$$

$$\text{Prob}(\delta|V = V_L, n) = \left(\frac{n}{\delta}\right)\theta^\delta(1 - \theta)^{n-\delta}. \quad (9)$$

Denote by $P_H$ and $P_L$ the insiders’ expectations of the secondary market price of a share in a high-value firm and in a low-value firm, respectively. Then, in any equilibrium with information production, we can use (9) to compute $P_H$ and $P_L$:

$$P_H = E[P|V = V_H, n] = \sum_{\delta=0}^{n} \left(\frac{n}{\delta}\right)\beta^\delta(1 - \beta)^{n-\delta} \times \left[ \frac{\beta^\delta(1 - \beta)^{n-\delta} \alpha V_H + \theta^\delta(1 - \theta)^{n-\delta} (1 - \alpha)V_L}{\beta^\delta(1 - \beta)^{n-\delta} \alpha + \theta^\delta(1 - \theta)^{n-\delta} (1 - \alpha)} \right],$$

$$P_L = E[P|V = V_L, n] = \sum_{\delta=0}^{n} \left(\frac{n}{\delta}\right)\theta^\delta(1 - \theta)^{n-\delta} \times \left[ \frac{\beta^\delta(1 - \beta)^{n-\delta} \alpha V_H + \theta^\delta(1 - \theta)^{n-\delta} (1 - \alpha)V_L}{\beta^\delta(1 - \beta)^{n-\delta} \alpha + \theta^\delta(1 - \theta)^{n-\delta} (1 - \alpha)} \right]. \quad (10)$$

**Lemma 2:** In any equilibrium with information production:

(a) $P_H$ is monotonically increasing in $n$; $P_L$ is monotonically decreasing in $n$.

(b) Denote by $P_H(n)$ the value of $P_H$ for $n$ investors producing information.

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15 The proof of this lemma is lengthy, and is omitted. It is available on request.
Then, \((P_H(n + 1) - P_H(n))\), the change in \(P_H\) due to another investor producing information, is decreasing in \(n\) and \(\alpha\).

(c) For any finite \(n\), \(P_H < V_H\); \(P_L > V_L\).

(d) \(\partial P_H / \partial \alpha > 0\).

The intuition behind part (a) is that the greater the amount of information produced by outsiders, the closer the expected secondary market price of either firm type is to its true value. Part (b) says that as the amount of information reflected in the secondary market price becomes greater, the impact of an additional investor producing information becomes smaller. Part (c) relies on the fact that as long as \(n\) is finite, there is always some uncertainty among outsiders about true firm type. Finally, as \(\alpha\) is closer to 1, \(P_H\) is closer to \(V_H\) and hence larger, which gives part (d). The variation of \(P_H\) and \(P_L\) with \(n\) is depicted in Figure 2.

![Figure 2](image_url)

**Figure 2.** The relationship between the level of information production (\(n\)) and the expected secondary market price of equity. The case illustrated is for the following values of the model parameters: \(\alpha\), outsiders' prior probability of the firm being of high value, is 0.5; \(\beta\), the probability of outsiders obtaining a good evaluation for a high-value firm, is 0.55; \(\theta\), the same probability for a low-value firm, is 0.4; \(V_H\), insiders' valuation of a share in the high-value firm, is 80; and \(V_L\), insiders' valuation of a share in the low-value firm, is 30. When no investor produces information, the expected secondary market price equals \(\alpha V_H + (1 - \alpha) V_L\), which is 55. As the number of information producers increases, the expected secondary market price of the high-value firm, \(P_H\), approaches \(V_H\), while that of the low-value firm, \(P_L\), approaches \(V_L\).
Denote by $J_H$ and $J_L$ the insiders' expectations of the combined proceeds from the two sales of equity for a high-value firm and a low-value firm, respectively.

$$J_H = S + \left( N - \frac{S}{F_H} \right) P_H,$$

$$J_L = S + \left( N - \frac{S}{F_L} \right) P_L. \quad (12)$$

The insiders of high-value firms set their offer price $F_H$ to maximize $J_H$, while those of low-value firms choose their offer price $F_L$ to maximize $J_L$. In either case, a lower share price in the IPO requires that more shares be sold in the initial offer to raise the amount $S$, leaving the insiders with a smaller number of shares to be sold at date 1.

