Proposition 1 (IPO waves even without a productivity shock) When the magnitude of the shock is moderate ($A_1 - A_2 < \Delta A \leq \Delta A_L$), the probability of the shock is large, the issuing cost is small, and the existing productivity levels of firm 1 and firm 2 are high, both firms will go public before the realization of a productivity shock (at time 1).

Proposition 2 (IPO waves only with a productivity shock) When the magnitude of a potential shock is large ($\Delta A > \Delta A_L$), in addition to the PBE in Proposition 1, we have two more possible equilibria:

(i) When the existing market share of firm 1 is moderately large ($m \geq (2 + s - \sqrt{s^2 + 4})/(2s)$), and the existing productivity levels of firm 1 and firm 2 are low, both firms will remain private before the realization of a productivity shock (at time 1) and go public only upon the realization of a shock (at time 2).

(ii) When the existing market share of firm 1 is small ($m < (2 + s - \sqrt{s^2 + 4})/(2s)$), the existing productivity of firm 1 is high, and the existing productivity of firm 2 is low, firm 1 will go public before the realization of a productivity shock (at time 1) and firm 2 will go public only upon the realization of a shock (at time 2).

Proposition 3 (IPOs off the wave) When the magnitude of a potential shock is not too large ($\Delta A < \Delta A_H$), we have two additional equilibria besides the above three PBEs:

(i) When the probability of a potential shock is low, the existing market share of firm 1 is small ($m < (2 + s - \sqrt{s^2 + 4})/(2s)$), the existing productivity of firm 1 is low, and the issuing cost is large, firm 1 will go public only upon the realization of a shock (at time 2) and firm 2 will remain private throughout.

(ii) When the probability of a potential shock is high, the existing market share of firm 1 is moderately large ($m \geq (2 + s - \sqrt{s^2 + 4})/(2s)$), the existing productivity of firm 1 is high, and the issuing cost is small, firm 1 will go public before the realization of a productivity shock (at time 1) and firm 2 will remain private throughout.
Figure 3: Evolution of market share and its relationship to the going public decision

**Proof of Propositions 1, 2, and 3:** Since these three propositions all involve solving the perfect Bayesian equilibrium (PBE) for the model, we put the proofs of them together. We first prove the more complicated case of purely idiosyncratic productivity shocks (studied in Appendix B) and then generalize the proofs to the case of industry-wide productivity shocks. After solving for the possible PBEs, we characterize the conditions under which they can occur.

**Part (a): Solving for all the PBEs in the model.**
To make our exposition simpler, we define the following operating profit functions per unit of

<table>
<thead>
<tr>
<th>Time 1</th>
<th>Time 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1’s market share</td>
<td>Firm 1’s market share</td>
</tr>
<tr>
<td>Go public</td>
<td>Go public</td>
</tr>
<tr>
<td>$m$</td>
<td>$1 - m$</td>
</tr>
<tr>
<td>Go public</td>
<td>Remain private</td>
</tr>
<tr>
<td>$m_i = m + (1 - m) sm$</td>
<td>$1 - m_b = (1 - m)(1 - sm)$</td>
</tr>
<tr>
<td>Remain private</td>
<td>Go public</td>
</tr>
<tr>
<td>$m_i = m - (1 - m) sm$</td>
<td>$1 - m_c = (1 - m)(1 + sm)$</td>
</tr>
<tr>
<td>Remain private</td>
<td>Remain private</td>
</tr>
<tr>
<td>$m$</td>
<td>$1 - m$</td>
</tr>
</tbody>
</table>
market share:

\[ \pi_1 = A_1(k_{1L}^e)^\gamma - ck_{1L}^e = (A_1)^{\frac{1}{\gamma}}c^{\frac{\gamma}{\gamma-1}}\gamma^{\frac{1}{\gamma-1}}(1 - \gamma) \]

\[ \pi_2 = A_2(k_{2L}^e)^\gamma - ck_{2L}^e = (A_2)^{\frac{1}{\gamma}}c^{\frac{\gamma}{\gamma-1}}\gamma^{\frac{1}{\gamma-1}}(1 - \gamma) \]

