Appendix A (not to be published): Theoretical models of risk-shifting and asymmetric information that can formally generate the predictions in the paper

A.1. How Putable Convertibles Can Control Risk-Shifting: A Simple Theoretical Example

The following is a simple theoretical example demonstrating how putable convertibles resolve the problem of risk-shifting better than ordinary convertibles by forcing putable convertible issuers to undertake the project with the highest NPV. There are two periods in this example with risk-neutral investors. At time $t = 0$ a firm must choose one of three available investment projects: $S$ (safe), $M$ (medium), or $R$ (risky), and simultaneously raise financing by issuing either straight debt, ordinary (non-putable) convertible debt, or putable convertible debt. Each of the available investment projects requires an initial investment at time $t = 0$ and pays off in two periods at time $t = 2$. There are two possible states of economy at $t = 2$, high ($H$) and low ($L$), occurring with probabilities $p$ and $1 - p$, respectively, which determine the payoff structure of the investment projects (see below). Debt, either straight or convertible, has a face value of $F$ and must return $F(1 + r)^2$ to debt holders at $t = 2$, where $r$ is the risk-free rate of return. If convertible debt is used, convertible debt holders have an option to convert to equity at $t = 2$ at a fixed conversion factor of $\gamma$ (thus convertible debt holders will receive a fraction $\gamma$ of the project’s payoff upon conversion).\(^1\) If putable convertibles are issued, putable convertible debt holders have an option to sell the debt back to the firm at $t = 1$ at a price of $F(1 + y)$, in addition to the conversion option.\(^2,3\)

The safe project $S$ requires an initial investment $I_S = 385$ at $t = 0$ and pays off $H_S = 2,150$ in the high state and $L_S = 1,150$ in the low state at time $t = 2$. Similarly, the initial investments and payoffs of projects $M$ and $R$ are as follows: $I_M = 530$, $H_M = 2,500$, $L_M = 500$; and $I_R = 540$, $H_R = 2,600$, $L_R = 100$.

\(^1\) We assume that, for reasons of corporate control, the firm cannot issue convertible debt with $\gamma$ exceeding a maximum value, denoted by $\gamma^\ast$. In the numerical illustration we have here, we assume $\gamma^\ast = 0.55$.

\(^2\) For example, if the put price exceeds the face value of debt by 5 percent, $y = 0.05$. Similarly, if the putable convertible debt is putable at face value, then $y = 0$.

\(^3\) The notion here is that, in a setting of incomplete contracting, the put option provides outside debt holders and opportunity to intervene and obtain their money back if they observe the firm pursuing suboptimal investment policies, before equity holders are able to dissipate the external capital they have provided to the firm. Thus, in a sense, putable convertibles allow the contracting between equity holders and debt holders to be “more complete.”
Project $S$ has the highest NPV and is the safest and project $R$ has the lowest NPV and is the most risky (i.e., it has the highest standard deviation of cash flows). The following summarizes the payoffs of the three investment projects.

<table>
<thead>
<tr>
<th>Project $S$</th>
<th>Project $M$</th>
<th>Project $R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_S = 2,150$</td>
<td>$H_M = 2,500$</td>
<td>$H_R = 2,600$</td>
</tr>
<tr>
<td>$L_S = 1,150$</td>
<td>$L_M = 500$</td>
<td>$L_R = 100$</td>
</tr>
<tr>
<td>$I_S = -385$</td>
<td>$I_M = -530$</td>
<td>$I_R = -540$</td>
</tr>
</tbody>
</table>

We also assume the following parameter values: $p = 0.5, F = 1,000, r = 0.1, y = 0.05, \text{ and } \gamma = 0.5$. With these parameter values, the net present value of project $S$ is $NPV_S = -315 + (0.5 \times 2,150 + 0.5 \times 1,150)/(1.10)^2 = 978.64$. Similarly, $NPV_M = 709.67$ and $NPV_R = 575.70$. The firm will benefit the most by accepting project $S$ as it has the highest net present value.