### III. Market Equilibrium

We define equilibrium as a set of offer prices, a system of beliefs formed by outsiders, and a level of information production in the economy (with the corresponding secondary market price of equity) such that:\textsuperscript{16}

1. The initial offer price set by insiders of each firm type maximizes their objective, given the equilibrium offer price set by the other firm type and the equilibrium beliefs formed by outsiders.
2. Outsider beliefs are rational, given the equilibrium choices made by firm insiders. Along the equilibrium path, outsiders form beliefs using Bayes' rule and, where appropriate, their knowledge of the pricing rule in the economy (i.e., they rationally learn from prices in the secondary market). Any deviation from equilibrium by insiders of either firm type is met by outsider beliefs which yield insiders a lower expected payoff compared to that obtained in equilibrium.
3. The number of information producers is the largest integer equal to or less than that value of $n$ for which all information producers obtain zero expected payoff from information production (given by the function (7)).\textsuperscript{17}

In the following, $F$ will denote the initial offer price set by either firm type: $F_H$ and $F_L$ will be used only where the type of the firm setting the offer price is relevant.

\textsuperscript{16} The equilibrium concept used here is essentially that of rational expectations equilibrium, with additional restrictions on firm insider's strategies and outsider beliefs in response to these strategies in the spirit of the sequential equilibrium of Kreps and Wilson (1982).

\textsuperscript{17} The value of $n$ at which the information production condition (6) holds exactly as an equality may not be an integer. However, the information production technology specified in this model is equivalent to sampling from a Bernoulli distribution, so that the functions $P$, $P_H$, and $P_L$ are defined only for integer values of $n$. Thus, for tractability, we will require the equilibrium number of information producers (for a given offer price) to be the largest integer value for which all information producers obtain a nonnegative expected payoff.
PROPOSITION 2 (Equilibrium): Both types of firms set the same offer price in equilibrium, \( F_H = F_L = F^* \), where \( F^* \) is that value of \( F_H \in (F, \bar{F}) \) which maximizes \( J_H \), with \( P_H \) as specified below.

Along the equilibrium path (i.e., for \( F = F^* \)),
\[
\text{Prob}(V = V_H|F) = \alpha; \quad \text{Prob}(V = V_H|F, e) \text{ is formed from } \text{Prob}(V = V_H|F) \text{ using Bayes' rule. The number of information producers } n^* \text{ is the largest integer equal to or below that value of } n \text{ at which all investors obtain zero expected payoff from information production, given an offer price } F^*. \quad P, P_H, \text{ and } P_L \text{ are given by setting } n = n^* \text{ in (8), (10), and (11), respectively. There is information production by outsiders only at date 0.}
\]

Off the equilibrium path (i.e., for \( F \neq F^* \)),
\[
\text{Prob}(V = V_H|F) = \text{Prob}(V = V_H|F, e) = 0, \quad P = P_H = P_L = V_L.
\]
There is no information production by outsiders either at date 0 or date 1.\(^{18}\)

The equilibrium offer price is the result of the dynamic tradeoff faced by insiders of the high-value firm. While they want to set the highest possible share price in the IPO, they also want to induce the greatest possible extent of information production about their firm, so that this information will be reflected in the secondary market price of their firm’s equity. However, in the interval \((F, \bar{F})\), a lower initial offer price results in a larger number of information producers. Consequently, insiders of the high-value firm pick that offer price in this interval which maximizes their expectation of the combined proceeds from the two sales of equity. The low-value firm, not wanting to reveal its type, mimics the high-value firm by setting the same offer price in equilibrium. Since, in this pooling equilibrium, outsiders cannot distinguish between the two firm types by observing the initial offer price, they produce information; the number of information producers is given by \( n^* \).