\[ \pi_{2H} = A_{2H}(k_{2H}^e)^\gamma - ck_{2H}^e = (A_{2H})^{\frac{1}{\gamma}}c^{\frac{\gamma}{\gamma-1}}\gamma^{\frac{1}{\gamma-1}}(1 - \gamma) \]

\[ \pi_{1H}^{IPO} = A_{1H}(k_{1H}^e)^\gamma - ck_{1H}^e = (A_{1H})^{\frac{1}{\gamma}}c^{\frac{\gamma}{\gamma-1}}\gamma^{\frac{1}{\gamma-1}}(1 - \gamma) \]

\[ \pi_{1H}^{PR}(m_{1H}^{I}) = A_{1H}(K_0/m_{1H}^{I})^{\gamma} - c(K_0/m_{1H}^{I}) \]

where \( \pi_1 \) and \( \pi_2 \) are the per-unit-market-share operating profits for firm 1 and firm 2 respectively if they do not experience the productivity shock; \( \pi_{2H} \) is the per-unit-market-share operating profit for firm 2 if it gets the shock (whether or not it is public); \( \pi_{1H}^{IPO} \) is the per-unit-market-share operating profit for firm 1 if it gets the shock and goes public; and \( \pi_{1H}^{PR}(m_{1H}^{I}) \), which depends on the level of \( m_{1H}^{I} \), is firm 1’s per-unit-market-share operating profit if it gets the shock but chooses to remain private. We then have the following relationship:

\[
\begin{cases} 
\pi_2 < \pi_1 & < \pi_{1H}^{PR}(m_{1H}^{I}) < \pi_{1H}^{IPO} \\
\pi_2 < \pi_{2H} & < \pi_{1H}^{PR}(m_{1H}^{I}) < \pi_{1H}^{IPO} 
\end{cases}
\]

Moreover, we have made the following assumption:

\[ m_{1H}^{IPO} - m_{1H}^{PR}(m_{1H}^{I}) > B \]

where the left handside denotes the benefit of going public (even if it does not enhance product market competitiveness) and the right handside is the cost.

Finally, we have also assumed that without productivity shocks, the maximum possible benefit each firm can derive from additional market share will be less than the cost of going public:

\[ B > [m_b + sm_c(1 - m_b)]\pi_1. \]

\[ B > [(1 - m_c)(1 + sm_c) - (1 - m)]\pi_2. \]

To solve for PBE, we use backward induction. Let \((a_1, a_2)\) denote the strategies adopted by firm 1 and firm 2 respectively. \( a_i \) can be either "IPO" (meaning "go public" or "remain public") or "PR" (meaning "remain private"). Then the strategy profile at time 1 can be one of the following four: \((IPO, IPO), (IPO, PR), (PR, IPO), (PR, PR)\). According to Figure 2, if the time 1 strategy profile is \((IPO, IPO)\), then the market share for firm 1 and 2 is \((m, 1 - m)\) at time 2 and the game is over because both firms cannot switch back to the private status. If the time 1 strategy profile is \((IPO, PR)\), then at time 2, firm 2’s market share will be \((1 - m_b)(1 - sm_b)\) if it stays private and \(1 - m_b\) if it goes public. The incremental market share it gets by IPO is \((1 - m_b)sm_b\). Similarly, if the time 1 strategy profile is \((PR, IPO)\), the incremental market share for firm 1 to go public rather than remaining private at time 2 is \((1 - m_c)sm_c\). Lastly, if the time 1 strategy profile is \((PR, PR)\), then either firm 1 or firm 2’s incremental market share by IPO (given rival’s strategy) at time 2 is \((1 - m)sm\). Hence, the firms will compare the incremental profit by IPO to the issuing cost, \( B \), to make their going public decision. Since we will make use of these incremental market
share, we need to rank them. Simple calculation yields
\[
\begin{cases}
(1 - m_b)m_b > (1 - m)m & \Leftrightarrow m < \frac{2^{s/2} - \sqrt{s^2 + 4}}{2s} (< \frac{1}{2}) \\
(1 - m_c)m_c > (1 - m)m & \Leftrightarrow m > \frac{s - 2^{s/2} - \sqrt{s^2 + 4}}{2s} (> \frac{1}{2}).
\end{cases}
\] (A1)

Since this model involves discrete choice of firms, we need to examine the parameter values in different ranges.

