Venture Capitalists versus Angels:
The Dynamics of Private Firm Financing Contracts

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Abstract

We develop a theoretical analysis of an entrepreneur’s choice between venture capital (VC) and angel financing at various stages in a private firm’s life, and characterize the dynamic evolution of the firm’s contract with its financier (VC or angel). In our model, an entrepreneur has information superior to a potential financier about his own firm; however, this information advantage diminishes as the financier interacts with the firm over time. Venture capitalists and angels differ in two ways. First, venture capitalists can add value to the firm by exerting effort, which, together with the entrepreneur’s effort, increases the chance of project success; the angel is unable to add significant value. Second, venture capital financing is scarce relative to angel financing. The equilibrium VC contract maximizes value-addition by ensuring that both the entrepreneur and the VC exert optimal effort. We develop the following results in the above setting. First, we characterize the optimal financing path of the firm: depending on firm characteristics, a firm may use angel financing in its early stages and switch to VC financing in later stages, or vice versa. Second, VC financing contracts resemble convertible debt, while angel financing contracts may resemble a variety of financial securities, including equity. Third, for firms which use venture financing in earlier as well as later rounds, earlier round financing contracts will have more of a fixed income component and less of a warrant (“upside”) component compared to later round financing contracts. Fourth, later round financing contracts between an entrepreneur and a VC who financed it in an earlier round will have a greater warrant component compared to such a contract between a VC and a previously angel-financed firm. Fifth, we characterize how the structure of VC financing contracts relate to: (i) the experience and productivity of the VC; (ii) the nature of the firm’s industry; and (iii) the scarcity of VC financing. Finally, we develop predictions for the relationship between the financing path of a firm and the probability of its having a successful exit (IPO or acquisition), and for differences in the compositions of VCs’ and angels’ investment portfolios.
1 Introduction

It is well known that angel financing is an important source of financing for private firms in the United States. However, beyond the fact that the annual amount of angel financing is much larger than that of venture capital financing, and that angels tend to be individuals who invest much smaller amounts than venture capitalists in individual firms, little is known about the important economic differences between venture capital and angel financing.\(^1\) One of the objectives of this paper is to bridge this gap in the literature by developing a theoretical analysis of the different roles played by venture capitalists and angels in funding private firms, and to develop an understanding of the situations under which firms make use of each type of financing.

The second objective of this paper is to develop an analysis of the dynamic features of financing contracts in the private equity market. The empirical evidence (as well as descriptions of individual cases) indicates that typically, firms undertake several rounds of private equity financing. Sometimes these different rounds of financing to a firm may come from the same source: for example, the same venture capital firm may provide multiple rounds of financing to a firm. In other situations, these different rounds of private equity financing may come from different sources: thus, a firm may be initially angel financed, and may later switch to venture capital financing; alternatively, a venture capitalist may provide funding initially, but may choose to sell his equity stake and leave the firm. The above situations lead us to ask several questions: First, are there any important differences between venture capital and angel financing contracts? Second, what motivates firms to switch from one form of private equity financing to another? Third, if firms make use of multiple rounds of financing from the same source, are there (and should there be) any systematic differences in the contracts between the entrepreneur and financier from one round to another (i.e., how do venture capital and angel financing contracts evolve over time?). We develop answers to many of these questions here.

Our analysis rests on a few assumptions based on certain stylized facts about the private equity market. First, we assume that in the early stages of a firm, the financier (venture capitalist or angel investor) is able to add value to the firm, at least in some situations.\(^2\) Second, we assume that, while both the venture capitalist and the angel

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\(^1\) Freear et al (1996) estimate that around 250,000 angels invest between $10 billion and $20 billion in around 30,000 firms annually. This compares with around $6.6 billion committed in the venture sector of the organized private equity market in 1995, making the angel market several times larger. See Fenn, Liang and Prowse (1997), Prowse (1998), Wetzel (1983, 1987) for good descriptions of the private equity market.

\(^2\) See, for example, Hellmann and Puri (2002), who find that venture capitalists play a significant role in the professionalization
may be able to add value in this way, the venture capitalist is more capable of adding value (or equivalently, the venture capitalist can add value in more situations) than the angel. Third, we assume that, while the financier is capable of adding value, he has to engage in costly effort to add this value, so that he has to be given the appropriate incentives to put forth effort optimally on behalf of the firm. Fourth, we assume that, prior to the financier getting involved with the firm, the entrepreneur has private information regarding the nature of his own project (including the likelihood of the financier being able to add value to the project). However, if he does provide funding to the entrepreneur’s project (thereby getting involved in its activities), the financier is able to learn more about the project over time, thus eliminating the information asymmetry between the entrepreneur and the financier. Finally, we assume that the entrepreneur’s effort is also required for the project to succeed, so that the contract provided to the financier must be such that the entrepreneur also has the appropriate incentives to make the project a success.

In the above setting, we derive a variety of interesting predictions about entrepreneurs’ equilibrium choice of private equity financing source and the structure of private equity financing contracts. First, we show why, in many situations, firms prefer venture capital financing over angel financing, even though venture capitalists require a greater rate of return from their investment in the firm. Second, we characterize the conditions under which firms switch financing sources across financing rounds (angel to venture capitalist or venture capitalist to angel). Third, we characterize the equilibrium financing contracts between venture capitalists and entrepreneurs on the one hand, and angels and entrepreneurs on the other, thus allowing us to make predictions regarding the differences between the two kinds of contracts. Fourth, we make predictions regarding how the structure of venture capital contracts will evolve over time. Fifth, we develop implications for the composition of projects (early versus later stage) financed by venture capitalists and angels, and how this composition varies with changes in the scarcity of venture capital financing relative to angel financing. Sixth, we develop predictions regarding the announcement effects of various forms of financing, and the relationship between the dynamic path of firm financing and the quality of firms’ projects.

An important trade-off driving our results is that the asymmetric information existing between the entrepreneur and the venture capitalist interferes with the provision of appropriate incentives to the venture capitalist of start-up firms in general, and in particular, in the hiring of their top management.
to create value for the firm. The venture capitalist learns more and more about the firm and the entrepreneur as he interacts with them over time. Therefore, the venture capitalist’s informational disadvantage relative to the entrepreneur will be significantly lower in later stage contracting compared to that in early stage contracting. This has two important consequences in our setting. First, later stage contracts between the entrepreneur and the venture capitalist will be better than early stage contracts at providing incentives to the venture capitalist to create value for the firm (while ensuring that the entrepreneur retains the appropriate incentives to exert effort as well). Second, the contract between the entrepreneur and the venture capitalist in a firm previously financed by the same venture capitalist will provide him with better incentives to create value compared to a contract with between a venture capitalist and a previously angel-financed firm. This trade-off also explains the venture capitalist’s equilibrium choice regarding whether to start funding a project at an earlier or later stage. On the one hand, starting to fund a project at an earlier stage allows the venture capitalist to create more value for the firm; on the other hand, if the venture capitalist starts funding a project at an earlier stage, there is a greater chance that he is investing in a firm where he cannot create significant value, and therefore has to exit the firm before project completion (earning a return lower than his opportunity cost of capital). Thus, we show that, during periods where venture capital funding is relatively scarce (so that the opportunity cost of venture capital is moderate or high), venture capitalists tend to finance more later stage projects.

In the process of developing the above results, we offer a somewhat different rationale for the widely observed use of convertible features in venture capital contracts compared to those provided by the existing corporate finance and contracting literature. These rationales can be grouped into several categories. The first category deals with conflicts between stockholders and bondholders, and the related incentives of insiders to take on excessively risky projects (see, e.g., Green (1984)). A second category of papers deal with how the use of convertibles may be driven by asymmetric information between firm insiders and outsiders (e.g., Stein (1992), Constantinides and Grundy (1989), and Brennan and Kraus (1987)). Both of the above rationales apply to the use of convertibles by public as well as private firms. A third set of papers argue that the use of convertibles in venture capital contracts arises from the incompleteness of contracts between the venture capitalist and the entrepreneur, and the ability of different financial contracts to optimally switch control between the two (see, e.g.,
A fourth literature argues that convertible features arise from issues related to providing the right incentives to the entrepreneur in a setting of moral hazard (Cornelli and Yosha (2003), Repullo and Suarez (2004)). Finally, Hellmann (2006) argues that a key feature of convertible securities in venture capital financing contracts is to create different cash flow rights for acquisitions and IPOs, and demonstrates how the convertible security implements an optimal trade-off between the need to allocate cash flows to the venture capitalist and the desire to make efficient exit decisions. In contrast to this literature, in our paper the rationale for the use of convertible features emerges from the need to provide incentives to the venture capitalist to exert effort to add value to the firm while maintaining the incentives of the entrepreneur to exert effort. Further, to the best of our knowledge, none of the above papers have analyzed how the contract between the venture capitalist and the firm should evolve across multiple rounds of financing. Thus, our model makes the novel prediction that, while convertible preferred equity or convertible debt will be used in both early and later stage financing, the relative magnitudes of the fixed income component and the upside (warrant) component will differ across financing rounds: while early stage financing with a venture capitalist will have more of a fixed income component and less upside, later rounds of financing with the same venture capitalist will feature a smaller fixed income component but a larger upside. Further, our analysis predicts that angel financing contracts are less likely to incorporate convertible features compared to venture capital contracts.

Our paper is also related to other strands of the theoretical and empirical literature on private equity financing. Like our paper, Repullo and Suarez (2004) also study the moral hazard problem on the part of both the venture capitalist and entrepreneur. However, unlike in our paper, the driving factor in their paper is the allocation of the refinancing cost of the project across states. Because the firm’s later stage financier has to buy back the firm’s financial contract from the initial financier, in their setting the optimal contract provides the initial financier a

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3 The incomplete contracting literature builds on the pioneering work of Grossman and Hart (1986). Three important papers in this literature are Aghion and Bolton (1992), Hart and Moore (1998), and Dewatripont and Tirole (1994). Many of the control theories of venture capital contracting make use of a modeling set-up similar to one or more of these papers.

4 See also Marx (1998), who argues that when the venture capitalist is risk-averse, convertible preferred equity motivates the venture capitalist to intervene in the firm in response to poor performance. Gompers (1996) argues that venture capital convertible debt contracts are quite different from convertible debt in large public corporations. See also Bergemann and Hege (1998).

5 The need to provide incentives to the entrepreneur to put forth optimal effort was argued in a number of early papers by Sahlman (see, e.g., Sahlman (1988, 1990)).

6 The practitioner literature indicates that both venture capital and angel contracts come in four basic forms: common stock, stock with warrants, convertible equity, and convertible notes (debt). Various additional provisions are added to these basic structures depending on the specifics of a given project or firm. See Bartlett (1995) for details.

7 Casamatta (2003) also studies the moral hazard problem on the part of both venture capitalist and entrepreneur. However, the focus of that paper is completely different from ours. See also Inderst and Muller (2002), who use a search model to demonstrate that the composition and the type of financial claims held by the venture capitalist and the entrepreneur depend on the market structure.
greater payoff when the state is high and a smaller payoff when the state is low, so that projects with smaller positive net present values can be financed. Apart from this difference in driving intuition, in their setting there is no asymmetric information between the entrepreneur and the outside financier; neither do they analyze the firm’s choice between different kinds of financiers (angels and venture capitalists). Admati and Pfleiderer (1994) study a setting in which a venture capitalist can observe the true state of a firm unlike the outside investors. They show that optimal investment decisions will be made by the firm in all states if and only if the venture capitalist is given a fixed-fraction equity contract, which eliminates his incentives to misrepresent the state to outside investors. Finally, our paper is related to the growing empirical literature providing detailed evidence on the structure of venture capital contracts in the U.S and other countries (prominent examples of this literature are Sahlman (1990), Gompers (1997), and Kaplan and Stromberg (2003)).

The rest of this paper is structured as follows. Section 2 describes the essential features of our model, while sections 3 and 4 characterize the equilibrium of the model and develop results. Section 5 describes the empirical implications of our model. Section 6 concludes. The proofs of all propositions are confined to the appendix.

2 The Model

The model has three dates (t= 0, 1, 2) and three kinds of agents (entrepreneur, venture capitalist, and angel), all of whom are risk neutral. The entrepreneur is endowed with a nondivisible project, which needs external financing $I$. Of the investment $I$, a minimum amount $I_0$ is required at time 0; the entrepreneur may raise the remaining amount $(I - I_0)$ either at time 0 or time 1. Thus, if the entrepreneur has invested an amount $I_0 \geq I_0$ at time 0, he will invest the remaining investment amount $(I - I_0)$ at time 1. We refer to the first period (time 0 to time 1) as the “earlier stage” of the firm’s project, and the second period (time 1 to time 2) as the “later stage” of that project. For simplicity, we normalize the risk-free rate of return to be zero.

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8 One paper which discusses the choice between different kinds of financiers is Leschinskii (1999). However, that paper is driven by the assumption that while the venture capitalist can fire the manager, business angels cannot, yielding the prediction that angel financing will be used only when replacing the manager is not optimal at any stage in the firm’s life.

9 See also Cumming (2000) for Canadian evidence, Bascha and Walz (2001) for German evidence, and Parhankangas and Smith (2000) for evidence from Finland.