First, we consider the case when straight debt is issued to finance an investment project and show that in such a case equity holders will implement the riskiest project $R$ with the lowest NPV. Next, we consider the case when ordinary convertible debt is issued and demonstrate that in such a case ordinary convertible debt only partially resolves the problem of risk-shifting, depending on the portfolio of projects available to the firm. If the firm’s choice is between projects $S$ and $M$, then the firm chooses the highest NPV project $S$. If, however, the firm has a riskier portfolio of projects, so that it has available to it a risky project $R$ as well (so that the firm’s choice is between $S$ and $R$, $M$ and $R$, or $S$, $M$, and $R$) then we can show that it will choose the lowest NPV project $R$, so that the risk-shifting incentive is not eliminated in this case. Finally, we consider the case when putable convertible debt is issued and demonstrate that in such a case a firm always chooses the highest NPV project $S$, regardless of the portfolio of projects available to it.

### A. Straight Debt

Consider the case where the firm issues debt with a face value $F = 1,000$, and promises to return $F(1 + r)^2 = 1,210$ at time $t = 2$. In the high state all three projects return more than $F(1 + r)^2$ so that...
straight debt holders are paid off in full. However, in the low state all three projects return less than \( F(1 + r)^2 \) so that the firm defaults and straight debt holders receive the project payoff \( L \). The firm can issue straight debt at time \( t = 0 \) at a price

\[
D_{\text{Straight}} = \frac{pF(1 + r)^2 + (1 - p)L}{(1 + r)^2}.
\]

(1)

If project \( S \) is chosen then \( D^S_{\text{Straight}} = (0.5 \times 1,210 + 0.5 \times 1,150)/1.10^2 = 975.21 \). Similarly, if project \( M \) is chosen then \( D^M_{\text{Straight}} = 706.61 \) and for project \( R \) the value of straight debt is \( D^R_{\text{Straight}} = 541.32 \).

We assume that, if the proceeds of the debt issue are more than the required initial outlay of an investment project, the extra amount raised will be returned to equity holders at \( t = 1 \) as a dividend. In addition to that, at \( t = 2 \) equity holders will receive the difference between the project’s payoff and the payment to debt holders in the high state (i.e., the residual cash flow) and nothing in the low state (since they default on the firm’s debt in this state). So the value of equity at time \( t = 0 \) is

\[
E_{\text{Straight}} = \frac{D_{\text{Straight}} - I}{1 + r} + \frac{p[H - F(1 + r)^2]}{(1 + r)^2}.
\]

(2)

We will demonstrate later that, in equilibrium, project \( R \) will be chosen so that outside investors will only pay \( D^R_{\text{Straight}} = 541.32 \) when the firm announces a straight debt issue. Thus, if project \( S \) is chosen then the equity value will be \( E^S_{\text{Straight}} = \frac{541.32 - 385}{1.10} + \frac{0.5 \times [2,150 - 1,210]}{1.10^2} = 530.54 \). Similarly \( E^M_{\text{Straight}} = 543.35 \) and \( E^R_{\text{Straight}} = 575.58 \). Since \( E^R_{\text{Straight}} > E^M_{\text{Straight}} > E^S_{\text{Straight}} \), equity holders accept project \( R \) when straight debt is issued, although it has the lowest NPV.

**B. Ordinary Convertible Debt**

When convertible debt is issued, its value at \( t = 0 \) depends on whether debt holders will convert to equity at \( t = 2 \) or not. With a conversion factor \( \gamma = 0.5 \), convertible debt holders will convert only in the high states of projects \( M \) and \( R \) as conversion values are greater than the payoff from debt: \( \gamma H_M > F(1 + r)^2 \) and \( \gamma H_R > F(1 + r)^2 \). In numbers, \( 0.5 \times 1,250 = 1,250 > 1,210 \) and \( 0.5 \times 1,300 = 1,300 > 1,210 \),
respectively. Convertible debt holders will not convert in the low states of all three projects $S$, $M$, and $R$, since by not converting they will receive full payoffs of these projects in the low states (the firm defaults on its debt in the low states), whereas conversion provides only a fraction of those payoffs: $\gamma L < L$. Convertible debt holders will not convert in the high state of project $S$ as well, as the conversion value $0.5 \times 2150 = 1075$ is less than the debt value $F(1 + r)^2 = 1210$. Thus, the price that equity holders can get for ordinary convertible debt issued at $t = 0$ if project $S$ is chosen is the same as that of straight debt determined by equation (1) and equals $D_{Ordinary}^S = 975.21$. If project $M$ or $R$ is chosen, the price of ordinary convertible debt at $t = 0$ is

$$D_{Ordinary} = \frac{p\gamma H + (1 - p)L}{(1 + r)^2}.$$  

Thus, if project $M$ is chosen the price of ordinary convertibles is $D_{Ordinary}^M = (0.5 \times 1250 + 0.5 \times 500)/1.10^2 = 723.14$ and if project $R$ is chosen the price is $D_{Ordinary}^R = (0.5 \times 1300 + 0.5 \times 100)/1.10^2 = 578.51$.