Case (i) \( \Delta A < A_1 - A_2 \)

1. \( 0 < m < \frac{2^{s/2} - \sqrt{s^2 + 4}}{2s} \Rightarrow (1 - m_b)m_b > (1 - m)m > (1 - m_c)m_c \)

By conditions (4) and (5), we then have:
\[
B > (1 - m)sm \pi_{2H} > (1 - m)sm \pi_1 > (1 - m)sm \pi_2,
\] (A2)

which means that if the time 1 strategy profile is \((PR, PR)\), firm 2 will continue to remain private at time 2. Firm 1 will not go public at time 2 if it does not get the productivity shock. However, if firm 1 gets the productivity shock, it pays off to go public. This is because given firm 2 IPO at time 2, firm 1 with higher productivity will gain net profit \(m \pi_1^{IPO} - m_c \pi_1^{PR} (m_c)\) by also going public. Similarly, given firm 2 remaining private at time 2, firm 1 with higher productivity will gain \(m_b \pi_1^{IPO} - m \pi_1^{PR} (m)\) by going public. Given assumption (3) in the main text, and the fact that \(m_1^{II} \pi_1^{PR} (m_1^{II})\) is increasing in \(m_1^{II}\) (proof of this is straightforward and thus omitted here), it directly follows that going public is a strictly dominant strategy for firm 1 at time 2 once it gets the productivity shock. Similarly, since \(\pi_{2H} < \pi_1\) and \([m_b + sm_b (1 - m_b) - m] \pi_1 < B\), we have:
\[
B > (1 - m_b)sm_b \pi_{2H} > (1 - m_b)sm_b \pi_2,
\] (A3)

which shows that remaining private is also dominant for firm 2 at time 2 if firm 1 has already gone public at time 1. Last, (3), (4), and (5) determine that:
\[
m_c \pi_1^{IPO} - (m_c - sm_c (1 - m_c)) \pi_1^{PR} (m_c - sm_c (1 - m_c)) > B > sm_c (1 - m_c) \pi_1.
\] (A4)

In this case, if the time 1 strategy profile is \((PR, IPO)\) and firm 1 has the productivity shock at time 2, it will go public. Otherwise it will remain private. Therefore, the expected payoffs for the four time 1 strategy profiles can be written as:
\[
\begin{align*}
(IPO, IPO): & \ (x_1, y_1) \\
(IPO, PR): & \ (x_2, y_2) \\
(PR, IPO): & \ (x_3, y_3) \\
(PR, PR): & \ (x_4, y_4)
\end{align*}
\] (A5)
where
\[
\begin{align*}
x_1 &= m(p\pi_1^{PO} + (1 - p)\pi_1) - B \\
x_2 &= (m_b + sm_b(1 - m_b))(p\pi_1^{PO} + (1 - p)\pi_1) - B \\
x_3 &= pm_c\pi_1^{PO} + (1 - p)[m_c - sm_c(1 - m_c)]\pi_1 - pB \\
x_4 &= p[m + sm(1 - m)]\pi_1^{PO} + (1 - p)\pi_1 - pB \\
y_1 &= (1 - m)(p\pi_2 + (1 - p)\pi_2) - B \\
y_2 &= (1 - m_b)(1 - sm_b)(p\pi_2 + (1 - p)\pi_2) \\
y_3 &= (p\pi_2 + (1 - p)\pi_2)(p(1 - m_c) + (1 - p)(1 - m_c)(1 + sm_c)) - B \\
y_4 &= (p\pi_2 + (1 - p)\pi_2)[p(1 - m_b) + (1 - p)(1 - m)]
\end{align*}
\]

(A6)

Now, given parametric conditions, it is easy to show that
\[
\begin{align*}
y_1 &< y_2 \\
y_3 &< y_4
\end{align*}
\]

(A7)

which means that remaining private is dominant for firm 2 at time 1. And the final PBE depends on
\[
x_2 - x_4 = sm_b(1 - m_b)(p\pi_1^{PO} + (1 - p)\pi_1) + (1 - m)sm(1 - p)\pi_1 - (1 - p)B.
\]

(A8)

When this condition is greater than 0, the PBE is that firm 1 goes public at time 1 and firm 2 remains private for both periods. When this condition is less than 0, the PBE is that both firms remain private at time 1 and firm 1 will go public at time 2 if the productivity shock is realized.