10 Private equity financing is often categorized into four stages. The “first stage” refers to firms in the start-up, R&D, testing and market research stage. The “second stage” refers to the prototype, further testing, and early expansion stage. The “third stage” refers to full scale manufacturing and marketing. Finally, the “fourth stage” refers to the financing of firms which are profitable. In our model, early stage (time 0) financing can be thought of as corresponding to the first stage in the above classification. On the other hand, later stage (time 1) financing corresponds to the second and the third stage in the above classification. In both of these
The entrepreneur has two potential sources of external financing: The venture capitalist (VC, from now on) or the angel. There are two differences between the VC and the angel in our setting. First, the VC contributes not only capital, but also effort, which helps in the successful implementation of the project. In contrast, the angel contributes only capital.\footnote{This argument is consistent with the survey evidence of Prowse (1995), who documents that a large proportion of angels tend to be unsophisticated investors, unable to add significant value to a firm. Empirical evidence consistent with this assumption is also provided by Wong (2001), who documents that angels are more passive compared to VCs, and Hochberg (2002), who documents that VCs tend to add more value than other institutional investors. Similar notions about the difference between venture capitalists and angels in term of their ability to add value to a firm are also prevalent among practitioners: see, e.g., the discussion of the differences between venture capitalists and angels in Qindlen (2000), Chapter 5: “Professional venture capital investing also usually involves a serious time commitment on the part of a partner and the partnership to help you, the entrepreneur, build your company. This time commitment relates to being your partner: helping you to recruit a great management team, strategizing with you on the company’s future direction, helping you to identify funding sources, introducing you to corporate or marketing partners-whatever is required to help you and the company be successful....Angel investors generally cannot make this type of long-term partnership commitment involving both time and money.” See also the case study on “Honest Tea” described in Gompers (2001), and the discussion there of the differences between venture capitalists and angels. Of course, there are also many “sophisticated” angels capable of adding considerable value to firms. In our setting, such angels can be included in the category of venture capitalists, since the defining characteristics of a venture capitalist in our setting is the ability to add significant value to a firm.} Second, VC financing is scarce relative to angel financing.\footnote{We will discuss the economic consequences of these two differences between VCs and angels later on in this paper.} At time 0 and time 1, the entrepreneur chooses between these two sources of financing depending on his private information and other relevant variables in the firm and the economy. We allow for the entrepreneur to refinance his project at time 1 (in case he decides to switch from an angel to a VC or vice versa). In other words, the amount raised from the time 1 financier can be more than the pure investment amount \((I - I_0)\) by the amount required to buy out the time 0 financier. Further, we allow for the entrepreneur to raise more money than the amount the firm needs at time 0 and 1. In this case, the amount in excess of the firm’s investment requirement goes to the entrepreneur.\footnote{Even though we allow for this possibility, the entrepreneur typically will not raise more than the required investment amount as long as this amount is reasonably large. The only scenario under which this occurs in equilibrium is when the required external financing amount is so small that if the VC provides only funding to this extent, his financial stake in the firm is not enough to motivate him to exert the optimal amount of effort to create value for the firm (so that the entrepreneur may choose to raise an amount in excess of this investment amount to induce the venture capitalist to exert optimal effort). In practice, there are several cases where entrepreneurs have raised financing in excess of firms’ investment requirements, with the excess cash going to the entrepreneurs: see, e.g., the article, “Startup Millionaires Even Before the IPO,” Business Week, May 9, 2005. Some examples of private (pre-IPO) firms where entrepreneurs received part of the funds raised from pre-IPO investors are Fastclick Inc., Netblue Inc., Datran Media, AzoogleAds, and Vendare Group.} The cash flows from the project are realized at time 2. We assume that there are only two possible outcomes for the project: “highly successful” (high cash flow \(X\)), or “less successful” (low cash flow \(\hat{X}\)). \footnote{The assumption that \(L_0 > X\) ensures that the project cannot be financed through risk-free debt at time 0.}
Entrepreneur chooses between angel and VC financing, and the amount to be raised at this time.

Entrepreneur and VC (in case of VC financing) exert effort.

Time 0

Entrepreneur decides between angel or VC financing to raise the remaining amount. Time 0 financing contract may be bought out at this date.

Time 1

The current financier (VC or the angel) and the entrepreneur observe the state of the project, \(n, p\).

Time 2

Final cash flows realized and allocated.

Entrepreneur and VC (in case of VC financing) exert effort.

Figure 1: Sequence of Events
Figure 2: Project Cash Flow Structure

\[ j = \begin{cases} 
1 & \text{if entrepreneur exerts high effort} \\
0 & \text{if entrepreneur exerts low effort} 
\end{cases} \]
2.1 The Venture Capitalist

We assume that the VC is able to add value to the firm’s project, depending on whether the entrepreneur exerts high or low effort, and also on the nature of the firm’s project. At time 0 (beginning of the first period), the firm’s project can be either of type G or type B. Similarly, at time 1 (beginning of second period) the firm’s project can be either in state p or in state n. We will first discuss value creation by the VC in the second period before going on to discuss how the VC can create value in the first period.

We define state p projects as those in which the VC’s effort in the second period is “productive”, and state n projects as those in which the VC’s effort is “not productive”. We model value creation by the VC at time 1 by assuming that, while the probability of the project achieving the high cash flow $X$ at time 2 is only $q$ for a state n project, it will be $q + f(c_1)$ for a state p project with VC financing, provided the entrepreneur also exerts high effort; here $c_1$ denotes the VC’s effort in the second period. We assume that the probability of a high cash flow for any project financed by an angel, or if the entrepreneur exerts only low effort, is $q$, regardless of the state the project is in (see figure 2).

We model value creation by the VC in the first period in a similar manner. We define a type G project as one in which the VC’s first period effort is productive, while we define a type B project as one in which this effort is not productive. Thus, while the probability of a project being in state p in the second period is only $\lambda$ for a type B project, it increases to $\lambda + f(c_0)$ for a venture financed type G project, provided that the entrepreneur exerts high effort in the first period ($c_0$ denotes the VC’s effort in the first period). We assume that the probability of any project being in state p in the second period is only $\lambda$ for any project (regardless of type) if financed by an angel, or if the entrepreneur exerts only low effort.

We assume that the VC’s effort, $c_i$, $i = 0, 1$, is a continuous variable, with the VC incurring a private cost of effort which is monotonically increasing in his effort level. For notational simplicity, we will use $c_i$, $i = 0, 1$, to denote not only the VC’s effort level, but also the corresponding effort cost incurred by him in each period. We assume that $f(c_i)$ is increasing and concave in $c_i$, with $f(0) = 0$, $q + f(\infty) < 1$, and $\lambda + f(\infty) < 1$.

Since VC financing is scarce relative to angel financing, the VC requires a minimum (threshold) NPV, denoted by $R > 0$, from investing in a firm’s project in each period (in other words, $R$ is the net present value of the
VC's alternative investment opportunity over two periods). One can think of $R_2$ as the NPV the VC can obtain in each period (i.e., the present value of cash flow net of all costs) from investing in his alternative investment opportunity for one period. This contrasts with the angel, who only insists that the NPV from any investment he makes be positive. Since $R_2$ reflects the current level of scarcity of VC financing in the economy, it varies according to the extent of this scarcity: $R_2$ will be high when VC financing is very scarce, and low when VC financing is less scarce. The objective of the VC in making his effort choices, as well as his investment decision, is to maximize the expected value of his total cash flows net of his effort costs.

2.2 The Angel

The angel is a pure supplier of capital; he cannot affect the probability of project success through his effort. Further, angel financing is abundant, so that the angel invests in all projects which yield him a positive NPV.

2.3 The Entrepreneur

As discussed in section 2.1, the project’s success is affected by the entrepreneur’s effort as well as that of the VC. We model the entrepreneur’s effort in the following way. In each period, the entrepreneur can either exert a low level of effort (in which case we normalize his cost of effort to be 0), or a high level of effort (cost of effort $k_i > 0$, $i = 0, 1$). We assume that the entrepreneur’s effort is complementary to that of the VC in each period, as follows. If the entrepreneur exerts only low effort in a particular period, the VC’s effort is not productive at all (i.e., whether or not the VC exerts effort does not affect the probability of project success). If, however, the entrepreneur exerts high effort in a given period, the VC is able to add value in that period, depending on the nature of the firm’s project, as discussed in detail in section 2.1. The entrepreneur’s objective in choosing between angel and VC financing, as well as in making his effort choice at each date, is to maximize the expected value of his total cash flow net of effort costs.

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15 Scarcity of VC financing is a natural assumption to make in our setting: while a large number of firms may be able to benefit from the value added by a VC’s effort, the supply of VCs capable of adding such value is likely to be much smaller. As Lerner (1998) has pointed out, the supply of venture capitalists is quite inelastic, since the effective oversight of young companies is a highly specialized skill that can only be developed with years of experience. Since the hallmark of venture capital financing is value-addition, this means that venture capital firms cannot rapidly increase the supply of such financing by hiring new venture capitalists. Variations in the scarcity of venture capital financing may also be driven by variation in the flow of investment into venture funds, which, in turn may be driven by prevailing economic conditions as well as variations in the risk aversion of venture fund investors and in the expected returns from alternative investment opportunities available to them.
2.4 Information Structure and Contracting

We assume that the entrepreneur has private information with respect to outsiders (including venture capitalists and angels) at time 0 and time 1. We model this private information at time 0 by assuming that, while the entrepreneur observes the type of his own project (G or B), outsiders observe only the prior probability $\theta$ of a project being of type G.

Similarly, at time 1, we assume that the realization of the state ($p$ or $n$) is observable by the entrepreneur and the firm’s current financier, but not by outsiders. Thus, if a financier was involved with a firm from time 0 itself, he has the same information about the firm at time 1 as the entrepreneur, so that any further financing undertaken by that financier at time 1 would not suffer from asymmetric information. In contrast, if a firm switches financiers at time 1, its time 1 financing would suffer from information asymmetry, since the new financier would not observe the true time 1 state of the firm.\footnote{We assume that there are a number of VCs, angels, and a number of projects of all stages, types and states in the economy. This implies that each VC or angel is able to select both the stage (time 0, time 1) and the nature (type G or type B for a time 0 project and state-$p$ or state-$n$ for a time 1 project) of the project he wants to invest in, provided it is in the interest of the corresponding entrepreneur to select such financing. Conversely, each entrepreneur will also have available to him the financing (VC or angel) of his choice, and the financing will proceed provided that it is in that financier’s interest to invest in such a firm.}

Since only the entrepreneur and the inside financiers observe the time 1 state, publicly enforceable contracts cannot be written on these states. Thus, we assume that all contracting is done on time 2 cash flow realizations.\footnote{In other words, we make the standard assumption in the incomplete contracting literature that not all economically relevant aspects of a situation can be contracted upon: see, e.g., Aghion and Bolton (1992), or Hart and More (1994).} However, we allow for the possibility that, after they observe the time 1 state, the entrepreneur and inside financier can renegotiate their original contract: in this case the entrepreneur makes a take-it-or-leave-it offer to the financier, and the financier can accept or reject this offer. The sequence of events is summarized in figure 1, and the project payoff and information structure is depicted in figure 2.

3 Equilibrium

\textit{Definition of equilibrium}: The equilibrium concept we use here is that of Pareto dominant or Efficient Perfect Bayesian Equilibrium (PBE).\footnote{Thus we look for Perfect Bayesian Equilibria which minimize the dissipative costs incurred due to asymmetric information by the higher quality firm type (i.e., the type G firm at time 0 and state-$p$ firm at time 1). See Milgrom and Roberts (1986) for an application of efficient PBE to signaling games.} An equilibrium consists of: (i) the entrepreneur’s time 0 and time 1 financing choices (between angel and VC), the contracts offered to these financiers, and the amounts raised; (ii) the entrepreneur’s choice of effort in each period; (iii) the VC’s choice of effort in each period, if VC financing is
chosen by the entrepreneur in that period, and (iv) the decision of the financier (VC or angel) to invest in the firm’s project or not. Each of the above choices must be such that: (a) the choices of each party maximize his objective, given the equilibrium beliefs and choices of others; (b) the belief of each party is consistent with the equilibrium choices of the others; further, along the equilibrium path, these beliefs are formed using Bayes rule. Any deviation from his equilibrium strategy by any party is met by beliefs by other parties which yield the deviating party a lower expected payoff compared to that obtained in equilibrium.

To facilitate exposition, we present the equilibrium in reverse order: we first discuss the equilibrium behavior of various parties at time 1 for a given financing choice at time 0, and then go on to discuss the overall equilibrium.

3.1 Later Round Financing Choices and Contracts For a Previously VC Financed Firm

There are two kinds of projects at time 1: those in state $p$ and those in state $n$. If the firm is in state $p$, the VC’s effort will be productive in the firm. Further, since the VC was the firm’s time 0 financier, he has the same information at time 1 as the entrepreneur (given that both agents observe the realized state at time 1). For both of these reasons, it is beneficial for a firm in state $p$ to obtain another round of financing from the same VC who funded it at time 0. In this case, not only can the VC provide the requisite effort to add value to the firm, but it can also be ensured that the contract between the VC and the entrepreneur does not suffer from asymmetric information. This, in turn, means that the contract between the entrepreneur and the VC can provide the latter with stronger incentives to add value to the firm. We assume that $f(\hat{c}_1)\Delta X - k_1 - \hat{c}_1 - \frac{R}{2} > 0$, where $\hat{c}_1$ is the VC’s equilibrium effort level. The above condition translates into the VC being able to add positive value to the firm: the first term above is the marginal value created by the VC and the entrepreneur, the next two terms are the entrepreneur and the VC’s effort costs, and the last term, $\frac{R}{2}$, is the incremental cost of VC financing (in equilibrium) over angel financing.

If, on the other hand, the firm is in state $n$, then the VC’s effort is not productive (i.e., he cannot add value to the firm). Therefore, if the firm is in state $n$, the firm will not choose VC financing, since VC financing is more expensive than angel financing, and a firm in state $n$ cannot obtain any additional benefit from using VC
financing. This means that an $n$-state firm will not mimic a $p$-state firm, which finds it optimal to get time 1 funding from the VC which financed it at time 0. In other words, it is not optimal for an $n$-state firm to sell overvalued securities by mimicking a $p$-state firm. Thus, in equilibrium, a firm in state $n$ will raise time 1 financing from an angel. These insights are embodied in proposition 1.\

**Proposition 1 (Later Round Financing Choice between VCs and Angels)**

The equilibrium actions of the entrepreneur, the VC and the angel at time 1 can be characterized as follows:

(i) At time 1, a firm in state $p$ will continue to use VC financing, with the contract specified in proposition 2.

(ii) A firm in state $n$ will use angel financing.

(iii) The VC will not continue to provide funding to any firm in state $n$, but will instead leave the firm, selling his stake to an angel.

We now turn to the optimal design of the financial contract between the entrepreneur whose firm is in state $p$, and a VC who is continuing to fund it at time 1. The objective of the contract design here is to ensure that both the VC and the entrepreneur put forth optimal effort. The entrepreneur designs the contract to maximize his objective, subject to: (i) the VC’s incentive compatibility (IC) constraint, which ensures that the VC puts forth the optimal amount of effort; (ii) the entrepreneur’s own incentive compatibility constraint, which ensures that the entrepreneur exerts the optimal amount of effort; (iii) the VC’s individual rationality (IR) constraint, which ensures that the VC obtains adequate compensation for the investment amount he provides to the firm and for his effort cost, and that the VC’s return is also larger than his opportunity cost of capital; (iv) The entrepreneur’s individual rationality constraint which guarantees that the entrepreneur gets a non-negative expected payoff; (v) limited liability constraints; (vi) the firm’s budget constraint, which ensures that the firm is able to raise the required second period financing $I - I_0$ from the VC.

Let $(a^p_1, b^p_1)$ specify the contract offered by the entrepreneur to the VC, where $a^p_1$ is the share of the total cash flow of the project to the VC if $X$ is realized at time 2 (i.e., the project is highly successful) and $b^p_1$ is the VC’s share if $X$ is realized (i.e., the project is less successful). By limited liability, $0 \leq a^p_1 \leq 1$ and $0 \leq b^p_1 \leq 1$. Denote by $V_p$ the value of the VC’s time 0 financial contract at time 1 when the firm is in state $p$; let $V_n$ be the value

---

\[19\] The out-of-equilibrium belief supporting this equilibrium is that, if a firm is seeking financing at time 1 from a new financier (rather than obtaining additional financing from the current VC), then the outsiders infer that it is in state $n$. 