The equilibrium involves pooling in the secondary market as well, since outsiders cannot distinguish between the two firm types with probability 1 even at date 1. However, the extent of pooling is lower in the secondary market, since the information produced by outsiders at date 0 is reflected in the secondary market price. Consequently, the stock price of high-value firms goes up, on average, in the secondary market, while that of low-value firms

\(^{18}\) Separating equilibria supported by “reasonable” outsider beliefs do not exist in this model. For an offer price such that \( V_L < F < V_H \), it is clearly optimal for the low-value firm to mimic the high-value firm, and it does so without cost. If, on the other hand, the offer price is in the range \( F \leq V_L \), no investor engages in information production, so that the low-value firm can get a price \( \alpha V_H + (1 - \alpha)V_L \) in the secondary market by mimicking the high-value firm, compared with a price of only \( V_L \) (in the IPO and in the secondary market) if they separate. Thus, the low-value firm will mimic the high-value firm in this range as well. Separating equilibria require a wedge between the two types, either in terms of the costs of various strategies or the benefits from them, which does not exist in our setting. However, the signaling models of Allen and Faulhaber (1989) and Welch (1989) introduce such a wedge and obtain separating equilibria by assuming the exogenous release of additional information about firm type between the IPO and the second offering. (Welch also posits an “imitation cost” that the low-value firm has to incur to mimic the high-value firm.)
goes down: the expected price change from the IPO to the secondary market increases with an increase in the number of information producers. Since the information produced by outsiders is noisy, and the secondary market price depends on the realization of \( \delta \), we can only talk about the expected price change: there will be some high-value firms with a lower equity value in the secondary market than in the IPO, and some low-value firms with a higher equity value.

Since \( n^* \) is an integer, the equilibrium offer price is that one of a finite set of offer prices in the interval \([F, \bar{F}]\) which maximizes the high-value firm insiders’ expectation of the combined proceeds from the two sales of equity. Depending on the parameters of the model, the equilibrium may be a corner solution \((F = \bar{F})\) or an interior solution \((F < F^* < \bar{F})\). The latter case is possible since, by Lemma 2(b), the marginal increase in \( P_H \) from having an additional investor produce information is declining in \( n \).

**Proposition 3 (Comparative statics):** The equilibrium offer price is:

(a) Increasing in the gross proceeds from the IPO, \( S \).

(b) Increasing in the prior probability, \( \alpha \), that the firm is of high value.

(c) Decreasing in the cost of information production, \( C \).

Increasing the amount raised in the initial offering has two effects. First, it increases the number of shares sold in the IPO and reduces the number sold in the secondary market. This means that firm insiders are relatively more concerned about the initial offer price and relatively less about the secondary market price. Second, a larger amount raised in the IPO increases the supply of shares in the initial offer, so that more outsiders produce information for any given offer price. Both effects lead to the equilibrium offer price increasing with \( S \).

A change in \( \alpha \) also has two effects. First, a higher \( \alpha \) leads to a larger number of outsiders producing information for any given initial offer price. Second, for any \( n \), a higher \( \alpha \) leads to a lower marginal benefit from an additional investor producing information. Both these effects lead to a higher equilibrium offer price corresponding to a higher \( \alpha \).

Finally, for any given offer price, a higher \( C \) leads to a lower number of outsiders producing information. Consequently, the equilibrium offer price is declining in \( C \).

In the following, we use the term “underpricing” for the price runup, on average across all new issues, from the IPO to the secondary market (consistent with the usage in the empirical literature).

**Proposition 4 (Underpricing equilibrium):**

(a) \( F^* < \alpha V_H + (1 - \alpha)V_L \) represents an underpricing equilibrium.

(b) For \( S \) small enough, there always exists an underpricing equilibrium.

The average share price in the secondary market (averaging across firm types and possible outcomes) can be shown to be \( \alpha V_H + (1 - \alpha)V_L \). Conse-
quently, underpricing results whenever the firm insiders’ tradeoff is so much in favor of including information production that they price stock in the IPO below $\alpha V_H + (1 - \alpha) V_L$. This happens, for instance, when the amount raised in the IPO is small enough that any sacrifice due to a lower IPO share price is overwhelmed by the benefits of inducing information production.

Combining the results in Proposition 4(a) and Proposition 3, we see that in an underpricing equilibrium, the extent of underpricing is decreasing in the gross proceeds from the IPO, decreasing in the prior probability $\alpha$, and increasing in the cost of information production. We now relate the extent of underpricing to the number of bidders in the IPO.