Similar to the case of straight debt, we assume that, if the proceeds of ordinary convertible debt issue are more than the required initial outlay of an investment project, the extra amount raised will be returned to equity holders at $t = 1$ as a dividend. In addition to that, if conversion is not optimal at $t = 2$, equity holders will receive the difference between the project’s payoff and the payment to debt holders in the high state and nothing in the low state (since the firm defaults on its debt in this state). However, if conversion is optimal at $t = 2$, equity holders will receive $(1 - \gamma)H$ in the high state and nothing in the low state (since the firm defaults on its debt in this state).

Here we need to consider three cases: a choice between project $S$ and project $M$, a choice between project $S$ and project $R$, and a choice between project $M$ and project $R$. Consider now a firm which has only projects $S$ and $M$ available to it. In this case we will demonstrate that project $S$ will be chosen in equilibrium, so that when the firm announces the issuance of convertible debt, outside investors will pay $D_{Ordinary}^S = 975.21$ for such convertibles. If project $S$ is chosen, then equity holders’ value is determined as in equation (2) and equals $E_{Ordinary}^S = 924.98$. If project $M$ is chosen then equity holders’ value is
\[ E_{\text{Ordinary}} = \frac{D_{\text{Ordinary}} - I}{1 + r} + \frac{p(1 - y)H}{(1 + r)^2}, \]  

so that \( E^M_{\text{Ordinary}} = 921.27 \). Thus \( E^S_{\text{Ordinary}} > E^M_{\text{Ordinary}} \) and equity holders will choose project \( S \) as they realize a greater value compared to project \( M \). In such a case convertible debt does resolve the problem of risk-shifting as the firm accepts the higher NPV project.

Consider now a firm which has only projects \( S \) and \( R \) available to it. In this case, we will demonstrate that project \( R \) will be chosen in equilibrium, so that outside investors will pay \( D^R_{\text{Ordinary}} = 578.51 \) for ordinary convertible debt when the firm announces its issuance. If project \( S \) is chosen then equity holders’ value is determined similar to that in equation (2) and equals \( E^S_{\text{Ordinary}} = 564.35 \). If project \( R \) is chosen then equity holders’ value is determined as in equation (4) and equals \( E^R_{\text{Ordinary}} = 572.20 \). Since \( E^R_{\text{Ordinary}} > E^S_{\text{Ordinary}} \), equity holders will choose project \( R \) as they realize a greater value compared to the case when the firm undertakes project \( S \). In this case convertible debt is not capable of resolving the problem of risk-shifting as the firm accepts the lowest NPV project.

Finally, consider a firm which has only projects \( M \) and \( R \) available to it. In this case, we will demonstrate that again project \( R \) will be chosen in equilibrium and outside investors will pay \( D^R_{\text{Ordinary}} = 578.51 \) for ordinary convertible debt when the firm announces its issuance. If project \( M \) is chosen then equity holders’ value is determined by equation (4) and equals \( E^M_{\text{Ordinary}} = 560.63 \). However, if project \( R \) is chosen then equity holders’ value is again determined by equation (4) and equals \( E^R_{\text{Ordinary}} = 572.20 \). Since \( E^R_{\text{Ordinary}} > E^M_{\text{Ordinary}} \), equity holders again choose project \( R \). Hence, ordinary convertible debt does not fully resolve the problem of risk-shifting, since the firm accepts the lowest NPV project in this case as well.\(^4\) In summary, we have demonstrated above that ordinary convertible debt is able to resolve the risk-shifting problem only if the portfolio of projects available to the firm is less risky: i.e., the riskiest project \( R \) is not available to the firm.