Similar analysis can be applied to the following three cases as well:

(II) \( \frac{2 + \sqrt{2} + 4}{2m} < m < \frac{1}{2} \) \( (1 - m)m > (1 - m_b)m_b > (1 - m_c)m_c \)

(III) \( \frac{1}{2} < m < \frac{2 + \sqrt{2} + 4}{2m} \) \( (1 - m)m > (1 - m_c)m_c > (1 - m_b)m_b \)

(IV) \( \frac{2 + \sqrt{2} + 4}{2m} < m < 1 \) \( (1 - m_c)m_c > (1 - m)m > (1 - m_b)m_b \)

The calculations are tedious but straightforward. If we use the following notations,
\[
\begin{align*}
x_1 - x_3 &= (*1) \\
y_3 - y_4 &= (*2) \\
x_2 - x_4 &= (*3) \\
y_1 - y_2 &= (*4)
\end{align*}
\]

(A9)

the results for case (i) can be succinctly presented below:

If \( (*3) > 0 \) or \( (*2) > 0 \) and \( m > \frac{2 + \sqrt{2} + 4}{2s} \), the time 1 equilibrium strategy is \( (IPO, PR) \) and the time 2 equilibrium strategy is \( (IPO, IPO, PR, PR) \).

Otherwise, the time 1 equilibrium strategy is \( (PR, PR) \) and the time 2 equilibrium strategy is \( (IPO, PR, PR, PR) \). We should note that \( (*1) \), \( (*2) \), \( (*3) \) and \( (*4) \) bear different forms and values under different parametric conditions, which means that \( x \)'s and \( y \)'s do not necessarily look like those given in (A5).

Case (ii) \( \Delta A \geq A_1 - A_2 \)

1The equilibrium strategy at time 1 is given in the form of \( (a_1, a_2, a_3, a_4) \) which correspond to the strategies adopted by firm 1 with shock, firm 1 without shock, firm 2 with shock, and firm 2 without shock, respectively.
Basically we follow the same steps to solve the model. This time the results are more complicated yet more interesting as well. Since there are many parametric ranges that lead to the same PBE, we do not provide the long list of those conditions here. The general form of solutions can be described as:\footnote{A complete list of all parametric conditions that lead to different PBEs can be provided to interested readers upon request.}

If $(\ast 1) > 0$ and $(\ast 4) > 0$, the time 1 equilibrium strategy is $(PR, PR)$ and the time 2 equilibrium strategy is $(IPO, IPO, IPO, PR)$.

- If $(\ast 3) > 0$
- $(\ast 4) < 0$
- $(1 - m_h)sm_h\pi_{2H} > B > (1 - m_h)sm_h\pi_2$

and the time 2 equilibrium strategy is $(IPO, IPO, IPO, PR)$.

- If $(\ast 3) > 0$
- $(\ast 4) < 0$
- $B > (1 - m_h)sm_h\pi_{2H} > (1 - m_h)sm_h\pi_2$

There may be rare cases where we have no PBEs or two PBEs (whose time 1 strategies are $(IPO, IPO)$ and $(PR, PR)$, respectively). For brevity, we do not provide detailed descriptions of these cases here. Last but not least, we can prove that when $x_1 > x_3$, we always have $y_3 < y_4$. This means that regardless of parameter values, $(PR, IPO)$ is never a time 1 equilibrium strategy.