13
when the firm is in state \( n \).

\[
V_p = a_0(q + f(c_1))X + b_0(1 - q - f(c_1))X, \quad (1)
\]

\[
V_n = a_0qX + b_0(1 - q)X. \quad (2)
\]

Let \((a_0, b_0)\) be the time 0 financial contract between the entrepreneur and the VC, defined similar to \((a'_p, b'_p)\). Recall that, at time 0, contracting is done on time 2 cash flow realizations, since the time 1 states cannot be contracted upon. Thus, a time 0 contract cannot distinguish between the case where the cash flow \( X \) is realized by a state-\( p \) or a state-\( n \) firm. In other words, the time 0 contract between the entrepreneur and the VC is a two-period contract on \( X \) and \( X \), which is renegotiable at time 1. For simplicity, we assume that renegotiation here takes the form of a buyout (or swap) at time 1, where the entrepreneur makes a take-it-or-leave-it offer to the time 0 financier. When the VC continues to fund the firm at time 1, the time 0 financial contract is swapped for a new contract at time 1 (in other words, the time 1 contract would compensate the VC for the value of the time 0 contract, in addition to compensating him for his second period investment and effort, and ensuring that he receives at least his opportunity cost of capital from his second period investment). \( V_p \) is the value of the VC’s time 0 security in state \( p \) if the VC rejects the entrepreneur’s contract offer. In other words, since the VC knows that the firm is in state \( p \), the VC’s reservation value for his security is \( V_p \).\(^{20}\) As a result, the VC will get \( V_p \) for his time 0 security. Similarly, if the state is \( n \), the VC will get \( V_n \) for his time 0 security.

The problem of an entrepreneur having a firm in state \( p \) can therefore be characterized as:

\[
Max \quad (1 - a'_p)qX + (1 - b'_p)(1 - q - f(c_1))X + I_p^p - (I - I_0) - k_1, \quad (3)
\]

s.t \( c_1 \in \arg \max \{a'_p(q + f(c_1))X + b'_p(1 - q - f(c_1))X - c_1 \}, \quad (4)\]

\[
(1 - a'_p)qX + (1 - b'_p)(1 - q)X \leq (1 - a'_p)(q + f(c_1))X + (1 - b'_p)(1 - q - f(c_1))X - k_1, \quad (5)
\]

\[
(1 - a'_p)(q + f(c_1))X + (1 - b'_p)(1 - q - f(c_1))X + I_p^p - (I - I_0) - k_1 \geq 0, \quad (6)
\]

\[
a'_p(q + f(c_1))X + b'_p(1 - q - f(c_1))X \geq I_p^p + \frac{R}{2} + c_1 + V_p, \quad (7)
\]

\(^{20}\) The qualitative nature of our results do not depend on the specific form of bargaining adopted here. As will become clear later, as long as the time 0 financier can get a share of the increased firm value in state \( p \) and this share can be affected by the time 0 financial contract, our results go through.
Here $I^p_1$ is the amount raised by the entrepreneur from the VC at time 1. In the above, the constraint (4) is the VC’s incentive compatibility constraint. The constraints (5) is the entrepreneur’s incentive compatibility constraints (recall that $k_1$ is the entrepreneur’s cost of high effort), which ensures that the entrepreneur has an incentive to exert high effort. The constraint (6) is the entrepreneur’s individual rationality constraint, which ensures that he gets a non-negative payoff. The constraint (7) is the VC’s individual rationality constraint. The constraints (8) are the limited liability constraints. The constraint (9) is the firm’s budget constraint, ensuring that the amount raised from the VC at least covers the firm’s second period investment requirement $(I - I_0)$.

It is clear that the VC’s individual rationality constraint is binding at the optimum (otherwise the entrepreneur increases $I^p_1$ and improves his objective). Using this to simplify the entrepreneur’s problem, this is equivalent to maximizing $f(c_1)\Delta X - c_1 - k_1 - \frac{R}{2}$ subject to the above constraints. If the VC’s effort $c_1$ were contractible, his (first best) effort level $c^{fb}$ would be given by the following first order condition:

$$f'(c^{fb}) = \frac{1}{\Delta X},$$

where $\Delta X \equiv X - X$. We assume that $c^{fb} > 0$.

However, since the VC’s effort level is not contractible in practice, it is determined by his IC constraint, which yields the first order condition:

$$(a^p_1X - b^p_1X)f'(c_1) = 1.$$
Rearranging (11), we get:

\[ f'(c_1) = \frac{1}{a_1^p X - b_1^p X} \]  

(12) defines the VC’s effort level \( c_1 \) as a function of \( P_1^p = a_1^p X - b_1^p X \). \( P_1^p \) can be thought of as measuring the “power” of the contract between the VC and the entrepreneur. Given our assumptions about \( f(c_1) \), the VC’s effort level, \( c_1 \), is a strictly increasing function of \( P_1^p \). The entrepreneur’s incentive compatibility condition (5) can then be simplified and rewritten as follows:

\[ f(c(P_1^p))(\Delta X - P_1^p) \geq k_1. \]  

(13)

It immediately follows that \( P_1^p < \Delta X \), since otherwise (13) will never be satisfied. Therefore, the first best effort from the VC, \( c^{fb} \), can never be achieved here. The following proposition summarizes the solution to the above contract design problem.

**Proposition 2 (Later Round Financing Contracts)** (i) The equilibrium financing contract between a firm in state \( p \) and the VC has the following features:

(a) The optimal \( P_1^{ps} \) is the maximum solution to (13) holding as an equality, so that \( \hat{c}_1 \), the VC’s equilibrium effort choice, is given by \( f'(\hat{c}_1)P_1^{ps} = 1 \).

(b) The equilibrium financing contract at time 1 is:

\[ a_1^{ps} = \frac{1}{X} [I_1^{ps} + \frac{R}{2} + \hat{c}_1 + V_p + (1 - q - f(\hat{c}_1))P_1^{ps}], \]  

(14)

\[ b_1^{ps} = \frac{1}{X} [I_1^{ps} + \frac{R}{2} + \hat{c}_1 + V_p - P_1^{ps}(q + f(\hat{c}_1))]; \]  

(15)

where \( I_1^{ps} \), the equilibrium amount of financing raised by the firm, satisfies \( I - I_0 \leq I_1^{ps} \leq X - [\frac{R}{2} + \hat{c}_1 + V_p - P_1^{ps}(q + f(\hat{c}_1))] \).

(c) If

\[ \frac{P_1^{ps}}{I - I_0 + \frac{R}{2} + \hat{c}_1 + V_p - P_1^{ps}(q + f(\hat{c}_1))} < \frac{\Delta X}{X}, \]  

(16)

then \( a_1^{ps} < b_1^{ps} \). In general, giving the VC equity alone \( (a_1^p = b_1^p) \) cannot implement the optimal outcome.
(d) In particular, if $I - I_0 = \sum - (\frac{R_2}{2} + \tilde{c}_1 + V_p - P^{p*}_1(q + f(\tilde{c}_1)))$, then $b^{p*}_1 = 1$.

(ii) A entrepreneur in state $n$ will use angel financing. Such an entrepreneur is indifferent to a variety of financing contracts to be given to the angel, as long as the value of the contract is $I^n_{1*} + V_n$.

Clearly, in order to induce the VC to put forth optimal effort, his payoff when the firm is highly successful (cash flow $X$) has to be greater than when the firm is less successful (cash flow $\sum$). This is ensured by setting the power of the contract, $P^{p}_1 = a^{p}_1X - b^{p}_1X > 0$. The contract in (c) can be implemented by giving the VC convertible preferred equity (or equivalently preferred equity with warrants), convertible debt (or equivalently debt with warrants).\footnote{It should be noted that, while we focus on ordinary convertibles (since they are the most commonly used), such contracts can also be implemented using Participating Convertible Preferred (PCP), a relatively new financing instrument (see Bartlett (1995) for institutional details of this relatively new security).} In general, such a contract dominates a contract which involves giving the VC equity alone ($a^{q}_1 = b^{q}_1$). Recall that the contracting here is between the entrepreneur and the VC undertaking a second-round financing, so that there is no asymmetric information between the two contracting parties (as will be the case in section 3.2). This, in turn, allows the entrepreneur to provide very strong incentives to the VC, which will not be possible in the presence of asymmetric information between the contracting parties. The equilibrium financing contract ensures that the entrepreneur also has an incentive to exert high effort. The entrepreneur is the residual claimant here, receiving the cash flow left over after paying the VC.

We now come to the design of the financial contract between a firm in state $n$ and its financier. As discussed before, in this case, the entrepreneur chooses angel financing. Let $(a^{p}_n, b^{p}_n)$ be the contract given to the angel. The entrepreneur’s problem is now to maximize his objective (17), subject to his own individual rationality constraint (18), which ensures that he gets a non-negative payoff, and the angel’s individual rationality constraint (19), which ensures that the angel is compensated for the investment amount he provides to the firm, as well as the amount he provides to the entrepreneur for buying out the VC who financed the firm at time 0. Similar to the case of a state-$p$ firm, the contract design here also needs to satisfy the limited liability constraints (20) and budget constraint (21). The entrepreneur’s problem is thus:
\[
\begin{align*}
    & \text{Max}_{a^n_1, b^n_1, I^n_1} \quad (1 - a^n_1)q\bar{X} + (1 - b^n_1)(1 - q)\bar{X} + I^n_1 - (I - I_0), \\
    \text{s.t.} \quad & 0 \leq (1 - a^n_1)q\bar{X} + (1 - b^n_1)(1 - q)\bar{X} + I^n_1 - (I - I_0), \\
    & a^n_1q\bar{X} + b^n_1(1 - q)\bar{X} \geq I^n_1 + V_n, \\
    & 0 \leq a^n_1 \leq 1, 0 \leq b^n_1 \leq 1, \\
    & I^n_1 \geq I - I_0.
\end{align*}
\]

The solution to this contract design problem is summarized in part (ii) of proposition 2. In this case, the angel cannot add any value through his effort, so that the form of the contract is irrelevant as long as the angel is compensated for the amount he provides to the firm. Thus, the entrepreneur is indifferent between providing the angel equity, convertible preferred equity, or convertible debt. Further, the entrepreneur exerts only low effort in equilibrium in this case. In equilibrium, the angel buys out the financing contract of the VC who financed the firm at time 0 at its time 1 full-information value, \(V_n\), so that the total financing raised by the entrepreneur from the angel will be \(I^n_{i1}^* + V_n\).

### 3.2 Later Round Financing Choices and Contracts For a Previously Angel financed Firm

We now discuss the time 1 financing choice of a firm whose time 0 financing was undertaken by an angel. As in section 3.1, there are two kinds of projects at time 1: those in state \(p\) and those in state \(n\). Further, in this case also, it is beneficial for a firm in state \(p\) to seek VC financing at time 1, since the VC can add value to the firm through his effort. Here, however, such VC financing is undertaken under asymmetric information: since the time 0 financier was an angel, any VC would be new to the firm and would therefore not have observed the realization of the firm’s state at time 1. As in section 3.1, the VC’s effort is not productive if the firm is in state \(n\). However, in contrast to the case discussed there, a previously angel financed firm in state \(n\) has an incentive to mimic a firm in state \(p\), since doing so allows the firm to sell overvalued securities to the VC who provides funding to the firm at time 1. Thus, an equilibrium which involves separation between state \(p\) and state \(n\) firms has to satisfy the incentive compatibility conditions which ensure that a previously angel financed firm in state \(n\)
will not find it profitable to mimic a firm in state \( p \) and vice versa. First, we present the incentive compatibility condition of an entrepreneur having a firm in state \( n \). This is given by:

\[
(1 - a_n^1)qX + (1 - b_n^1)(1 - q)X + I_n^1 - (I - I_0) \geq (1 - a_n^0)qX + (1 - b_n^0)(1 - q)X + V_n + I_n^0 - V_n - (I - I_0). \tag{22}
\]

The right hand side of (22) gives the state-\( n \) entrepreneur’s payoff if he mimics a state-\( p \) entrepreneur by obtaining VC financing. The sum of the first two terms on the right hand side gives the residual cash flow to the entrepreneur. The sum of the next two terms is the money raised by the entrepreneur, \( V_p + I_p^1 \). The sum of the last two terms is the cost incurred by the entrepreneur: he has to pay \( V_n \) to buyout the angel and invest \( I - I_0 \). Similar to section 3.1, here \( V_n \) and \( V_p \) are the reservation values of the angel’s time 0 security in states \( n \) and \( p \), respectively (which are the equilibrium values of the security if the project is in states \( n \) and \( p \), respectively).

\[
V_p = a_0(q + f(c_1^p)X + b_0(1 - q - f(c_1^p))X, \quad \text{and} \tag{23}
\]

\[
V_n = a_0qX + b_0(1 - q)X, \tag{24}
\]

where \( c_1^p \) is the VC’s equilibrium effort if the project is in state \( p \). The left hand side of (22) gives payoff to an entrepreneur when his firm is in state \( n \), and he does not mimic a firm in state \( p \). In this case, the firm will reveal itself to be an \( n \)-state firm and thus obtain only angel financing. The sum of the first two terms is the residual cash flow to the entrepreneur; \( I_n^1 \) is the money raised by the entrepreneur; and \( I - I_0 \) is the firm’s investment requirement. In sum, the entrepreneur’s incentive compatibility constraint (22) ensures that he is better off revealing that his firm is in state \( n \) rather than mimicking a state-\( p \) entrepreneur.

We now consider a firm in state \( p \). The incentive compatibility constraint of this firm’s entrepreneur is given by (25). If the firm deviates by seeking angel (rather than VC) financing, the angel would believe that this firm is in state \( n \). The entrepreneur’s payoff is now given by the right hand side of (25). If, however, it does not deviate, it will reveal itself as a \( p \)-state firm, and is thus able to obtain VC financing, yielding the entrepreneur a
payoff given by the left hand side of (25).

\[(1 - a^p_1)(q + f(c_1))X + (1 - b^p_1)(1 - q - f(c_1))X + I^p_1 - (I - I_0) - k_1 \geq (1 - a^n_1)qX + (1 - b^n_1)(1 - q)X + I^n_1 - (I - I_0).\]  \hspace{1cm} (25)

As before, we assume that \(f(c_1^*)\Delta X - c_1^* - \frac{R_2}{k_1} > 0\), i.e., the VC is able to add positive value to the firm in equilibrium. We will show in the appendix that this assumption implies that the above incentive compatibility condition for the \(p\)-state firm holds. If, in addition, (22) also holds, we have the following separating equilibrium.

**Proposition 3 (Later Round Financing Choice between VCs and Angels)**

(i) A firm in state \(p\) at time 1 will obtain its second round financing from a VC (with the contract specified in proposition 4) and buy out the time 0 financier, the angel.

(ii) A firm in state \(n\) at time 1 will continue to use angel financing.

We now turn to the optimal design of the financing contract between the entrepreneur and the VC (in the case of a \(p\)-state firm) and the entrepreneur and the angel (in the case of an \(n\)-state firm) in the above separating equilibrium. We first consider the contract between the \(p\)-state entrepreneur and the VC.