\(^4\) It can also be shown that, if the firm has all three projects (i.e., \( S, M, \) and \( R \)) available to it, it will choose \( R \) if ordinary convertible debt is issued to finance the project.
C. Putable Convertible Debt

Consider now a firm which has all three projects \( S, M, \) and \( R \) available to it. The price of putable convertible debt at \( t = 0 \) depends not only on whether putable convertible debt holders will convert at \( t = 2 \), but also on whether they will put the debt back to the firm at \( t = 1 \). The optimal conversion policy of putable convertible debt is the same as that of ordinary convertible debt. Further, putable convertible debt will be put back to the firm at \( t = 1 \) at a price of \( F(1 + y) = 1,050 \) if the expected payoff from holding the debt until \( t = 2 \) is less than 1,050. If project \( S \) is chosen, the expected payoff of putable convertible debt holders at \( t = 1 \) from waiting until \( t = 2 \) is \( [pF(1 + r)^2 + (1 - p)L_S] / (1 + r) = 1,072.73 \). If project \( M \) is chosen, the expected payoff of putable convertible debt holders at \( t = 1 \) from waiting until \( t = 2 \) is \( [pM'H_M + (1 - p)L_M] / (1 + r) = 795.45 \). If project \( R \) is chosen, the expected payoff of putable convertible debt holders at \( t = 1 \) from waiting until \( t = 2 \) is \( [pR'H_R + (1 - p)L_R] / (1 + r) = 636.36 \). Thus putable convertible debt will be put back to the firm at \( t = 1 \) only if either project \( M \) or project \( R \) is chosen. Therefore, if project \( S \) is chosen, the price of putable convertible debt issued at \( t = 0 \) is the same as that of straight debt or ordinary convertible debt as determined by equation (1) and equals \( D^S_{\text{Putable}} = 975.21 \). For projects \( M \) and \( R \) the price of putable convertible debt at \( t = 0 \) is

\[
D_{\text{Putable}} = \frac{F(1 + y)}{(1 + r)},
\]

which is 954.55.

Similar to the previous two cases, we assume that, if the proceeds of putable convertible debt issue are more than the required initial outlay of an investment project, the extra amount raised will be returned to equity holders at \( t = 1 \) as a dividend. However, if debt holders decide to put the debt back to the firm at \( t = 1 \), the firm needs to use this extra amount (which is less than \( F(1 + y) \)) in addition to some borrowing from a bank at a rate of \( r_1 \) to buy the debt back as promised.\(^5\) Thus the firm needs to borrow \[F(1 + y) - (D_{\text{Putable}} - I)] \text{ from a bank at } t = 1 \text{ if the put is exercised. At } t = 2, \text{ in the high state, the firm}

\(^5\) We use the word “bank” to stand for any outside source of financing that the firm may rely upon to raise additional funds to honor the put.
will pay the bank back the amount borrowed with interest $[F(1 + y) - (D_{\text{Putable}} - I)](1 + r_1)$; however in the low state the firm will default on the loan, and the bank will receive $L$. For the bank to break-even in this lending transaction it has to set the borrowing rate $r_1$ high enough so that the following holds:

$$F(1 + y) - (D_{\text{Putable}} - I) = \frac{p[F(1 + y) - (D_{\text{Putable}} - I)](1 + r_1) + (1 - p)L}{1 + r}.$$  \hspace{1cm} (6)

Solving for $r_1$ we have

$$r_1 = \frac{[F(1 + y) - (D_{\text{Putable}} - I)](1 + r) - (1 - p)L}{p[F(1 + y) - (D_{\text{Putable}} - I)]} - 1.$$ \hspace{1cm} (7)

We will demonstrate later that, in equilibrium, project $S$ will be chosen, so that when a firm announces a putable convertible debt issue, outside investors will pay $D_{\text{Putable}}^S = 975.21$ for such debt. However, if the firm chooses to accept project $M$, the bank will charge $r_{M1} = 0.3733$ for the firm to borrow at $t = 1$, and if the firm chooses to accept project $R$, the bank will charge $r_{R1} = 1.0373$. With these borrowing rates at $t = 2$, the firm must pay back to the bank $[1,000 \times (1 + 0.05) - (975.21 - 530)] \times 1.3733 = 830.56$ if project $M$ is chosen and $[1,000 \times (1 + 0.05) - (975.21 - 540)] \times 2.0373 = 1,252.55$ if project $R$ is chosen. The firm will have enough in the high states to pay off these amounts; however it will default in the low states.