When the productivity shock is industry-wide, the analysis is very similar to the case of idiosyncratic shocks. The only difference is that when the time 1 strategy profile is $(PR, PR)$, the uncertain situations faced by the two firms are reduced to two rather than four as before. With probability $p$, both firms’ technology parameters $A_i$ will increase by $\Delta A$ and with probability $1 - p$ the parameters $A_i$ will remain unchanged for the whole industry. All other analysis follows the same procedure as outlined above. To save space, we do not present the detailed list of all possible equilibria here, but they are available upon request.

**Part (b): Characterizing the conditions under which different PBEs may occur.**

After solving for all five possible PBEs in the model, we continue to characterize them in this section. For brevity, we only illustrate the case of industry-wide productivity shocks (Propositions 1, 2, and 3). The characterization of PBEs in the case of idiosyncratic productivity shocks is similar. To make our notations easy to follow, we define the following two boundaries for the magnitude of...
the shock ($\Delta A$):

$$
\Delta A_L = \left( \frac{c}{\gamma} \right)^7 \left( \frac{B}{(1-\gamma)\text{Max}\{sm(1-m), sm_b(1-m_b)\}} \right)^{1-\gamma} - A_2.
$$

$$
\Delta A_H = \left( \frac{c}{\gamma} \right)^7 \left( \frac{B}{(1-\gamma)\text{Min}\{sm(1-m), sm_b(1-m_b)\}} \right)^{1-\gamma} - A_2.
$$

Figure 2 in the main text lists all possible PBEs under different ranges of market share distribution and the magnitude of the productivity shock. As we can see, when the magnitude of the shock is very large ($\Delta A \geq \Delta A_H$), we only observe IPO-wave equilibria (Eq1, Eq2, and Eq3), whereas when the magnitude of the shock is small ($\Delta A \leq A_1 - A_2$), only off-the-wave IPOs may occur in equilibrium (Eq4 and Eq5). Eq1 may occur only when the magnitude of the shock is greater than $A_1 - A_2$ (i.e., at least moderate), while Eq2 and Eq3 may occur only when $\Delta A$ is at least large ($\Delta A > \Delta A_L$). When the magnitude of the shock is large but not very large ($\Delta A_L < \Delta A < \Delta A_H$), the market share distribution matters: if $m$ is below $\frac{2+\sqrt{2^2+4}}{24}$, only Eq3 and Eq4 may occur rather than Eq2 or Eq5.

To differentiate the PBEs within each category of market share distribution and the magnitude of productivity shock, we need to look at the specific conditions for them to occur. From our proof in part (A), we know that the general rule is that Eq1 will occur when both $(*)1 > 0$ and $(*)4 > 0$; Eq2 and Eq4 will occur when both $(*)3 < 0$ and $(*)2 < 0$; and Eq3 and Eq5 will occur when $(*)3 > 0$ & $(*)4 < 0$. A complete analysis of $(*)1$, $(*)2$, $(*)3$, and $(*)4$ shows that all these four conditions increase with $\Delta A$ and decrease with $B$. $(*)1$ and $(*)3$ increase with $A_1$, while $(*)2$ and $(*)4$ increase with $A_2$. $(*)1$ and $(*)4$ always increase with $p$, but $(*)2$ and $(*)3$ may increase or decrease with $p$. When $p$ is close to 1 and $m < \frac{2+\sqrt{2^2+4}}{24}$, $(*)3$ will decrease with $p$; and when $p$ is close to 1 and $m > \frac{2+\sqrt{2^2+4}}{24}$, $(*)2$ will decrease in $p$. In all other situations, both $(*)2$ and $(*)3$ increase in $p$. We find out these results by listing $(*)1$, $(*)2$, $(*)3$, and $(*)4$ for each parameter range and calculate their derivatives with respect to model parameters, $\Delta A$, $B$, $A_1$, $A_2$, and $p$. For brevity, the specific calculations case by case will not be presented here, but they are available upon request.

Q.E.D.

Proposition 4 (Average pre-IPO productivity and IPO timing) Consider an industry with parameter values such that all five possible PBEs in Propositions 1, 2, and 3 occur with positive probabilities. Then:

(i) firms that go public in an IPO wave will have lower average pre-IPO productivity than those that go public off the wave.