The entrepreneur designs the contract to maximize his objective (26) subject to: (i) the VC’s incentive compatibility constraint (27), which ensures that he puts forth optimal effort; (ii) the entrepreneur’s own incentive compatibility constraints (28), which ensures that he also puts forth optimal effort; (iii) the entrepreneur’s individual rationality constraint (29), which ensures that he gets a non-negative payoff; (iv) the VC’s individual rationality constraint (30), which ensures that the VC obtains adequate compensation for the investment amount he provides to the firm and for his effort cost, and the VC’s return is also larger than his opportunity cost of capital; (v) the firm’s and the VC’s limited liability constraints; (vi) the firm’s budget constraint ensuring that the amount raised from the VC at least covers the firm’s second period investment requirement \((I - I_0)\) and; (vii) the incentive compatibility or “truth telling” conditions, (22) and (25), ensuring that \(n\)-state and \(p\)-state firms respectively are better off revealing their true type rather than mimicking the other type. Thus the entrepreneur’s
contract design problem is given by:

\[
\begin{align*}
\max_{a_1, b_1, I_1} & \quad (1 - a_1^p)(q + f(c_1))X + (1 - b_1^p)(1 - q - f(c_1))X + I_1^p - (I - I_0) - k_1, \\
\text{s.t} & \quad c_1 \in \arg\max\{a_1^p(q + f(c_1))X + b_1^p(1 - q - f(c_1))X - c_1\}, \\
& \quad (1 - a_1^p)qX + (1 - b_1^p)(1 - q)X \leq (1 - a_1^p)(q + f(c_1))X + (1 - b_1^p)(1 - q - f(c_1))X - k_1, \\
& \quad (1 - a_1^p)(q + f(c_1))X + (1 - b_1^p)(1 - q - f(c_1))X + I_1^p - (I - I_0) - k_1 \geq 0, \\
& \quad a_1^p(q + f(c_1))X + b_1^p(1 - q - f(c_1))X \geq I_1^p + \frac{R}{2} + c_1 + V_p, \\
& \quad (22), (25), (8) \text{ and } (9).
\end{align*}
\]

Recall that, unlike in section 3.1, the contracting here is done under asymmetric information between the VC and the entrepreneur, so that we need to impose the incentive compatibility conditions (22) and (25), in addition to the constraints (i)-(vi) imposed on the entrepreneur’s maximization problem in section 3.1. In summary, the objective of the contract design here is to ensure that both the VC and the entrepreneur put forth optimal effort, while ensuring separation between \textit{n-state} and \textit{p-state} firms.

Since the VC’s IR (30) is binding, the \textit{n-state} entrepreneur’s incentive compatibility constraint (22) can be simplified to:

\[
R \frac{R}{2} + c_1 - Pf(c_1(P_1^p)) \geq 0.
\]

It should be clear from the VC’s first order condition (12) that the VC’s effort level \(c_1\) is a function only of \(P_1^p\). The first two terms of the left hand side of (31) represent the cost of having VC financing for the \textit{n-state} firm. The last term is the \textit{n-state} firm’s benefit from selling overpriced securities to the VC. Thus condition (31) says that if the cost of having VC financing is greater than the benefit, then the \textit{n-state} firm will not mimic a \textit{p-state} firm by seeking VC financing. We now summarize the solution to the above contract design problem in the following proposition:

\textbf{Proposition 4 (Later Round Financing Contracts)} (i) The equilibrium financing contract between a firm
in state $p$ and the VC has the following features:

(a) The optimal $P^*_p$ is the maximum solution to (31) holding as an equality. $c^*_1$, the VC’s equilibrium effort choice, is such that $f'(c^*_1)P^*_p = 1$.

(b) The financing contract at time 1 is:

\[ a^*_1 = \frac{1}{X}[I^*_p + V_p + (1 - q)P^*_p], \]  

\[ b^*_1 = \frac{1}{X}[I^*_p + V_p - qP^*_p]. \]  

$p^*_1$, the equilibrium fund raised by the firm, satisfies $I - I_0 \leq p^*_1 \leq X - V_p + P^*_p q$.

(c) If

\[ \frac{P^*_p}{I - I_0 + V_p - P^*_p q} < \frac{\Delta X}{X}, \]  

then $a^*_1 < b^*_1$.

(d) In particular, if $I - I_0 = X - V_p + P^*_p q$, then $b^*_1 = 1$.

(e) Compared with the case where the firm is previously VC financed, the VC exerts a lower level of effort in the second period when the firm is previously angel financed.

(ii) A firm in state $n$ will continue to use angel financing. An entrepreneur in state $n$ is indifferent to a variety of financing contracts to be given to the angel, as long as the value of the contract is $I^*_n + V_n$.

Notice that the above equilibrium contract provides weaker incentives for the VC to exert effort compared to the case of a previously VC financed firm (in section 3.1), where the contracting proceeds under symmetric information between the entrepreneur and the VC. Asymmetric information prevents the provision of stronger incentives to the VC. This is because the greater the incentive given to the VC to exert effort, the greater also is the incentive of the $n$-state firm to mimic the $p$-state firm. This arises from the fact that the benefit to the $n$-state firm from mimicking the $p$-state firm is given by $(a^*_1 X - b^*_1 X)f(c^*_1)$, which is an increasing function of $(a^*_1 X - b^*_1 X)$. This, in turn, means that the power of the contract given to the VC in equilibrium, $P^*_p$, has to be smaller than that in the case of a previously VC financed firm, thus leading the VC to underinvest in effort here. The above contract can be implemented by giving the VC convertible preferred equity (or equivalently,
preferred equity with warrants), convertible debt (or equivalently, debt with warrants), or equity with warrants (equity with warrants can be used, however, only when \( a_1^{*} > b_1^{*} \)). However, the value of the “upside” to the VC will be smaller here compared to the case of a previously VC financed firm. Controlling for the total value of the securities issued to the VC, this implies that the value of the conversion option (or warrant value when preferred equity or debt with warrants are issued) will be lower that in the case of a previously VC financed firm. As in that case, the equilibrium financing contract here also ensures that the entrepreneur is motivated to exert high effort in equilibrium. The design of the financing contract between an entrepreneur in state \( n \) and the angel is similar to that in section 3.1. We will therefore not discuss that contract design in detail here: the solution to that design problem is summarized by part (ii) of proposition 4.

4 Early Round Financing and The Overall Equilibrium

We now describe the overall equilibrium of the model. While our primary focus here will be on characterizing the early round financing choices made by firms and the nature of early round financing contracts between the firm and its financiers, we will also discuss how these early round financing choices affect the later round financing choices described earlier, thus characterizing the overall equilibrium. Early round financing choices and contracts depend on the extent of scarcity of VC financing. One can think of three scarcity regimes: First, consider the case where VC financing is very scarce (“high scarcity”). In this case, the threshold NPV, \( R_2 \), required by the VC to invest in projects will be relatively high. At the other extreme, where the VC financing is freely available (“low scarcity”), the threshold NPV, \( R_2 \), required by the VC will be relatively low. Finally, when VC financing is moderately scarce, the threshold NPV, \( R_2 \), will lie between the above two extremes.

At time 0, the VC suffers from asymmetric information, in that he cannot a priori distinguish between the type G and type B firms. Recall that the only difference between the two types of firm at time 0 is that the VC can add value through his effort to a type G firm, but cannot do so in the first period for a type B firm. Since both types of entrepreneurs benefit from having the VC finance their firms if it ends up in state \( p \) at time 1 (since the VC can add more value in that case through his effort in the second period), the VC is a more desirable financier than the angel for both types of firms. On the other hand, VC financing is more expensive than angel
financing for both types of firms at time 0. Further, since a type B cannot benefit from the VC’s effort in the first period, it is less advantageous for a type B firm to use VC financing than for a type G firm. Given this, we will show below that the nature of early projects financed by the VC in equilibrium will differ across different scarcity regimes of VC financing. In particular, we will show that in the high-scarcity regime, VCs will fund only later stage (time 1) projects in state p, leaving all earlier stage (time 0) projects, as well as later stage projects in state n to the angel in equilibrium. In the moderate-scarcity regime, VCs will fund only type G earlier stage projects and later stage projects in state p; they will leave type B earlier stage projects, as well as later stage projects in state n, to the angel. Finally, in the low-scarcity regime, the VCs will fund all earlier stage projects (both type G and type B), and later stage projects in state p, leaving later stage projects in state n to the angel.

In the next section, we will formally define the three regimes of VC financing scarcity, and develop some intuition regarding the relationship between the scarcity regimes of VC financing and the nature of the firms seeking VC (rather than angel) financing. We will then go on to characterize the equilibrium under the moderate, low, and high scarcity regimes in section 4.2, 4.3, and 4.4, respectively.

4.1 Relationship Between the Scarcity of VC Financing and the Nature of Firms Seeking VC Financing

We define the “low scarcity regime” of VC financing as one where the opportunity cost to VC of financing a firm, \(\frac{R}{2}\), is low enough that it satisfies the following condition:

\[
0 < \frac{R}{2} \leq \lambda \delta \leq f\left(c_0^\ast\right) f\left(\hat{c}_1\right) \Delta X - \hat{c}_1 - \frac{R}{2} - k_1 - c_0^\ast - k_0 + \lambda \delta \leq f\left(c_1^\ast\right) \Delta X - c_1^\ast - k_1.
\]

(35)

Intuitively, we can think of the low scarcity regime of VC financing as one where all firms which can potentially benefit from using VC rather than angel financing will choose to do so. As discussed earlier, while both the type G and the type B firms can benefit from obtaining VC financing at time 0, the benefit to a type B firm is lower, since it comes only from the fact that if the firm ends up in state p at time 1, the VC can create value more efficiently (since value-creation can proceed under symmetric information) if the VC has already invested in the firm at time 0. A type G firm can benefit more than a type B firm from VC financing, since the VC can, by
exerting effort, increase the probability of such a firm being in state $p$ at time 1. Finally, at time 1, while a firm in state $p$ can benefit from using VC financing with probability 1, a firm in state $n$ has no benefit at all from using VC rather than angel financing. Condition (35) implies that under a low VC financing scarcity regime, both the type G and type B firms will use VC financing at time 0; further, at time 1, a firm in state $p$ will also use VC financing, while a firm in state $n$ will use only angel financing.

In (35), $\delta \equiv (f(\hat{c}_1)\Delta X - \hat{c}_1) - (f(c^*_1)\Delta X - c^*_1)$, is the incremental value created by the VC and the entrepreneur in the absence of asymmetric information at time 1, and $c^*_0$ is the VC’s equilibrium effort in both types of firm. From (35), we can see that in a low-scarcity regime the VC’s opportunity cost, $R_2$, is below the benefit to either a type G or a type B firm from VC financing at time 0, which are $f(c^*_0)(f(\hat{c}_1)\Delta X - \hat{c}_1 - \frac{R}{2} - k_1) - c^*_0 - k_0 + \lambda \delta$ and $\lambda \delta$, respectively. Therefore, in the low-scarcity regime, both types of firms choose VC financing at time 0 in equilibrium. Since the VC finances both types of projects at time 0, the equilibrium is a pooling one at this date. Further, the inequalities $0 < \frac{R}{2} < f(c^*_1)\Delta X - c^*_1 - k_1$ in (35) imply that, at time 1, a firm in state $n$ will not benefit from VC financing (and will therefore use only angel financing), while a firm in state $p$ will use VC financing.

At the other extreme, we define the “high scarcity regime” of VC financing as one where the opportunity cost of VC financing, $\frac{R}{2}$, is so high that it satisfies the following condition:

$$\lambda \delta \leq f(c^*_0)(f(\hat{c}_1)\Delta X - \hat{c}_1 - \frac{R}{2} - k_1) - c^*_0 - k_0 + \lambda \delta \leq \frac{R}{2} \leq f(c^*_1)\Delta X - c^*_1 - k_1$$

Intuitively, one can think of the high scarcity regime of VC financing as one where the cost of VC financing is so high that only firms with the greatest benefit from VC financing choose to do so, with other firms using angel financing. From (36), we can see that in a high-scarcity regime, the cost of VC financing, $\frac{R}{2}$, is higher than the benefit to a type G firm from VC financing at time 0, but lower than the benefit to a state $p$ firm from VC financing at time 1. Thus, in the high-scarcity regime, only later-stage (time 1) projects in state $p$ use VC financing in equilibrium; no early stage projects are funded by the VC. Both types of early stage projects will now be financed by the angel, thus making the equilibrium at time 0 a pooling one.

We define the moderate scarcity regime as one where the scarcity of VC financing is between the above two
extremes, so that the opportunity cost of VC financing, \( \frac{R}{2} \), satisfies the following condition:

\[
\lambda \delta \leq \frac{R}{2} \leq f(c_0^G)(f(\bar{c}_i)\Delta X - \bar{c}_i - \frac{R}{2} - k_1) - c_0^G - k_0 + \lambda \delta \leq f(c_1^*)\Delta X - c_1^* - k_1.
\]  

(37)

\( c_0^G \) in (36) above and in (37) is the equilibrium level of VC effort in a type G firm in a moderate scarcity regime.

Intuitively, the VC’s opportunity cost of capital in a moderate scarcity regime is low enough that a type G early stage firm and a later stage firm in state \( p \), find it optimal to use VC financing; however, it is high enough that an early stage type B firm will not find it optimal to use VC financing and will instead use angel financing (we will show this to be the case in equilibrium in section 4.2). Notice that, regardless of the scarcity regime, we are imposing the parameter restriction that \( 0 < \frac{R}{2} < f(c_1^*)\Delta X - c_1^* - k_1 \). This ensures that the time 1 equilibrium is not affected by the scarcity regime: while firms in state \( p \) will use VC financing at time 1, those in state \( n \) will use angel financing.\(^{25}\)

We now formally characterize the equilibrium in the three regimes of VC financing defined above. Since we believe that the moderate scarcity regime is the most interesting and the one most likely to prevail in practice, we will first analyze the equilibrium in the moderate scarcity regime in detail, and then go on to briefly discuss the equilibria in the low scarcity and the high scarcity regimes.

### 4.2 Early Round Financing Choices and Contracts under Moderate Scarcity of VC Financing

In a moderate-scarcity regime, the equilibrium is separating at both time 0 and time 1. In equilibrium, a type G firm chooses VC financing and a type B firm chooses angel financing at time 0. At time 1, a state \( p \) firm uses VC financing while a state \( n \) firm uses angel financing, as characterized in sections 3.1 and 3.2.

**Proposition 5** (Early Round Financing Choice between VCs and Angels under Moderate Scarcity)

At time 0:

(i) The type G entrepreneur seeks (and receives) VC financing;

\(^{25}\) Condition that \( \frac{R}{2} > 0 \) follows automatically from the fact that venture capital financing is scarcer than angel financing, as discussed earlier. The last inequality in (37), \( \frac{R}{2} < f(c_1^*)\Delta X - c_1^* - k_1 \), ensures that the incremental value created by the VC over and above his opportunity cost by funding a firm in state \( p \) is positive. Clearly, if this condition is violated, no VC would choose to start financing a firm at time 1, which is an uninteresting range of values of the VC’s opportunity cost.
(ii) The type B entrepreneur seeks (and receives) angel financing.