If project $S$ is chosen, neither conversion nor putting is optimal, so that the equity holders’ value at $t = 0$ is determined by equation (2) and equals $E_{\text{Putable}}^S = 924.98$. If project $M$ or project $R$ is chosen and put option is exercised at $t = 1$, the equity holders’ value is

$$E_{\text{Putable}} = \frac{p[H - (F(1 + y) - (D_{\text{Putable}} - I))(1 + r_1)]}{(1 + r)^2}.$$ \hspace{1cm} (8)

Thus, if project $M$ is chosen, $E_{\text{Putable}}^M = [0.5 \times (2,500 - 830.56) + 0.5 \times 0]/1.10^2 = 689.86$ and, if project $R$ is chosen, $E_{\text{Putable}}^R = [0.5 \times (2,600 - 1,252.55) + 0.5 \times 0]/1.10^2 = 556.80$. Since $E_{\text{Putable}}^S > E_{\text{Putable}}^M > E_{\text{Putable}}^R$, project $S$ will be implemented in equilibrium. Thus issuing putable convertible debt fully resolves the problem of risk-shifting as equity holders accept the highest NPV project $S$, even when the risky project $R$ is available to the firm. It can be also demonstrated that, if the firm’s choice is only between projects $M$
and $R$, the issuance of putable convertible debt forces the firm to accept the higher NPV project $M$ (unlike in the case with ordinary convertible debt).

### A.2. Issuing Putable Convertibles under Asymmetric Information: A Simple Theoretical Example

This simple theoretical example demonstrates that firms with favorable private information will issue putable convertibles in equilibrium to separate themselves from firms with unfavorable private information, which will issue ordinary convertibles in equilibrium. Similar to the risk-shifting example, there are two periods in this example with risk-neutral investors. There are two types of firms: good ($G$), with favorable private information, and bad ($B$), with unfavorable private information. Both types of firms have assets in place (about which insiders have private information) and an investment project (about which there is no private information: the nature of this project is the same for type $G$ and type $B$ firms).

At $t = 0$ a firm must issue either ordinary convertibles or putable convertibles to finance the investment project. For type $G$ firms, the value of assets in place will be low $A_L$ with probability $p_G$ and it will be high $A_H$ with the complimentary probability $1 - p_G$ at $t = 2$. Similarly, type $B$ firms will realize the low cash flow $A_L$ from assets in place with probability $p_B$ and the high cash flow $A_H$ with probability $1 - p_B$ (where $p_B > p_G$). Information on whether the value of assets in place is going to be high or low at $t = 2$ will be revealed to outsiders at $t = 1$. Insiders of both types of firms know the above probabilities at $t = 0$ (at which point this information is private). In equilibrium, outsiders will be able to infer these probabilities from the type of convertible debt a firm decides to issue. The investment project requires an initial outlay of $I$ and has either a high payoff $H$ at $t = 2$ with probability $q$ or a low payoff $L$ with the complimentary probability $1 - q$. Thus there are four possible states for a firm at $t = 2$ summarized as follows.

<table>
<thead>
<tr>
<th>State</th>
<th>Probability</th>
<th>Firm value at $t = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-high</td>
<td>$p(1 - q)$</td>
<td>$A_L + H$</td>
</tr>
<tr>
<td>Low-low</td>
<td>$pq$</td>
<td>$A_L + L$</td>
</tr>
<tr>
<td>High-high</td>
<td>$(1 - p)(1 - q)$</td>
<td>$A_H + H$</td>
</tr>
<tr>
<td>High-low</td>
<td>$(1 - p)q$</td>
<td>$A_H + L$</td>
</tr>
</tbody>
</table>
Firms issue either ordinary or putable convertible debt with a face value of \( F \), and promise to pay back at \( t = 2 \) an amount of \( F(1 + r)^2 \), where \( r \) is the risk-free rate of return. Convertible debt holders have an option to convert to equity at \( t = 2 \) at a fixed conversion factor of \( \gamma \) (in other words, convertible debt holders may choose to receive a fraction \( \gamma \) of the firm value as indicated above at \( t = 2 \) instead of \( F(1 + r)^2 \)). If putable convertible debt is issued, putable convertible debt holders have an option to sell the debt back to the firm at \( t = 1 \) at a price of \( F(1 + y) \), in addition to the conversion option. Figure 2 depicts the sequence of events in this example.