**Proposition 5:**

(a) In an underpricing equilibrium, the extent of underpricing is increasing in the average number of outsiders bidding for shares across firms.

(b) For any given firm, the price runup (if it occurs) is increasing in the number of bidders in its IPO.

The intuition behind part (a) is that more outsiders choose to produce information when the equilibrium offer price is lower, so that a greater extent of underpricing is associated with a larger number of bidders (on average across all new issues). The intuition behind part (b) is that a larger number of bidders in the IPO of any given firm indicates more good evaluations about that firm, giving a higher secondary market price.\(^{19}\)

In equilibrium, the high-value firm effectively pays outsider information acquisition costs: not only the fixed cost $C$, but also the expected loss arising from buying shares in low-value firms (when the evaluation of the firm is incorrect). Thus, all outsiders obtain zero expected net payoff in equilibrium.\(^{20}\) Further, when outsiders produce information, the expected payoff of insiders of high-value firms is higher, and that of low-value firm insiders lower, than in a setting without information production.

**IV. Empirical Implications**

We now summarize the empirical implications of the model, relating them to the existing evidence.

**Implication 1:** The larger the number of bidders in the IPO, the greater the extent of underpricing. Thus, IPOs that are oversubscribed to a greater degree will be associated with a greater extent of underpricing. This is consistent with the evidence of Beatty and Ritter (1986).

\(^{19}\) Thus, investment bankers and other practitioners may be justified in considering oversubscribed IPOs a “success.”

\(^{20}\) In contrast, signaling models (e.g., Allen and Faulhaber (1989) and Welch (1989)) imply that investors can obtain large positive abnormal returns by bidding across the board on all IPOs.
Implication 2: The greater the cost information production, the greater the extent of underpricing. Consequently, firms that are relatively obscure, or those with projects that are costlier to evaluate, will have a greater extent of underpricing. Consistent with this, Muscarella and Vetsuypens (1987) and Ritter (1991) find a significant inverse relationship between underpricing and the availability of information about the firm (as proxied by the age of the firm).

Implication 3: It is often in the issuers' interest to set the IPO share price below the highest price at which they can sell. Further, firms that plan to approach the secondary market soon after the IPO will have a lower equilibrium offer price than those that do not intend to do so, since the benefits of information production are obtained only on shares sold in a seasoned offering.

Implication 4: The greater the probability that the firm is of high value, the smaller the extent of underpricing. Thus, there may be time periods and industries in which the proportion of high-value firms making IPOs is larger than it is in others; our model predicts less underpricing in such periods and industries. Consistent with this, Ibbotson and Jaffe (1975), Ritter (1984), and Ibbotson, Sindelar, and Ritter (1988) document the existence of "hot issue" and "cold issue" markets, across which the extent of underpricing differs significantly.

Implication 5: The greater the gross proceeds from the IPO, the smaller the extent of underpricing. This is consistent with the evidence of Ritter (1991).

V. Conclusion

In this paper, we developed a model of IPO pricing in which firm insiders sell equity both in the new issues market and in the secondary market, have private information about their firm’s prospects, and outsiders may produce information at a cost about the firm. Underpricing results from insiders inducing information production in order to obtain a more precise valuation of their firm in the secondary market. We related the extent of underpricing to the various characteristics of the new issues market, generating implica-

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21 This implication is exactly the opposite of that obtained in the Rock (1986) setting, where the only effect of informed outsiders is to bias the allocation of shares against uninformed outsiders, forcing issuers to underprice. In that context, the number of informed outsiders decreases as the cost of information production increases (see Beatty and Ritter (1986) and Carter and Manaster (1990)), resulting in a reduction in the required level of underpricing. In our setting, however, issuers benefit from inducing information production, and consequently, costlier information results in a greater extent of underpricing.

22 This prediction also gives a rationale for “road shows” and other marketing efforts by investment bankers as ways of reducing outsider information acquisition costs. For instance, investment bankers may use these occasions to answer potential investors’ questions about the firm, making it easier for them to evaluate the information already provided in the prospectus.
tions consistent with some of the recent evidence and suggesting further empirical tests.