A separating equilibrium of the above nature will exist if the benefit of VC financing at time 0 exceeds its cost for a type G firm but not for a type B, so that the former chooses VC financing while the latter chooses angel financing. The incentive compatibility conditions of the type B firm, which ensure that the type B is better off using angel financing rather than VC financing, is given by (38).

\[
\begin{align*}
\lambda[W_p(c_1^*) - V^B_p(c_1^*) - (I - I_0) - c_1^* - \frac{R}{2} - k_1] + (1 - \lambda)[W_n - V^B_n - (I - I_0)] + I_0^B - I_0 & \geq 0, \\
\lambda[W_p(c_1) - V^G_p(c_1) - (I - I_0) - \hat{c}_1 - \frac{R}{2} - k_1] + (1 - \lambda)[W_n - V^G_n - (I - I_0)] + I_0^G - I_0 & \geq 0. 
\end{align*}
\]

(38)

The left hand side of (38) gives the payoff to the type B firm from not mimicking the type G firm, thereby revealing its type and obtaining only angel financing. The right hand side gives the payoff to the type B firm from mimicking the type G, and thus obtaining VC financing.

In (38), \( W_p(c_1^*) \equiv (q + f(c_1^*))X + (1 - q - f(c_1^*))\overline{X} \) and \( W_n \equiv qX + (1 - q)\overline{X} \) are the firm’s time 1 expected values in equilibrium if it is in state \( p \) and state \( n \), respectively, provided that it is financed by an angel at time 0. Remember that, since the firm makes a take-it-or-leave-it offer to the time 0 financier when it switches financing sources, the value of the time 0 financier’s security at time 1 is determined by his reservation value of these securities. Therefore, \( V^B_p \equiv (q + f(c_1^*))a_0^B_X + (1 - q - f(c_1^*))b_0^B_X \), and \( V^B_n \equiv a_0^B qX + b_0^B (1 - q)\overline{X} \) are the time 1 values of the angel’s contract \((a_0^B, b_0^B)\) when the state of the firm is \( p \) and \( n \), respectively. \( W_p(c_1) \) and \( W_n \) are the firm’s time 1 value in state \( p \) and in state \( n \) if the type B firm mimics the type G by using VC financing at time 0. Because there is no asymmetric information at time 1 if the firm is financed by a VC at time 0, in equilibrium the VC will exert effort \( \hat{c}_1 \), and the entrepreneur will exert effort \( k_1 \) if the firm is in state \( p \). Therefore, \( W_p(c_1) \equiv (q + f(\hat{c}_1))X + (1 - q - f(\hat{c}_1))\overline{X} \) and \( W_n \equiv qX + (1 - q)\overline{X} \). Similarly, \( V^G_p(c_1) \) and \( V^G_n \) are the time 1 values of the VC’s contract \((a_0^G, b_0^G)\) when the state of the firm is \( p \) and \( n \), respectively. \( V^G_p(c_1) \equiv (q + f(\hat{c}_1))a_0^G_X + (1 - q - f(\hat{c}_1))b_0^G_X \) and \( V^G_n \equiv qa_0^G X + (1 - q)b_0^G \).
B (and obtaining angel financing). Thus,

\[(\lambda + f(c_{G}^{*}))[W_p(\tilde{c}_1) - V_p^{G}(\tilde{c}_1) - (I - I_0) - \tilde{c}_1 - \frac{R}{2} - k_1] + (1 - \lambda - f(c_{G}^{*}))[W_n - V_n^{G} - (I - I_0)] + I_0^{G} - I_0 - k_0 \]

\[\geq \lambda[W_p(c_{1}^{*}) - V_p^{B}(c_{1}^{*}) - (I - I_0) - c_{1}^{*} - \frac{R}{2} - k_1] + (1 - \lambda)[W_n - V_n^{B} - (I - I_0)] + I_0^{B} - I_0, \quad (39)\]

If (38) and (39) are satisfied simultaneously, the separating equilibrium characterized in proposition 5 exists.

We now discuss the design of the time 0 financial contract between the entrepreneur and the VC under moderate scarcity of VC financing. The objective of contract design by the type G entrepreneur at time 0 is to maximize his payoff while ensuring separation between the two types of firms, subject to the VC’s individual rationality condition. The entrepreneur designs the contract to maximize his objective (40) subject to: (i) the VC’s incentive compatibility constraint (41), ensuring that the VC puts forth the optimal level of effort; (ii) the entrepreneur’s own incentive compatibility constraint (42), which ensures that he exerts high effort; (iii) the VC’s individual rationality constraint (43), which ensures that the VC obtains adequate compensation for the investment amount he provides to the firm and his effort cost, and also the VC’s return is larger than his opportunity cost of capital; (iv) the firm’s and the VC’s limited liability constraints (44); (v) the firm’s budget constraint (45) which ensures that the firm can raise at least the required amount of money for its investment; (vi) the incentive compatibility conditions, (38) and (39), which ensure that the type G and the type B firms respectively are better off revealing their true type rather than mimicking the other type; and (vii) the entrepreneur’s own individual rationality constraint which ensures that his payoff is non-negative. Thus the entrepreneur’s contract design problem is summarized by:

\[\text{Max}_{a_{G}^{0}, b_{G}^{0}, I_0^{G}} \quad (\lambda + f(c_0))[W_p(\tilde{c}_1) - V_p^{G}(\tilde{c}_1) - (I - I_0) - \tilde{c}_1 - \frac{R}{2} - k_1] + (1 - \lambda + f(c_0))[W_n - V_n^{G} - (I - I_0)] + I_0^{G} - I_0 - k_0 \]

\[s.t \quad c_0 \in \arg\max_{c_0} \{\lambda_{G} V_p^{G} + (1 - \lambda_{G}) V_n^{G} - c_0\}, \quad (41)\]

\[26\] Notice that, while we do not allow for contingent contracts based on time 1 states, p and n respectively, this is without loss of generality in our setting. This is because, in a contract design problem with contingent claims, what matters are the equilibrium values of $V_p^{G}$ and $V_n^{G}$. But for each pair of $(V_p^{G}, V_n^{G})$, we can always find a contract in our setting, $(a_{G}^{0}, b_{G}^{0})$, that implements them.
\[(\lambda + f(c_0))(W_p(c_1) - V^G_p(c_1) - (I - I_0) - \tilde{c}_1 - \frac{R}{2} - k_1)\]
\[+ (1 - (\lambda + f(c_0)))(W_n - V^G_n - (I - I_0)) + I^G_0 - I_0 - k_0\]
\[\geq \lambda(W_p(c_1) - V^G_p(c_1) - (I - I_0) - \tilde{c}_1 - \frac{R}{2} - k_1) + (1 - \lambda)(W_n - V^G_n - (I - I_0)) + I^G_0 - I_0, \quad (42)\]
\[(\lambda + f(c_0))W^G_p + (1 - (\lambda + f(c_0)))W^G_n \geq I^G_0 + \frac{R}{2} + c_0, \quad (43)\]
\[0 \leq a^G_0 \leq 1, 0 \leq b^G_0 \leq 1, \quad (44)\]
\[a^G_0 \geq I_0 \geq I^G_0 \quad (38), (39).\]

We assume that the type B and the type G entrepreneur’s IRs, limited liability, and budget constraints are not binding at the optimum at time 0. Further, under the equilibrium time 0 contracts, the entrepreneur’s IRs, limited liability, and budget constraints are also assumed to be not binding at time 1. The parameter restrictions (A-9) given in the appendix guarantee this to be the case. The following proposition summarizes the solution to the entrepreneur’s contract design problem under the above assumptions.

**Proposition 6 (Early Round financing Contracts under Moderate Scarcity)** (i) The equilibrium financing contract between a type G entrepreneur and the VC has the following features:

(a) The optimal \(P^G_0^*\) is the maximum solution to (A-7) holding as an equality. \(a^G_0^*\), the VC’s equilibrium effort choice, is such that \(f'(a^G_0^*)f(c_1)P^G_0^* = 1\).

(b) The financing contract at time 0 is:

\[a^G_0^* = \frac{1}{X}[I^G_0^* + \lambda \delta + P^G_0^*(1 - \lambda f(c_1) - q)], \quad (46)\]
\[b^G_0^* = \frac{1}{X}[I^G_0^* + \lambda \delta - P^G_0^*(\lambda f(c_1) + q)]. \quad (47)\]

\(I^G_0^*, \) the equilibrium external financing raised by the firm at time 0, satisfies \(L_0 \leq I^G_0^* \leq X + P^G_0^*(\lambda f(c_1) + q) - \lambda \delta.\)

(c) In particular, if \(L_0 = X + P^G_0^*(\lambda f(c_1) + q) - \lambda \delta, \) then \(b^G_0^* = 1.\)
(d) If
\[
\frac{P^G_0}{L_0 + \lambda \delta - P^G_0 (\lambda f(\hat{c}_1) + q)} < \frac{\Delta X}{\lambda},
\]
then \(a_0^{G*} < b_0^{G*}\).

(e) At this stage, the VC exerts a lower level of effort than at the later stage, i.e., \(c_0^{G*} < c_1 < \hat{c}_3\).

(ii) A type B firm will use angel financing. It is indifferent between a variety of financing contracts to be given to the angel, as long as the value of the contract is \(I_0^{B*}\), and the conditions \(L_0 \leq I_0^{B*} \leq X + \lambda f(c_1^*)P_0^{B*}\) and \(f(c_1^*)(\Delta X - P_0^{B*}) - c_1^* - \frac{R^2}{2} - k_1 \geq 0\) hold.

(iii) The time 1 equilibrium choice of financing, the equilibrium financing contract, and the VC’s choice of effort are as specified in section 3.1 and 3.2 respectively, depending upon whether the time 0 financing is done by the VC or the angel. The entrepreneur will choose to exert the appropriate level of effort at time 1, depending on the state the firm is in.

The equilibrium VC financing contract can be implemented by giving the VC convertible debt (or equivalently, debt with warrants), convertible preferred equity (or equivalently, preferred equity with warrants), or equity with warrants. Under the equilibrium contract, the VC exerts a lower level of effort than in later stage financing (regardless of whether the existing VC is financing a second round or a new VC is entering the firm at time 1). This is because, in earlier stage contracting, the extent of asymmetric information between the entrepreneur and the VC is higher. In other words, the benefit to a type B entrepreneur from mimicking a type G entrepreneur is higher than the benefit to a state \(n\) entrepreneur from mimicking a state \(p\) entrepreneur, since there is an incremental benefit from getting VC financing at an earlier stage (namely the entrepreneur can get more efficient VC financing at a later stage). In summary, the earlier stage contract between the VC and the entrepreneur has less power than a later stage contract, since the greater severity of asymmetric information at time 0 precludes the contract from providing stronger incentives to the VC. As a consequence, the VC exerts a lower equilibrium level of effort in the first period (early stage).
4.3 Early Round Financing Choices and Contracts under Low Scarcity of VC Financing

In the low scarcity regime, the equilibrium is a pooling one at time 0. In equilibrium, both the type G and the type B firm choose VC financing at time 0. At time 1, a state $p$ firm uses VC financing while a state $n$ firm uses angel financing, as characterized in sections 3.1 and 3.2. We summarize the equilibrium financing contract in the low-scarcity regime in the following proposition.

Proposition 7 (Early Round Financing Contracts under Low Scarcity) (i) At time 0, both types of firms seek (and receive) VC financing:

(a) $c_0^*$, the VC’s equilibrium effort choice in both type of firms, is such that $f'(c_0^*)f(\hat{c}_1)P_0^* = 1$. The optimal $P_0^*$ is given in the appendix.

(b) The financing contract at time 0 is:

$$a_0^* = \frac{1}{X} \left[ I_0^* + \frac{R}{2} + c_0^* + P_0^* (1 - (\lambda + \theta f(c_0^*))f(\hat{c}_1) - q) \right],$$  \hspace{1cm} (49)

$$b_0^* = \frac{1}{X} \left[ I_0^* + \frac{R}{2} + c_0^* - (\lambda + \theta f(c_0^*))f(\hat{c}_1)P_0^* - P_0^* q \right].$$  \hspace{1cm} (50)

$I_0^*$, the equilibrium external financing raised by the firm, satisfies $L_0 \leq I_0^* \leq X + P_0^* ((\lambda + \theta f(c_0^*))f(\hat{c}_1) + q) - \frac{R}{2} - c_0^*$.

(c) If

$$\frac{P_0^*}{L_0 + \frac{R}{2} + c_0^* - (\lambda + \theta f(c_0^*))f(\hat{c}_1)P_0^* - P_0^* q} < \frac{\Delta X}{X},$$  \hspace{1cm} (51)

then $a_0^* < b_0^*$.

(d) In particular, if $L_0 = X + P_0^* ((\lambda + \theta f(c_0^*))f(\hat{c}_1) + q) - \frac{R}{2} - c_0^*$, then $b_0^* = 1$.

(e) At this stage, the VC exerts a lower level of effort than at the later stage, i.e., $c_0^* < \hat{c}_1$.

(ii) The time 1 equilibrium choice of financing, the equilibrium financing contract, and the VC’s choice of effort are as specified in section 3.1. The entrepreneur will always choose to exert an appropriate level of effort at time 1, depending on the state the firm is in.
We now briefly discuss the contract design in the low-scarcity regime. The contract design here is similar to that in the moderate-scarcity regime, i.e., the objective of contract design is to maximize the type G entrepreneur’s payoff. Since the VC finances both the type G and the type B firm at time 0, this optimization problem is now constrained by his individual rationality constraint, which is different from that in the moderate-scarcity regime, and is given by (A-14) in the appendix. Analogous to (A-9) in the other two regimes, the parametric restrictions (A-18) in the appendix ensures that the contract offered at time 0 by the entrepreneur to the VC is such that the entrepreneur’s individual rationality constraints, limited liability constraints, and the firm’s budget constraints at time 0 and time 1 are satisfied.

Note that, in the low-scarcity regime, VCs fund a greater proportion of projects compared to the proportion they fund in the other two regimes, though the average quality of their projects will be lower than in the other regimes. The equilibrium VC financing contract can be implemented by giving the VC convertible debt (or equivalently, debt with warrants), convertible preferred equity (or equivalently, preferred equity with warrants), or equity with warrants. Under the equilibrium contract, the VC exerts a lower level of effort than in later stage financing. This is because some of the projects financed by the VC in this regime cannot benefit from the VC’s effort.

4.4 Early Round Financing Choices and Contracts under High Scarcity of VC Financing

In the high scarcity regime, the cost of VC financing, \( K^2 \), is so high that both the type G and the type B firm find it optimal to choose angel financing. That is, the equilibrium at time 0 is now a pooling one. At time 1, a state \( p \) firm uses VC financing while a state \( n \) firm uses angel financing, as characterized in section 3.2. The following proposition characterizes the equilibrium early stage financing choices and contracts under high scarcity of VC financing.

**Proposition 8 (Early Round Financing Choices and Contracts under High Scarcity)**

(i) At time 0, both the type G and the type B entrepreneur seek (and receive) angel financing.