\[\text{Figure 2. Sequence of events in the asymmetric information model.}\]

We assume the following parameter values: \( p_G = 0.4, p_B = 0.6, A_L = 100, A_H = 1,200, L = 300, H = 1,400, I = 700, q = 0.5, F = 1,000, r = 0.1, \gamma = 0.5, \) and \( y = 0.05 \). With these parameter values, it is optimal

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6 Due to space limitations, we do not introduce straight debt into the menu of securities offered by the firm in the asymmetric information setting. However, it can be shown that straight debt will be dominated by putable convertibles in an asymmetric information setting for two reasons. First, in the absence of costs of financial distress, the type \( B \) firm always has an incentive to mimic the type \( G \) firm if it issues straight debt, so that a separating equilibrium does not exist (recall that there is no advantage in this case for the type \( G \) firm to issue straight debt over the type \( B \)). Second, even if we introduce costs of financial distress, it can be shown that it is more efficient for the type \( G \) firm to signal its type by issuing putable convertibles rather than straight debt. This is because of the feature of putable convertibles which makes them cheaper for a type \( G \) firm to issue under asymmetric information compared to ordinary convertibles, while issuing ordinary convertibles is cheaper for a type \( B \) firm (see our example for details). Straight debt does not have this feature, requiring excessively large amount of debt to be issued to signal firm type, so that even the type \( G \) firm has to incur a significant probability of financial distress if it were to attempt to signal firm type by issuing straight debt. See Stein (1992) for a three-type model where the two higher types signal using straight debt and convertible debt, respectively, in an environment of asymmetric information and costs of financial distress.
to convert convertible debt (either ordinary or putable) to equity at \( t = 2 \) only in the high-high state since the conversion value \( \gamma(A_H + H) = 1,300 \) is greater than the payoff from the debt \( F(1 + r)^2 = 1,210 \). In the high-low and low-high states conversion is not optimal as conversion values \( \gamma(A_H + L) = 750 \) and \( \gamma(A_L + H) = 750 \) are less than \( F(1 + r)^2 = 1,210 \). In the low-low state conversion is not optimal as well, since a firm will default in this state \( (A_L + L < F(1 + r)^2) \) and convertible bond holders will get \( A_L + L \) if they don’t convert and only \( \gamma(A_L + L) \) if they do.

First we consider the case where the firm issues ordinary convertibles. The value of ordinary convertibles at \( t = 0 \) is the expected value of the payoff convertible debt holders will realize at \( t = 2 \):

\[
D_{\text{Ordinary}} = p \left[ \frac{(1-q)F(1+r)^2 + q(A_L + L)}{(1+r)^2} \right] + (1 - p) \left[ \frac{(1-q)\gamma(A_H + H) + qF(1+r)^2}{(1+r)^2} \right].
\]

The only difference between the value of ordinary convertible debt issued by type \( G \) and type \( B \) firms is the probability \( p \) of the assets in place realizing a low value \( (p_G \text{ for type } G \text{ and } p_B \text{ for type } B) \) and thus\( D^G_{\text{Ordinary}} = 888.43 \) and \( D^B_{\text{Ordinary}} = 814.05 \).

Similar to the risk-shifting example, we assume that, if the proceeds of convertible debt issue are more than the required initial outlay of the investment project, the extra amount raised will be returned to equity holders at \( t = 1 \) as a dividend. In addition to the above cash flow at \( t = 1 \), at \( t = 2 \) equity holders will receive a cash flow of \( (1 - \gamma)(A_H + H) \) in the high-high state (when convertible debt holders convert to equity), \( A_H + L - F(1 + r)^2 \) in the high-low state and \( A_L + H - F(1 + r)^2 \) in the low-high state (recall that conversion is not optimal in these two states and equity holders have enough cash to pay off convertible debt holders), and nothing in the low-low state (since the firm defaults on its debt in this state). The value of equity at \( t = 0 \) is thus:

\[
E_{\text{Ordinary}} = \frac{D_{\text{Ordinary}} - I}{1 + r} + p \left[ \frac{(1-q)(A_L + H - F(1+r)^2)}{(1+r)^2} \right] +
\]

\[
+ (1 - p) \left[ \frac{(1-q)(1-\gamma)(A_H + H) + q(A_H + L - F(1+r)^2)}{(1+r)^2} \right].
\]
The only difference between the value of equity for type $G$ and type $B$ firms is the probability $p$ of the assets in place realizing a low value ($p_G$ for type $G$ and $p_B$ for type $B$) and thus $E^G_{\text{Ordinary}} = 613.45$ and $E^B_{\text{Ordinary}} = 438.39$.

Next we consider the case where the firm issues putable convertible debt. Now, in addition to the conversion option, putable convertible debt holders have an option to sell the debt back to the firm at $t = 1$ at a price of $F(1 + y)$. At $t = 1$ putable convertible debt holders will know whether the value of the assets in place is going to be high or low at $t = 2$ and will exercise the put option optimally (as we will demonstrate below) if they receive information that the value of the assets in place is going to be low. In this case, they will get $F(1 + y) = 1,050$ if they exercise the put option, which is more than the value of waiting until $t = 2$, which is $[(1 - q)F(1 + r)^2 + q(A_L + L)]/(1 + r) = 731.82$. However, if at $t = 1$ convertible debt holders receive information that the value of the assets in place is going to be high at $t = 2$, then the put option will not be exercised: by exercising the put option they will get $F(1 + y) = 1,050$, which is less than the value of waiting until $t = 2$, which is $[(1 - q)\gamma(A_H + H) + qF(1 + r)^2]/(1 + r) = 1,140.91$. Thus the price at which putable convertible debt can be issued at $t = 0$ is

$$D_{\text{Put}} = p \frac{F(1 + y)}{1 + r} + (1 - p) \left[ (1 - q)\gamma(A_H + H) + qF(1 + r)^2 \right]/(1 + r)^2.$$

The only difference between the price of putable convertible debt for type $G$ and type $B$ firms is the probability $p$ of the assets in place realizing low value ($p_G$ for type $G$ and $p_B$ for type $B$) and thus $D^G_{\text{Put}} = 1,004.13$ and $D^B_{\text{Put}} = 987.60$.

If the put option is exercised, the firm needs to borrow from a bank at $t = 1$ an amount equal to $F(1 + y) - (D_{\text{Put}} - I)$ for one period at an interest rate of $r_1$ (the firm does not need to borrow the full amount of $F(1 + y)$ as it also has the extra amount left from the issuance of putable convertible debt after investing in the initial outlay of the investment project). At $t = 2$, if the project pays $H$, the firm will have enough money to repay the borrowed amount with interest; however, if the project pays $L$, the firm will default on its loan to the bank. For the bank to break-even in this lending transaction it has to set the lending rate $r_1$ high enough so that the following holds:
\[
F(1 + y) - (D_{\text{Putable}} - I) = \frac{(1-q)[F(1+y) - (D_{\text{Putable}} - I)](1 + r_1) + q(A_L + L)}{1 + r}.
\]

Solving for \(r_1\) we have
\[
r_1 = \frac{[F(1+y) - (D_{\text{Putable}} - I)](1 + r) - q(A_L + L)}{(1-q)[F(1+y) - (D_{\text{Putable}} - I)]} - 1.
\]

Thus, if putable convertible debt is issued by a type G firm, \(r^G_1 = 0.6637\), and if putable convertible debt is issued by a type B firm, \(r^B_1 = 0.6753\). Therefore at \(t = 2\), a type G firm needs to repay \([1,050 - (1,004.13 – 700)]\times1.6637 = 1,240.91\) and a type B firm needs to repay \([1,050 - (987.60 – 700)]\times1.6753 = 1,277.27\). Clearly, both types will have enough cash to repay the loan if the project pays \(H\) at \(t = 2\), since \(A_L + H = 1,500\), and both types will default on the bank loan if the project pays \(L\) at \(t = 2\), since \(A_L + L = 400\) (we will show later, however, that only type G firms will issue putable convertibles in equilibrium).