Appendix

Proof of Proposition 1: Since investors know the pricing rule in this economy, they can compute the number of information producers \( n \) for any offer price \( F \). At date 1, all investors know \( \delta \), since the number of bidders in the IPO is public knowledge at this time. Then, using Bayes’ rule, we get (13):

\[
\text{Prob}(V = V_H | n, \delta) = \frac{\text{Prob}(\delta | V = V_H) \text{Prob}(V = V_H)}{\text{Prob}(\delta | V = V_H) \text{Prob}(V = V_H) + \text{Prob}(\delta | V = V_L) \text{Prob}(V = V_L)} \\
= \frac{n^{\delta} \beta^\delta (1 - \beta)^{n - \delta} \alpha^{\delta}}{n^{\delta} \beta^\delta (1 - \beta)^{n - \delta} \alpha + n^{\delta} \theta^\delta (1 - \theta)^{n - \delta} (1 - \alpha)},
\]

substituting for \( \text{Prob}(\delta | V = V_H) \) and \( \text{Prob}(\delta | V = V_L) \) from (9). Since all agents are risk neutral, the secondary market price \( P(n, \delta) \) of a share of stock is given by \( E[V | n, \delta] \), which can immediately be shown using (13) to equal (8).

To demonstrate that (8) is a fully revealing rational expectations equilibrium price functional, we need to show two things. First, we need to show that \( P(n, \delta) \) is one-to-one in \( \delta \): i.e., investors can infer \( \delta \) by observing \( P \). This is true by inspection of (8), noting that \( \delta \) can take on only integer values. Second, we need to show that \( \delta \) is a sufficient statistic for the information produced by all investors. This is the case, since the information production technology postulated in the model is statistically equivalent to sampling from a Bernoulli distribution with an unknown parameter, and we therefore appeal to the standard result that in this case, the number of successes (i.e., \( \delta \)) is a sufficient statistic for the information contained in the entire sample (see DeGroot (1970), page 156). Q.E.D.

Proof of Proposition 2: We will first show that, given outsider equilibrium beliefs, the offer price set by the insiders of each firm type maximizes their objective. High-value firm insiders are always better off in an equilibrium with information production than when on investor produces information. If no investor produces information, their share price will be \( \alpha V_H + (1 - \alpha) V_L \) in the IPO as well as in the secondary market, giving combined proceeds of \( N(\alpha V_H + (1 - \alpha) V_L) \). Since \( \bar{F} \geq \alpha V_H + (1 - \alpha) V_L \) and \( P_H > \alpha V_H + (1 - \alpha) V_L \) as long as at least one investor produces information (Lemma 2(a)), high-value firm insiders can pick an offer price from the interval \([F, \bar{F}]\), for which they obtain higher combined proceeds than the above by inducing information production. Accordingly, they pick that offer price \( F^* \) from this interval which induces the optimal degree of information production (i.e., which maximizes \( J_H \)). Since \( P_L > V_L \) and \( F^* > \bar{F} > V_L \), \( NV_L < S + (N - \)
$S/F^*)P_L$; mimicking the high-value firm is therefore the dominant strategy for low-value firm insiders, so that $F_L = F^*$.

Since $F^*$ is common knowledge, and both firm types set the same offer price $F = F^*$ in equilibrium, outsiders assign the probability $\alpha$ that a firm which sets an offer price $F = F^*$ is a high-value firm. In contrast, if they observe an offer price $F \neq F^*$, they assign zero probability that it is a high-value firm and value its shares at $V_L$, at both date 0 and date 1. Corresponding to these beliefs, the number of investors producing information is $n^*$ for an offer price $F = F^*$ and zero for $F \neq F^*$. Given the above system of insider strategies and outsider beliefs, it follows that $P_L$ are also formed as specified above, with the equilibrium secondary market pricing functional $P$ having the properties discussed in section II.B. It also follows that there is information production in equilibrium only at date 0. Q.E.D.

**Proof of Proposition 3:** We will provide only an outline of the proof of this proposition and will provide computational details on request. This proposition indicates only the direction of changes in the equilibrium offer price $F^*$ with changes in the exogenous parameters $S$, $\alpha$, and $C$. We will therefore use the following methodology for the proof.