(ii) An entrepreneur is indifferent between a variety of financing contracts to be given to the angel, as long as
the value of the contract is $I^*_0$, and the conditions $L_0 \leq I^*_0 \leq X + \lambda f(c_1^*) P_0^*$ and $f(c_1^*) (\Delta X - P_0^{B*}) - c_1^* - \frac{R}{2} - k_1 \geq 0$ hold.

(iii) The time 1 equilibrium choice of financing, the equilibrium financing contract, and the VC’s choice of effort are as specified in section 3.2. The entrepreneur will always choose to exert an appropriate level of effort at time 1, depending on the state the firm is in.

Because in equilibrium the VC does not finance any firm at time 0, both the type G firm and type B firm have the same probability of being in state $p$ at time 1. Therefore, the actual form of angel financing contract does not matter in the sense that the angel faces no asymmetric information at time 0. Notice that, in the high-scarcity regime, the VC chooses to finance only later stage projects, since his threshold NPV, $\frac{R}{2}$, is higher in this case. By financing only later-stage projects, the VC is guaranteed that his effort will be productive. By doing this, however, the VC is able to add only less value to the project, since the contracting proceeds under a greater degree of asymmetric information, given that the VC enters the firm at only a later stage in its life cycle.

4.5 Composition of Projects Financed by VCs and Angels

We now characterize the proportion of early and later stage projects funded by VCs and angels, respectively, under the moderate and high-scarcity regimes of VC financing.\(^{27}\)

**Proposition 9** (i) If $\theta < \frac{1}{2}$, then in periods of moderate or high scarcity of VC financing, a greater proportion of earlier stage projects are funded by angels than by VCs.

(ii) Under the high-scarcity regime of VC financing, angels finance all earlier stage projects, while VCs finance only later stage projects.

As we discussed in previous sections, as the scarcity of VC financing relative to angel financing goes up from low to high, the stage at which the VC first starts financing the firm goes from early to late. The moderate-scarcity regime reflects a situation between these two extremes, where the VC finances later stage projects as well as a small fraction of earlier stage projects. As discussed before, when the scarcity of VC financing is greater, the

\(^{27}\) We believe that it is the high and moderate-scarcity regimes which reflect the relative scarcity of VC financing versus angel financing in the U.S. as well as in most other developed economies in practice. We characterize the equilibrium in the low-scarcity regime only for completeness.
VCs are able to get their required return only by focusing on later stage projects, where their effort is guaranteed to be able to add value (thus creating a higher return for their investment). In contrast, since angels supply only capital but no effort, this choice between earlier and later stage projects does not apply to them, so that angels finance a larger fraction of early-stage projects in equilibrium, in the high and moderate-scarcity regimes of VC financing.

4.6 Announcement Effects of VC and Angel Financing

We now characterize the announcement effects of early round as well as later round financing of projects by VCs and angels.

Proposition 10 (Announcement Effects)

(i) Under conditions of moderate-scarcity of VC financing, the announcement of a firm obtaining VC financing at time 0 conveys favorable information to outsiders, leading them to revise their expectation of the firm’s value upward.

(ii) If a firm receives a new round of financing at a later stage from a VC, it conveys further positive information to outsiders, regardless of whether its earlier stage financing was undertaken by an angel or a VC.

(iii) If a VC exits a firm funded by him earlier, it conveys negative information about the firm to outsiders.

In the above proposition, by “announcement effect” we refer to the revision of a firm’s valuation (upward or downward) by private equity investors (since the securities of the private firms we are analyzing are not publicly traded). If a firm receives financing from the VC, outsiders infer that there is a greater probability of the VC adding value to that firm, compared to the case where the firm is angel financed. As a result, outsiders revise their valuation of the firm upward. Similarly, if a firm receives later stage funding from a VC, it conveys further positive information to outsiders, who infer that the state of the firm is such that the VC will indeed be able to add additional value to the firm. In contrast, if a VC who financed an earlier round exits the firm at a later stage, it conveys to outsiders that the VC’s assessment of his ability to add value to that firm is negative, thus leading them to revise the firm’s value downward.
4.7 Numerical Illustration: The Dynamic Evolution of Financing Contracts

In this section we give a numerical example illustrating the situations we characterized before. We illustrate only the moderate-scarcity regime of VC financing. Let the model parameters be the following: \( \theta = 0.05 \), \( \lambda = 0.1 \), \( I = 107 \), \( q = 0.2 \), \( k_0 = 1 \), \( k_1 = 55 \). \( R = 20 \) and \( I_0 = 101 \). Further, the VC’s production function is: \( f(c) = \beta c^\alpha \), where \( \alpha = 0.2 \) and \( \beta = 0.4^{28} \). Also \( Y = 300 \) and \( X = 100 \). It immediately follows that the first best effort from the VC at time 1 is \( c_{1b}^* = 32 \) and \( f(c_{1b}) = 0.8 \). Also, \( \bar{c}_1 = 17.29 \), \( f(\bar{c}_1) = 0.707 \), \( c_1^* = 5 \), and \( f(c_1^*) = 0.552 \). At time 0, \( c_0^* = 4.53 \) and \( f(c_0^*) = 0.541 \).

**Time 0 financing undertaken by the type G firm:**

From propositions 5 and 6, we know that G type firm will choose VC financing in equilibrium and the financing contract is \( a_0^{G*} = 0.487 \), \( b_0^{G*} = 0.869 \). This contract can be implemented by convertible debt or convertible preferred equity, but not using equity. The face value of the convertible preferred is 86.9, so that 86.9% cash flow to the firm is given to the preferred equity if \( X \) is realized. The preferred equity can be converted to 0.487 share of the firm’s common equity at time 2. It is clear that the contract holder will not convert to equity if \( X \) is realized. But if \( X \) is realized at time 2, the contract holder will convert for sure, since 0.487 share of 300 is greater than 86.9. The financing raised by the firm at time 0 is 101. This contract will take on one of two values at time 1, depending on the state of the project. If the state is \( n \), the value of this security is \( V_n^{G} = 98.694 \); if the state is \( p \), the value is \( V_p^{B} = 140.553 \). The contract is worth 125.53 at time 0, which covers the VC’s investment cost of 101, the VC’s opportunity cost of capital 20, and the VC’s effort cost 4.53. The entrepreneur exerts high effort at a cost of 1.

**Time 0 financing undertaken by the type B firm:**

The type B firm will choose angel financing at time 0. The entrepreneur and the angel are indifferent to a wide range of securities and also indifferent to the amount raised at time 0, since the financial contract has no impact on the value created. We assume that the firm raises 101 at time 0, the same as the type G firm does.\(^{29}\)

The contract given to the angel is \( b_0^{B*} = 1 \), and \( a_0^{B*} = 0.346 \). The contract can be implemented by convertible debt or convertible preferred equity.\(^{28} \) We assume that \( f(c) = f(c^{lb}) \) for \( c > c_1^{lb} \), so that the probability of project success in the model is always well defined, i.e., less than one.

\(^{28}\) This is only for concreteness. Since the equilibrium in the moderate-scarcity regime is separating, the type B firm is indifferent to the amount raised as long as it is at least 101 (see proposition 6 (ii)).
debt or convertible preferred equity, for the same reason as before. It should be clear that the fraction of cash flow that the firm gives to the angel when $X$ is realized is less than the corresponding fraction the type G firm gives to the VC (this fraction is larger for type B firms when $X$ is realized). This contract will be worth $V^B_n = 100.784$ if state $n$ is realized, and $V^B_p = 102.946$ if state $p$ is realized.

**Time 1 financing if the time 0 financing is done by a VC:**

*Case-1: The firm is in state $p$.*

By propositions 1 and 2, we know that the firm will continue to use VC financing and the contract given to the VC is $a_1^p = 0.651$, $b_1^p = 0.729$ (note that $a_1^p < b_1^p$ here). This contract can be implemented by convertible preferred or convertible debt; it cannot be implemented using equity alone, or using equity plus call options (warrants). The firm raises an additional amount of investment, $I - I_0 = 6$, from the VC at time 1. The VC swaps the old (time 0) contract for a new contract at time 1. The VC will exert an amount of effort $c_1 = 17.29$, and the entrepreneur will choose to exert high effort. The power $P^p_1$ of the contract $(a_1^p X - b_1^p X) = 122.246$.

*Case-2: The firm is in state $n$.*

The VC will leave the firm at time 1. The firm therefore has to raise new financing from an angel. The angel will have to pay $104.694$ to the firm. The firm will use $98.694$ to buy out the VC and use the remainder to invest. This contract can be implemented using a variety of securities, including equity ($a_1^p = b_1^p = 0.968$), convertible preferred, or convertible debt, as long as the contract’s value is $104.694$. The entrepreneur will exert low effort.

**Time 1 financing if the time 0 financing is done by an angel:**

*Case-1: The firm is in state $p$.*

By propositions 3 and 4, we know that the firm will switch to VC financing and buy out the angel. The contract given to the VC is: $a_1^p = 0.484$, $b_1^p = 0.999$ ( $a_1^p < b_1^p$ here). The power of the contract is $45.299$; notice that the power of the contract is less here than for a previously VC financed firm. This contract can be implemented by convertible preferred, but this time, the face value is $0.999 \times 100 = 99.99$. The firm raises from the VC $108.946$. The firm pays the angel $102.946$ to buy out his contract and invests an amount $I - I_0 = 6$. The VC will exert effort $c_1^e = 5$, and the entrepreneur will also choose to exert high effort at a cost of $55$.

*Case-2: The firm is in state $n$.*
The firm will continue to use angel financing in this case. The firm raises an additional amount of investment \( I - I_0 = 6 \) from the angel. The contract given to the angel can take a variety of forms, including equity \( (a_1 = b_1^* = 0.763) \) as long as its value is 106.784. The entrepreneur will exert low effort in equilibrium.

Finally, in this equilibrium, the angel finances 95\% of the earlier stage firms while VCs finance the remaining 5\%. Further, among the later-stage firms financed by the VC, 74.8\% are previously angel-financed firms financed by VCs for the first time, while 25.2\% are second-round financings of firms previously financed by the same VC.

5 Empirical Implications

We discuss some of the empirical implications of our model below.

(i) Entrepreneurs’ choice between venture capital and angel financing: Our model predicts that venture capitalists will finance firms in industries where the potential for adding value is the greatest, while angels will fund firms in industries where the potential for adding value is more limited. This means that venture capitalists will tend to finance firms in technologically sophisticated and knowledge-intensive areas, where they can create the most value. In contrast, angels will tend to finance projects in industries which are less technologically sophisticated and knowledge-intensive, with less opportunities for the financier to add value. By the same token, entrepreneurs who are technologically sophisticated themselves will tend to have self-financing or angel financing, since the incremental value that can be added by the financier will be smaller in such firms.

(ii) Differences between venture capital financing and angel financing contracts: Convertible features (convertible debt or convertible preferred equity) in venture capital financing contracts have two important roles in our setting. First, the fixed income portion, which offers downside protection to the financier, minimizes the pricing effects of the entrepreneurs’ private information. Second, the “upside” of the convertible (i.e., the warrant component) motivates the venture capitalists to put forth optimal effort to add value to the firm. (In contrast, when convertible debt or convertible preferred are used in angel contracts, they only have the first role, namely, minimizing the pricing effects of the entrepreneurs’ private information). Thus, our model predicts that angel financing contracts are less likely to have convertible features, and, when present, will have a smaller upside.
(ceteris paribus) compared to venture capital financing contracts.\(^{30}\)

(iii) The dynamic evolution of venture capital contracts: Our model predicts that, as the firm ages and engages in several rounds of venture capital financing, the nature of its venture capital financing contract will be altered across these financing rounds. This is because the venture capital contract trades-off the need to minimize the impact of asymmetric information between the entrepreneur and the venture capitalist and the competing need to provide high-powered incentives to the venture capitalist to put forth effort to add value to the firm. Thus, in the earlier financing rounds (when the extent of asymmetric information is greater), the contract would give much greater down-side protection (i.e., the fixed income component contract will be greater) to the venture capitalist. In later financing rounds, when there is less asymmetric information between the venture capitalist and the entrepreneur, the need to provide incentives to the venture capitalist dominates, so that the fixed income component of the contract will be less, while the upside (warrant) component of the contract will be more. We predict that this will be true regardless of the extent of scarcity of venture capital financing relative to angel financing.\(^{31}\) Evidence consistent with this is provided by Kaplan and Stromberg (2003), who document that venture capitalists’ state contingent cash flows increase over financing rounds at the expense of entrepreneurs.\(^{32}\)

(iv) Cross-sectional differences in later-round venture capital financing contracts: Our model predicts that there will be significant differences between the venture capital contracts involving firms which were previously angel financed (so that the venture capitalist is financing the firm for the first time) and those involving firms financed by the same venture capitalist in previous rounds: Ceteris paribus, the financing contract in the latter situation will have a relatively larger upside (warrant) component and a smaller downside (fixed-income) component compared to the former. This is because the former contracting situation is characterized by a greater

\(^{30}\) Some preliminary evidence supporting this prediction is provided by Prowse (1998), who presents evidence on the angel market gathered from field research. He comments that angel financing contracts are more likely to involve the use of equity. When angels do take convertible preferred, they are motivated by the objectives of minimizing downside investment risk and ensuring performance by management, which is also consistent with the rationale developed here. Similar evidence is provided by Wong (2001).

\(^{31}\) Of course, as the firm matures further, so that the venture capitalist’s effort becomes less important for creating value relative to that of the entrepreneur’s, the venture capital contract in subsequent financing rounds will again have a smaller “upside” (for example, this may be the case immediately prior to the firm going public), thus ensuring that the entrepreneur has the appropriate incentives to add value at this later stage in the firm’s life (see implication v).

\(^{32}\) Our analysis also has implications for the contracting in R&D alliances, where typically a large corporation contracts with a smaller research firm (with the former outsourcing some of its R&D to the latter). Similar to the relationship between a venture capitalist and an entrepreneurial firm, such strategic alliances are also characterized by asymmetric information, with the large corporation (which acts as the financier) being able to contribute to the success of the research firm through its effort. Similar to our implication for venture capital contracting, our analysis predicts that, when the research firm is in its earlier stages, the contract between the two firms should have a greater fixed income component; as the research firm matures, the warrant component of the contract should become larger, with the fixed income component becoming smaller. Evidence consistent with this prediction is provided by Robinson and Stuart (2004) in the context of biotech strategic alliances.
extent of asymmetric information, so that the equilibrium contract has to focus more on dissipating the effects of this asymmetric information and less on motivating the venture capitalist to put forth optimal effort.

(v) Relationship between firm and industry characteristics and venture capital financing contracts: Our model predicts that, in situations where there is more asymmetric information between the venture capitalist and the entrepreneur, the venture capitalist’s financing contract would give him greater down-side protection (i.e., the fixed-income component will be greater). Further, the fixed-income component of venture capitalist’s contract will also be increasing in the importance of the entrepreneur’s effort relative to that of the venture capitalist for project success (ensuring that the entrepreneur has incentive to exert more effort).