Equity holders’ value at \(t = 0\) is as follows:
\[
E_{\text{Putable}} = p \left[ \frac{(1-q)(A_L + H - [F(1+y) - (D_{\text{Putable}} - I)](1 + r_1))}{(1+r)^2} \right] + (1-p) \left[ \frac{D_{\text{Putable}} - I}{1 + r} + \frac{(1-q)(1-\gamma)(A_H + H) + q(A_H + L - F(1+r)^2)}{(1+r)^2} \right].
\]

In (14), the value of the assets in place is low with probability \(p\) and the put option is exercised, so that equity holders will borrow from a bank at \(t = 1\) (to honor the put) and are left with \(A_L + H - [F(1+y) - (D_{\text{Putable}} - I)](1 + r_1)\) at \(t = 2\) in the low-high state after paying the bank back with interest; they are left with nothing in the low-low state at \(t = 2\), since they default to the bank in this state. The value of the assets in place is high with probability \(1 - p\) and the put option is not exercised, so that at \(t = 1\) equity holders receive the amount left from putable convertible debt issuance after paying for the initial investment of the project, and at \(t = 2\) they are left with \((1-\gamma)(A_H + H)\) when putable convertible debt holders convert to equity in the high-high state; they are left with \(A_H + L - F(1+r)^2\) in the high-low state, since in this state conversion is not optimal and debt holders are paid off. The only difference between the value of equity for type G and type B firms is the probability \(p\) of the assets in place realizing a low value (\(p_G\) for type G and \(p_B\) for type B) and thus \(E^G_{\text{Putable}} = 602.93\) and \(E^B_{\text{Putable}} = 422.61\).
We will demonstrate below that, in equilibrium, type \( G \) firms will issue putable convertible debt and type \( B \) firms will issue ordinary convertible debt. Thus, if outside investors observe a firm issuing putable convertible debt, they will infer that it is a type \( G \) firm, and price its debt accordingly using equation (11), so that \( D_{\text{Putable}} = D^G_{\text{Putable}} \). Setting this debt value in equation (14), the firm’s equity will then be priced at \( E^G_{\text{Putable}} = 602.93 \). On the other hand, if outsiders observe a firm issuing ordinary convertible debt, they will infer that it is a type \( B \) firm and will price its debt at \( D_{\text{Ordinary}} = D^B_{\text{Ordinary}} \) using equation (9). Setting this value in equation (10), the firm’s equity will then be priced at \( E^B_{\text{Ordinary}} = 438.39 \).

Note that, if a type \( B \) firm tries to mimic a type \( G \) firm by issuing putable convertibles, its equity holders’ value will be determined by equation (14) with \( D_{\text{Putable}} = D^G_{\text{Putables}} \), since outside investors in this case will pay the price of putable convertible debt issued by a type \( G \) firm, while firm insiders’ private information is that \( p = p_B \). This mimicking value \( E^B_{\text{Putable-Mimic}} = 437.64 \) is less than the value a type \( B \) firm realizes by issuing ordinary convertibles, \( E^B_{\text{Ordinary}} = 438.39 \), so that a type \( B \) firm will issue ordinary convertibles in equilibrium. On the other hand, if a type \( G \) firm attempts to mimic a type \( B \) firm and issues ordinary convertibles, its equity holders’ value will be determined by equation (10) with \( D_{\text{Ordinary}} = D^B_{\text{Ordinary}} \), since outside investors in this case will infer that the firm is of type \( B \) and will pay the price of ordinary convertible debt issued by a type \( B \) firm, while firm insiders’ private information is that \( p = p_G \). This mimicking value \( E^G_{\text{Ordinary-Mimic}} = 545.83 \) is less than the value a type \( G \) firm realizes by issuing putable convertibles, \( E^G_{\text{Putable}} = 602.93 \), thus a type \( G \) firm will issue putable convertibles in equilibrium.8

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7 Thus, the beliefs supporting the above equilibrium are such that if a firm issues putable convertible debt, outsiders infer it to be of type \( G \) with probability 1, and if it issues ordinary convertible debt, outsiders believe it to be of type \( B \) with probability 1.

8 Note that, we have thus demonstrated that the incentive compatibility conditions hold for the type \( G \) as well as the type \( B \) firms, supporting this fully separating equilibrium.