First, relax the requirement that the equilibrium value of $n$ be an integer, allowing $n$ to take on the exact value given by the function (7). We will assume that the values of $P_H$ are then given by the continuous extension of the $P_H$ function (10) to $R^+$, defined as follows: (a) This function has the same arguments as (10), and takes on the same values as (10) whenever $n$ is an integer; (b) when $n$ is not an integer, the function takes on values between those given by (10) at adjacent integer values of $n$; (c) the function has continuous derivatives in its arguments, with $\partial P_H / \partial n > 0$, $\partial^2 P_H / \partial n^2 < 0$, $\partial^3 P_H / \partial \alpha \partial n < 0$, $\partial P_H / \partial \alpha > 0$. Clearly, a function according to the above specifications exists, since these properties are simply the continuous analog of the properties of the original $P_H$ function shown under Lemma 2.

With $P_H$ as specified above, the objective of the insiders of the high-value firm, $J_H$, is continuous in all arguments, with continuous derivatives, and is concave in $n$ and $F$, using the assumptions made in footnote 11. The equilibrium offer price, which is that offer price that maximizes $J_H$, is then characterized by the following first-order conditions (for an interior solution):

$$\frac{\partial J_H}{\partial F} = N \left( \frac{\partial P_H}{\partial F} \right) - \frac{S}{F^2} \left( F \cdot \frac{\partial P_H}{\partial F} - P_H \right) = 0,$$

where $\partial P_H / \partial F = (\partial P_H / \partial n)(\partial n / \partial F)$. Applying the envelope theorem to (14), we can compute the derivatives of the equilibrium $F$ with respect to $S$, $\alpha$, and $C$, and show that $dF^*/dS > 0$, $dF^*/\alpha > 0$, and $dF^*/dC < 0$. This gives us the direction of the change in the equilibrium offer price $F^*$ corresponding to changes in $S$, $C$, and $\alpha$ respectively, when there is no integer requirement on the equilibrium $n$. 

Now reimpose the integer requirement on \( n \). Given the nature of this integer requirement, there will be no change in the direction in which the equilibrium offer price \( F^* \) moves with changes in \( S \), \( \alpha \), and \( C \) respectively (except that in some instances, there will not be any movement in \( F^* \) at all.) We thus obtain the comparative static results stated in parts (a), (b), and (c). Q.E.D.

**Proof of Proposition 4:** (a) The average price across all new issues in the secondary market is \( \alpha P_H + (1 - \alpha)P_L \), averaging across all possible outcomes for both types. Substituting for \( P_H \) and \( P_L \) in this expression from (10) and (11) respectively and simplifying, we get:

\[
\alpha V_H \left[ \sum_{\delta = 0}^{n} \binom{n}{\delta} \beta^\delta (1 - \beta)^{n-\delta} \right] + (1 - \alpha)V_L \left[ \sum_{\delta = 0}^{n} \binom{n}{\delta} \theta^\delta (1 - \theta)^{n-\delta} \right].
\]

(15) reduces to \( \alpha V_H + (1 - \alpha)V_L \), since each of the terms in square brackets is equal to 1, being the sums of binomial probabilities over the entire event space. Consequently, whenever \( F^* \) is below this value, there will be a price runup, on average, from the IPO to the secondary market.

(b) By Proposition 3, the equilibrium offer price is decreasing in \( S \). For \( S \) small enough, the offer price falls below \( \alpha V_H + (1 - \alpha)V_L \), and we have an underpricing equilibrium by part (a) of this proposition.

**Proof of Proposition 5:** (a) The average number of investors bidding in an IPO across firms is \( n^*(\alpha \beta + (1 - \alpha)\theta) \), which is increasing in \( n^* \). Since, in the interval \( [F, \tilde{F}] \), the number of information producers is decreasing in \( F \), a larger \( n^* \) results in a lower \( F^* \), giving a greater extent of underpricing in equilibrium.

(b) For a given firm, a larger number of bidders in the IPO indicates a larger \( \delta \). From (8), \( P \) is monotonically increasing in \( \delta \). Consequently, the price runup from the IPO to the secondary market is higher for a larger number of bidders in the IPO. Q.E.D.

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