(vi) Relationship between the productivity of the venture capitalist and the nature of venture capital contracts: Our model predicts that in situations where the venture capitalist is more productive (able to add more value), the financing contract used will have more of an “upside” (warrant component). Thus, venture capitalists with greater experience will use convertibles with more upside than those with less experience. Further, a given venture capitalist will use convertibles that have less of an upside in situations where he is less productive.

(vii) Financing of early versus late stage projects: Our model predicts that there will be systematic differences in the nature of projects financed by venture capitalists and angels (stage of financing as well as project quality). Thus, in periods of high scarcity of such financing, venture capital firms will fund only later stage projects, while angels will fund early stage projects. In periods of moderate scarcity of venture capital funds, venture capitalists will fund later stage projects and a small fraction of early stage projects, leaving the rest to angel financing. In summary, our model makes two predictions regarding the stage of financing of projects by venture capitalists and angels. First, our model predicts that angels will fund the majority of early stage projects (see Wong (2001) for evidence consistent with this). Second, as the scarcity of venture capital financing relative to angel financing goes up from low to high (as measured by lower flows into venture capital funds), the fraction of early stage projects funded by venture capitalists becomes smaller. Primack (2002) provides evidence consistent with the latter prediction of our model. He documents that from 1995 to 2001, when venture capital financing was relatively abundant, early-stage investing accounted for about a quarter of all venture capital investments. In contrast,

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33 This implication follows from Proposition 4.
34 What matters for our results is the magnitude of the venture capitalists’ opportunity cost of capital relative to the average return of projects available for investment. Thus the proportion of early stage projects funded by venture capitalists will also become smaller if, holding venture capital availability constant, the average productivity of the projects available for investment falls.
this fraction dropped to 18% in the third quarter of 2001 (when venture capital financing became quite scarce) and down to 10% in the fourth quarter of the same year. In terms of project quality, our prediction is that the average quality of projects funded by venture capitalists will be greater than those funded by angels. Further, the average quality of projects funded by venture capitalists will go up during periods of high scarcity of venture capital financing (as measured by lower flows into venture capital funds).

(viii) The concentrated nature of venture capitalists’ portfolios relative to that of angels: Our model predicts that (despite the risk-reducing advantages of diversification) venture capital firms will take concentrated positions in a few firms, while angels will invest significantly smaller amounts in a number of firms. Recall that the defining feature of venture capital financing here is value-addition, and if the venture capital firm makes an investment in a firm below a certain threshold amount, it will not be incentive compatible for the firm to allocate any venture capitalist to that firm (thus, generating this effect requires the additional assumption of the indivisibility of human capital). Since angels do not engage in significant value addition, they do not face such a minimum-investment constraint, thus enabling them to take small positions in a number of firms. Further, we predict that venture capitalists will tend to concentrate their investment portfolios in a limited number of industries, since venture capitalists tend to develop expertise in a few industries which allows them to add value. In contrast, since angels are not as concerned with value addition, their investments tend to be more dispersed across industries.

(ix) The financing path of firms, announcement effects, and the probability of successful exit: Our model makes the following predictions regarding the relationship between the financing path of firms and firm quality, which, in turn, affects their probability of successful exit (IPO or acquisition versus write-off).35 Firms which are venture-financed at their early stages, and continue to attract further venture financing are of the highest quality (most likely to go public or be acquired). Firms which are angel-financed initially and attract subsequent rounds of venture financing are of lower quality (and will therefore be less likely to have a successful exit compared to those which are continuously venture financed). Finally, firms which are initially as well as subsequently angel-financed, or those which start out as venture-backed firms but attract only angel financing in subsequent rounds, will be of the lowest quality (and therefore least likely to go public or be successfully acquired). Consistent with

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35 The IPO literature indicates that, in an environment of asymmetric information, the highest intrinsic value firms are the ones that are most likely to have successful IPOs (see, e.g., Chemmanur (1993), Allen and Faulhaber (1989), or Welch (1989)).
this, our model also predicts that, if a firm successfully obtains venture capital financing in its early stages, this will convey favorable information to outside private-equity investors about that firm, who will revise its value upward. Further rounds of venture capital financing will serve as additional favorable signals to outsiders (as are later rounds of venture capital financing for a firm which is initially angel financed). Finally, exit by venture capitalists from initially venture-backed firms will serve as a negative signal to outsiders.

### 6 Conclusion

We have developed a theoretical analysis of an entrepreneur’s choice between venture capital (VC) and angel financing at various stages in a private firm’s life, and characterized the dynamic evolution of the firm’s contract with its financier (VC or angel). In our model, an entrepreneur had information superior to a potential financier about his own firm; however, this information advantage diminished as the financier interacted with the firm over time. Venture capitalists and angels differed in two ways in our model. First, venture capitalists could add value to the firm by exerting effort, which, together with the entrepreneur’s effort, increased the chance of project success; the angel was unable to add significant value. Second, venture capital financing was scarce relative to angel financing. The equilibrium VC contract in our setting maximized value-addition by ensuring that both the entrepreneur and the VC exerted optimal effort. We developed the following results in the above setting. First, we characterized the optimal financing path of the firm: depending on firm characteristics, a firm may use angel financing in its early stages and switch to VC financing in later stages, or vice versa. Second, we showed that VC financing contracts resemble convertible debt, while angel financing contracts may resemble a variety of financial securities, including equity. Third, we showed that, for firms which use venture financing in earlier as well as later rounds, earlier round financing contracts will have more of a fixed income component and less of a warrant (“upside”) component compared to later round financing contracts. Fourth, we showed that later round financing contracts between an entrepreneur and a VC who financed it in an earlier round will have a greater warrant component compared to such a contract between a VC and a previously angel-financed firm. Fifth, we characterized how the structure of VC financing contracts relate to: (i) the experience and productivity of the VC; (ii) the nature of the firm’s industry; and (iii) the scarcity of VC financing. Finally, we developed predictions
for the relationship between the financing path of a firm and the probability of its having a successful exit (IPO or acquisition), and for differences in the compositions of VCs’ and angels’ investment portfolios.
References


Appendix: Proofs of Propositions

Proof of proposition 1. We prove this proposition using the optimal contract characterized in proposition 2. If the state $p$ firm continues to use financing from the same VC, by proposition 2, the entrepreneur gets

$$qX + (1 - q)X + f(\hat{c}_1)\Delta X - \hat{c}_1 - \frac{R}{2} - V_p - (I - I_0) - k_1.$$  \hspace{1cm} \text{(A-1)}$$

On the other hand, if the firm deviates by using angel financing, let the contract be $(a^n_1, b^n_1)$ and the firm raises $I^n_1$ from the angel. The entrepreneur gets

$$(1 - a^n_1)qX + (1 - b^n_1)(1 - q)X + I^n_1 + V_n - V_p - (I - I_0).$$  \hspace{1cm} \text{(A-2)}$$

The first two terms are the entrepreneur’s residual cash flow. The next two terms are the total money raised by the entrepreneur from the angel. The entrepreneur has to pay the VC $V_p$, and invest $(I - I_0)$. Simplifying (A-2) by using the angel’s IR which is binding, $a^n_1 qX + b^n_1 (1 - q)X = I^n_1 + V_n$, it becomes $qX + (1 - q)X - V_p - (I - I_0)$. The difference between the two payoffs is $f(\hat{c}_1)\Delta X - \hat{c}_1 - \frac{R}{2} - k_1$, positive by assumption. Therefore the entrepreneur is worse off by deviating. If a state $n$ firm deviates by continuing to use VC financing, because there is no asymmetric information between the entrepreneur and the VC, by the VC’s binding IR, the entrepreneur gets

$$qX + (1 - q)X - \frac{R}{2} - V_p - (I - I_0).$$  \hspace{1cm} \text{(A-3)}$$

If a state $n$ firm uses angel financing instead, by proposition 2, the entrepreneur gets

$$qX + (1 - q)X - V_p - (I - I_0),$$  \hspace{1cm} \text{(A-4)}$$

which is clearly better than the case where the firm deviates.

Proof of proposition 2: Part (i): Simplifying the objective function using the VC’s IR, the problem becomes that of maximizing $f(c_1)\Delta X - c_1 - k_1 - \frac{R}{2}$. Because $\hat{c}_1 < c^f$, as we have shown in the main text, the entrepreneur’s IC (13) has to be binding. Otherwise, the entrepreneur can increase $P^n_1$ to $P^{n'}_1$ by increasing $a^n_1$ and fixing $b^n_1$.
to induce the VC to exert more effort $c'_1$, $c'_1 > \hat{c}_1$. At the same time he can increase $P^*_1$ such that the VC’s IR is just satisfied. The new contract improves the value of his objective from $f(\hat{c}_1)\Delta X - \hat{c}_1$ to $f(c'_1)\Delta X - c'_1$ ($f(c'_1)\Delta X - c'_1 > f(\hat{c}_1)\Delta X - \hat{c}_1$, because $\hat{c}_1 < c'_1 < c(f^p)$). Solving (13) as an equality and choosing the maximal solution gives us the optimal $P^*_1$. Thus we have shown (a). The optimal $b^*_p$ is solved by setting the VC’s IR (7) as an equality and then substituting $a^*_1 = b^*_1 \bar{X} + P^*_1$ into the equation.

For limited liability to be not binding, we need $b^*_1 \leq 1$ and $b^*_1 \geq 0$. $b^*_1 \leq 1$ is equivalent to $I^*_1 + R + \hat{c}_1 + V_p - P^*_1(q + f(\hat{c}_1)) \leq \bar{X}$. Together with the budget constraint, we get $I - I_0 \leq I^*_1 \leq \bar{X} - [\frac{R}{2} + \hat{c}_1 + V_p - P^*_1(q + f(\hat{c}_1))]$. (c) follows because this condition implies that the above inequalities become equalities. $b^*_1 \geq 0$ is equivalent to $I^*_1 + \frac{R}{2} + \hat{c}_1 + V_p - P^*_1(q + f(\hat{c}_1)) \geq 0$, which requires that $I^*_1 \geq P^*_1(q + f(\hat{c}_1)) - \frac{R}{2} - \hat{c}_1 - V_p$. This constraint is never binding since the entrepreneur can always increase $I^*_1$ to satisfy it. Part (d) follows because $a^*_1 < b^*_1$ if and only if $\frac{P^*_1}{X} < \frac{\Delta X}{\bar{X}}$, but $\frac{P^*_1}{X} = \frac{P^*_1}{I^*_1 + \frac{R}{2} + \hat{c}_1 + V_p - P^*_1(q + f(\hat{c}_1))} < \frac{P^*_1}{I - I_0 + \frac{R}{2} + \hat{c}_1 + V_p - P^*_1(q + f(\hat{c}_1))}$. (e) follows from (13) as shown in the main text.

For part (ii), it is clear that the angel’s IR has to be binding. But if that is the case, the entrepreneur’s objective is thus $q\bar{X} + (1-q)\bar{X} - (I - I_0) - V_n$, which doesn’t depend on the contract anymore. We assume and verify later that the entrepreneur’s IR (which implies limited liability and budget constraints here) is satisfied in equilibrium.

**Proof of proposition 3:** We prove this proposition using the optimal contract characterized in proposition 4. The out-of-equilibrium belief is that the firm is of type $n$. The state $n$ firm will not have the incentive to mimic the state $p$ by obtaining VC financing by (22), which is satisfied by the contract design as specified in proposition 4. A state $p$ firm will not benefit from obtaining angel financing because (25) is satisfied (using the VC and the angel’s IRs to simplify (25) we get $f(c^*_1)\Delta X - k_1 - c^*_1 - \frac{R}{2} \geq 0$, which is true by assumption). If a state $p$ deviates by offering an out-of-equilibrium contract to the VC, the VC would believe he is of state $n$ and the VC would not exert effort. As a result, the firm is worse off so doing. If a state $n$ offers an out-of-equilibrium contract to a VC, it reveals its type. Because a state $n$ firm cannot benefit from VC financing and it costs more, the firm is better off using angel financing.

**Proof of proposition 4:** As in the proof of proposition 2, it is easy to see that the entrepreneur’s objective is to maximize $f(c_1)\Delta X - c_1 - k_1 - \frac{R}{2}$. 


First, at the optimum, \( c_1^* < c^{fb} \). This is because of the entrepreneur’s IC, (28). (28) can be simplified as:

\[
f(c_1^*)(\Delta X - (a_1^{ps}X - b_1^{ps}X)) \geq k_1. \tag{A-5}
\]

It is clear that \( a_1^{ps}X - b_1^{ps}X < \Delta X \), otherwise (A-5) will not be satisfied.

Second, at the optimum, the \( n \)-state firm’s incentive compatibility constraint implies (A-5), the entrepreneur’s IC. Therefore, the entrepreneur’s IC is not binding. The \( n \)-state firm’s incentive compatibility constraint is (31), which is equivalent to:

\[
\frac{R}{2} + c_1^* \geq P_1^{ps}f(c_1^*(P_1^{ps})). \tag{A-6}
\]

By assumption \( f(c_1^*)\Delta X - k_1 - c_1^* - \frac{R}{2} \geq 0 \), so that (A-6) implies (A-5). This result implies that (A-5) is not binding at the optimum. Thus \( \hat{c}_1 \geq c_1^* \) follows because if this is not true, then (i) \( c_1^* \) satisfies all the constraints in the problem in section 3.1; (ii) the two problems have the same objective function \( f(c_1)\Delta X - c_1 \); and (iii) \( \hat{c}_1 < c_1^* < c^{fb} \). Therefore, \( c_1^* \) would be a better solution than \( \hat{c}_1 \) for the problem in section 3.1, which violates the optimality of \( \hat{c}_1 \). Further, if \( f(c_1^*)\Delta X - k_1 - c_1^* - \frac{R}{2} > 0 \), then \( \hat{c}_1 > c_1^* \) because \( \hat{c}_1 \) will violate (A-6) for sure.

Third, the \( p \)-state firm’s IC is satisfied as long as \( f(c_1^*)\Delta X - k_1 - c_1^* - \frac{R}{2} \geq 0 \), which is true by assumption. Therefore, at the optimum, (A-6) is binding. Otherwise, the entrepreneur can increase \( P_1^{ps} \) to \( P_1^{ps} \) by increasing \( a_1^{ps} \) and fixing \( b_1^{ps} \) to induce the VC to exert more effort \( c_1^*, c_1' > c_1^* \). At the same time he can increase \( P_1^{ps} \) such that the VC’s IR is just satisfied. The new contract improves the value of his objective. Solving (A-6) as an equality and choosing the maximal solution gives us the optimal \( P_1^{ps} \). Thus we have shown (a). The optimal \( b_1^{ps} \) is solved by setting the VC’s IR (30) as an equality and then substituting \( a_1^{ps}X = b_1^{ps}X + P_1^{ps} \) and the binding (A-6).

For limited liability to be not binding, we need \( b_1^{ps} \leq 1 \) and \( b_1^{ps} \geq 0 \). \( b_1^{ps} \leq 1 \) is equivalent to \( I_1^{ps} + V_p - P_1^{ps}q \leq X \). Together with the budget constraint, we get \( I - I_0 \leq I_1^{ps} \leq X - V_p + P_1^{ps}q \). (c) follows because the condition implies that the inequalities become equalities. \( b_1^{ps} \geq 0 \) is equivalent to \( I_1^{ps} \geq P_1^{ps}q - V_p \), which is never binding because the entrepreneur can always raise \( I_1^{ps} \). Part (d) follows because \( a_1^{ps} < b_1^{ps} \) if and only if \( \frac{P_1^{ps}}{b_1^{ps}} < \frac{\Delta X}{X} \), but \( \frac{P_1^{ps}}{b_1^{ps}} \leq \frac{P_1^{ps}}{I_1^{ps} + V_p - P_1^{ps}q} \leq \frac{P_1^{ps}}{I_0 - I_0 + V_p - P_1^{ps}q} \). (e) is already shown. Thus we have proven part (i). The proof of part
(ii) is similar to the proof of part (ii) in proposition 2.

**Proof of proposition 5:** We prove this proposition using the optimal contract characterized in proposition 6. The out-of-equilibrium belief is that the firm is of type B. The type B firm will not have the incentive to mimic a type G by obtaining VC financing by (38), which is satisfied by the contract design as specified in proposition 6. If the type B firm proposes an out-of-equilibrium contract to the VC and thereby reveals its type, the VC would not exert any effort. Because VC financing costs $\frac{R}{2}$ more, the type B entrepreneur is worse off if he does so. The type G has no incentive to seek angel financing because (39) is satisfied (using the VC and the angel’s IRs to simplify (25) we get $\lambda \delta + f(c_0^G)(f(\tilde{c}_1)\Delta X - \tilde{c}_1 - \frac{R}{2} - k) - c_0^G - k_0 - \frac{R}{2} \geq 0$, which is true by assumption). The type G has no incentive to deviate by offering an out-of-equilibrium contract to the VC either, because the VC would believe he is of type B and thus would not exert any effort to create value for the firm. Thus, the type G adopts VC financing using the equilibrium contract, and the type B adopts angel financing.

**Proof of proposition 6:** First, let’s look at the type B’s incentive compatibility condition, (38). Simplifying it by using the VC’s IR and the angel’s IR, $V_p^B \lambda + (1 - \lambda)V_n^B = I_0^B$, we get:

$$\lambda[(f(\tilde{c}_1)\Delta X - \tilde{c}_1) - (f(c_1^*)\Delta X - c_1^*)] - \frac{R}{2} - c_0 + f(c_0)f(\tilde{c}_1)(a_0^G X - b_0^G X) \leq 0. \quad (A-7)$$

The first term is the value created by having VC financing earlier if the firm is in state $p$, which is the benefit of a more efficient financing at time 1, $\delta \equiv (f(\tilde{c}_1)\Delta X - \tilde{c}_1) - (f(c_1^*)\Delta X - c_1^*)$. The second term is the extra financing cost by choosing VC financing. The third term is the profit from selling overvalued securities to the VC, $V_p^G - V_n^G = f(\tilde{c}_1)(a_0^G X - b_0^G X)$.

To prove (i) (a), (b), (c), and (d), we need to first show that the entrepreneur’s IC to exert high effort (42) is not binding. Simplifying (42), we get

$$f(c_0)(f(\tilde{c}_1)\Delta X - \tilde{c}_1 - \frac{R}{2} - k) - k_0 \geq f(c_0)(f(\tilde{c}_1)(a_0^G X - b_0^G X)). \quad (A-8)$$

Because $f(c_0)(f(\tilde{c}_1)\Delta X - \tilde{c}_1 - \frac{R}{2} - k_1) - c_0 - k_0 - \frac{R}{2} + \lambda \delta \geq 0$ for $c_0 = c_0^*$ by assumption, we can conclude that at the optimum, (A-7) implies (A-8). That is, (42) is not binding at the optimum.
This result characterizes the optimal $P^G_0 = a_0^G X - b_0^G X$ at time 0, i.e., $P^G_0$ is the maximum solution to the binding (A-7). The optimal $c^G_0$ is determined by $f'(c^G_0) = \frac{1}{\nu_{c_0}} = \frac{1}{f(c_0) + \lambda_c X - b_0^G X}$. And then we can get $a_0^G$ and $b_0^G$ using the VC’s IR (43) and the binding (A-7). Thus we have proved (a) and (b).

For limited liability to be not binding, we need $b_0^G \leq 1$ and $b_0^G \geq 0$. $b_0^G \leq 1$ is equivalent to $I_0^G + \lambda \delta - P^G_0 (\lambda f(\bar{c}_1) + q) \leq 0$. Using the budget constraint, we get $L_0 \leq I_0^G \leq X + P^G_0 (\lambda f(\bar{c}_1) + q) - \lambda \delta$. (c) follows because it implies that the inequalities become equalities. $b_0^G \geq 0$ is equivalent to $I_0^G \geq P^G_0 (\lambda f(\bar{c}_1) + q) - \lambda \delta$, which is never binding because the entrepreneur can always increase $I_0^G$. Part (d) follows because $a_0^G < b_0^G$ if and only if $P^G_0 \leq \frac{a_0^G X}{b_0^G X} < \frac{\lambda \delta}{\lambda f(\bar{c}_1) + q}$, but $P^G_0 \geq \frac{a_0^G X}{b_0^G X}$ because $I_0^G \geq P^G_0 (\lambda f(\bar{c}_1) + q) - \lambda \delta$.

To prove (i) (e), we need to show that $c_0^G < c_1^* < \bar{c}_1$.

The second inequality is proven; we need to show only the first. Suppose $c_0^G \geq c_1^*$. By the F.O.C. of the VC at time 0, $f'(c^G_0) = \frac{f(\bar{c}_1) a_0^G X - b_0^G X}{f(\bar{c}_1) a_0^G X - b_0^G X}$, we can find $a_1^*$ and $b_1^*$ such that $f'(c^G_0) = \frac{1}{a_1^* X - b_1^* X}$. We then have $f(c^G_0) f(\bar{c}_1) (a_0^G X - b_0^G X) = (a_1^* X - b_1^* X) f(c^G_0)$. Since $c_0^G \geq c_1^*$, state n’s IC (A-6) is violated (weakly). That is, $(a_1^* X - b_1^* X) f(c^G_0) - \frac{R}{\nu_{c_0}} \geq 0$. Using the above two conditions to simplify the left hand side of (A-7), we have $\lambda \delta - \frac{R}{\nu_{c_0}} - c_0^G + f(c^G_0) f(\bar{c}_1) (a_0^G X - b_0^G X) \geq \lambda \delta > 0$. That is, type B’s IC (A-7) cannot hold, which is a contradiction. Thus we have (e).

The proof of part (ii) is similar to the proof of part (ii) in proposition 2.

To prove (iii), we need also to show that the entrepreneur’s IR and limited liability constraints are satisfied at time 0 and time 1.

First we look at the type G firm. After some algebra we can show that if $I \leq X - \frac{R}{\nu_{c_0}} + \bar{c}_1 + \frac{R}{\nu_{c_0}} + c_0^G + (1 - \lambda - f(c^G_0)) f(\bar{c}_1) P^G_0 - P^G_0 (q + f(\bar{c}_1))]$ and $q X + (1 - q) X - I - (\frac{R}{\nu_{c_0}} + c_0^G - (\lambda + (f(c^G_0)) P^G_0) \geq 0$, the entrepreneur’s time 1 IRs (which are implied by limited liability and his IC), limited liability, and budget constraints can be satisfied by choosing $I^G_0 = I^B_0 = L_0$ and $I^G_1 = I^B_1 = I - L_0$. At time 0, the entrepreneur’s payoff is at least $q X + (1 - q) X - I$, which is greater than zero if the above two conditions hold. The limited liability and budget constraints can be satisfied if $L_0 \leq X + P^G_0 (\lambda f(\bar{c}_1) + q) - \lambda \delta$, as is given in part (i) (b).

Next, we look at the type B firm. By an argument similar to that used for the type G, the type p firm’s limited liability and budget conditions can be satisfied at time 1 if $I - I_0 \leq X - (I^B_0 + (1 - \lambda) f(c^*_1) P^B_0) + P^B_0 q$. The entrepreneur’s IR in state n is satisfied if $q X + (1 - q) X - (I - I_0) - (I^B_0 - \lambda f(c^*_1) P^B_0) \geq 0$, which can be
satisfied if \( qX + (1 - q)X - I \geq 0 \). In state \( p \), it is implied by limited liability and his IC assuming that he exerts high effort. Finally, at time 0, the entrepreneur’s IR is satisfied, since his payoff is at least \( qX + (1 - q)X - I \).

Limited liability and budget constraints at time 0 can be satisfied if \( I_0 \leq \sum + \lambda f(c_0^*)P_0^{B_0} \).

To summarize, if

\[
I \leq \min \{ X - \frac{R}{2} + \check{c}_1 + \frac{R}{2} + c_0^{G*} + (1 - \lambda - f(c_0^{G*})f(\check{c}_1)P_0^{G*} - P_1^{p*}(q + f(\check{c}_1))], \\
qX + (1 - q)X - (\frac{R}{2} + c_0^{G*} - (\lambda + f(c_0^{G*})P_0^{G*}) - X - (1 - \lambda f(c_0^*)P_0^{B*} + P_1^{p*}q) \},
\]

(A-9)

the equilibrium holds. Thus (iii) is proved.

**Proof of proposition 7:** The optimal contract design problem of the type G is the following:

\[
\max_{c_0, b_0, I_0} \quad (\lambda + f(c_0))[W_p(\check{c}_1) - V_p(\check{c}_1) - (I - I_0) - \check{c}_1 - \frac{R}{2} - k_1] \\
+ (1 - \lambda - f(c_0))[W_n - V_n - (I - I_0)] + I_0^* - I_0 - k_0
\]

\[\text{s.t.} \quad c_0 \in \arg\max_{c_0} \{(\lambda + \theta f(c_0))V_p + (1 - (\lambda + \theta f(c_0)))V_n - c_0\}, \quad \text{(A-12)}\]

\[
\lambda[W_p(\check{c}_1) - V_p(\check{c}_1) - (I - I_0) - \check{c}_1 - \frac{R}{2} - k_1] \\
+ (1 - \lambda)[W_n - V_n - (I - I_0)] + I_0^* - I_0 - k_0
\]

\[\geq \lambda[W_p(\check{c}_1) - V_p(\check{c}_1) - (I - I_0) - \check{c}_1 - \frac{R}{2} - k_1] \\
+ (1 - \lambda)[W_n - V_n - (I - I_0)] + I_0^* - I_0, \quad \text{ (A-13)}\]

\[
(\lambda + \theta f(c_0))V_p + (1 - (\lambda + \theta f(c_0)))V_n \geq I_0^* + \frac{R}{2} + c_0, \quad \text{ (A-14)}
\]

\[0 \leq a_0 \leq 1, 0 \leq b_0 \leq 1, \quad \text{ (A-15)}\]

\[I_0^* \geq I_0 \geq I_0, \quad \text{ (A-16)}\]

Since (A-14) is binding (otherwise the entrepreneur can increase \( I_0^* \)), the entrepreneur’s objective can be simplified to:

\[
f(c_0)(f(\check{c}_1)\Delta X - \check{c}_1 - \frac{R}{2} - k_1) - c_0 - \frac{R}{2} - k_0 - (1 - \theta)f(c_0)(V_p - V_n). \quad \text{ (A-17)}
\]
The first four terms in (A-17) are the value created by the VC and the entrepreneur’s effort. The last term is the extra financing cost the type G has to pay the VC because of asymmetric information. The VC’s F.O.C. is
\[ f'(c_0) = \frac{1}{\theta f(c_1) P_0}. \]
There are two cases. First, let (A-13) be binding. Then \( f(c_0)(f(c_1)\Delta X - \hat{c}_1 - \frac{R}{2} - k_1) - k_0 = f(c_0)f(\hat{c}_1)P_0 \) and \( f'(c_0) = \frac{1}{\theta f(c_1) P_0} \) determine the optimal \( P_0^* \) and \( c_0^* \). In this case, \( (a_0^*, b_0^*) \) are solved by using (A-14). Second, let (A-13) be not binding. In this case, the optimal \( P_0^* \) and \( c_0^* \) is determined by taking first order condition of (A-17), \( f'(c_0)(f(\hat{c}_1)\Delta X - \hat{c}_1 - \frac{R}{2} - k_1) - 1 - (1 - \theta)f'(c_0)f(\hat{c}_1)P_0 = 0 \), and \( f'(c_0) = \frac{1}{\theta f(c_1) P_0} \). In this case, \( a_0^* \) and \( b_0^* \) are solved for using (A-14). \( c_0^* < \hat{c}_1 \) follows by comparing (A-13) and (13), as in the proof of proposition 6 (i) (e).

Next, we check the entrepreneur’s incentive to deviate. The out-of-equilibrium belief is that the firm is of type B. If a type G firm deviates and offers another contract to the VC, the VC would exert zero effort. By our assumption (35), in this case the type G is worse off. The type G firm clearly has no incentive to seek angel financing either, by (35). The type B firm would not seek angel financing since \( \lambda \delta \geq \frac{R}{2} \). Comparing the type B’s payoff if it offers an out-of-equilibrium contract to the VC and thereby reveals his type to his equilibrium payoff, we find that type B has no incentive to deviate if and only if \( \lambda \delta - \frac{R}{2} \leq \lambda \delta - \frac{R}{2} + f(c_0^*)\theta f(\hat{c}_1)P_0^* - c_0^* \). By the VC’s IC (A-12), we get \( f(c_0^*)\theta f(\hat{c}_1)P_0^* - c_0^* \geq 0 \). Therefore, the type B will not deviate.

Finally, similar to the proof of proposition 6, it can be shown that the entrepreneur’s IR, limited liability, and budget constraints at time 0 and time 1 are satisfied in equilibrium. The condition corresponding to (A-9) is:

\[
I \leq \min \{ \bar{X} - [\frac{R}{2} + \hat{c}_1 + \frac{R}{2} + c_0^* + (1 - \lambda - \theta f(c_0^*))f(\hat{c}_1)P_0^* - P_1^{ps}(q + f(\hat{c}_1))], \\
q\bar{X} + (1 - q)\bar{X} - (\frac{R}{2} + c_0^* - (\lambda + \theta f(c_0^*))P_0^*) \}.
\]

(A-18)

**Proof of proposition 8**: The out-of-equilibrium belief is that the firm is of type B. Clearly the type B firm has no incentive to deviate since, if it does, by assumption (36), the marginal benefit of VC financing, \( \lambda \delta \), is lower than the marginal cost, \( \frac{R}{2} \), and the firm is worse off. If the type G firm deviates by seeking VC financing, the VC would believe that he is of type B and thus not exert any effort. Therefore, the firm is worse off because it has to pay the VC an additional amount, \( \frac{R}{2} \). The entrepreneur’s IRs, limited liability, and budget constraints are satisfied by (A-9).
Proof of proposition 9: This follows directly from propositions 5, 6, and 7.

Proof of proposition 10: This follows directly from propositions 5, 6, 7, and 8.