(ii) firms that go public earlier in an IPO wave will have higher average pre-IPO productivity than those that go public later in the wave.

Proof: Part (i) of the proposition is straightforward to prove. Consider an industry where all five possible PBEs in Propositions 1, 2, and 3 will occur with positive probabilities, then if we observe a stand-alone IPO, it must be conducted by firm 1 (possibly from Eq3, Eq4, and Eq5). That’s why the pre-IPO productivity of such a firm is $A_1$. Similarly, if we observe an IPO wave in the sense that
two IPOs occur during the period of time 1 and 2, then it must be the case that both firm 1 and 2 have gone public so the average pre-IPO productivity for such a firm is \((A_1 + A_2)/2\), which is lower than \(A_1\).

Part (ii) of the proposition follows from the observation that if an IPO wave spans both time 1 and 2, then it must be from Eq3, which says that firm 1 will go public at time 1 and firm 2 will do so at time 2 when the productivity shock occurs. That’s why the pre-IPO productivity for a firm that goes public earlier in such a wave \((A_1)\) is higher than that for a later comer in the IPO wave \((A_2)\).

Q.E.D.

**Proposition 5 (Average post-IPO operating performance and IPO timing)** Consider an industry with parameter values such that all five possible PBEs in Propositions 1, 2, and 3 occur with positive probabilities. Then, conditional on whether an industry-wide productivity shock has occurred:

(i) an average firm that goes public in an IPO wave will have lower post-IPO operating performance than an average firm that goes public off the wave.

(ii) an average firm that goes public earlier in an IPO wave will have higher post-IPO operating performance than an average firm that goes public later in the wave.

**Proof:** We also consider an industry where all five possible PBEs in Propositions 1, 2, and 3 will occur with positive probabilities. The post-IPO operating performance for firm \(i\) is defined as:

\[
ROA_{III}^i = \frac{m_{III}^i (A_{III}^i)^{\gamma \theta} c^{\eta \gamma} (1 - \gamma) - B}{K_0 + E},
\]

(\(A10\))

where \(A_{III}^i\) can be either \(A_i\) or \(A_{iH}\), depending on whether the industry-wide shock occurs. If two firms have identical initial capital level \((K_0)\), raise the same amount of equity \((E)\), pay the same issuing costs \((B)\), and have the same average post-IPO market share \((m_{III}^i = \bar{m}, i = 1, 2)\), then their post-IPO operating performance \((ROA_{III}^i)\) depends mainly on their post-IPO productivity \((A_{III}^i)\). If these two average firms go public at different times of "market hotness", then the one that goes public alone will have an average post-IPO productivity of \(A_{iH}\) if the industry-wide productivity shock takes place (Eq4 and Eq5) and an average post-IPO productivity of \(A_i\) if the productivity shock does not occur (Eq3 and Eq5). Thus the average post-IPO productivity for a stand-alone IPO firm based on the above updated posterior belief will be \(pA_{iH} + (1 - p)A_i\), where \(p\) is the ex ante belief that the industry-wide productivity shock will occur. Similarly, the average post-IPO productivity for an on-the-wave IPO firm based on updated posterior belief will be \(p(A_{iH} + A_{2H})/2 + (1 - p)(A_1 + A_2)/2\). This completes the proof for part (i) of the proposition.

Part (ii) of the proposition can be proved in a similar fashion. Conditional on the occurrence of the productivity shock, the post-IPO productivity for a firm that goes public earlier in a "sequential IPO wave" (firm 1 in the model) will be \(A_{1H}\) whereas the post-IPO productivity for a firm that goes public later in the wave will be \(A_{2H}\). Similarly, conditional on no occurrence of the productivity shock, the post-IPO productivity for a firm that goes public earlier in a "sequential IPO wave" (firm 1 in the model) will be \(A_1\) whereas the post-IPO productivity for a firm that goes public later in the wave will be \(A_2\). In both cases, the equilibrium that results in such a sequential wave is Eq3, and the post-IPO productivity for a firm that goes public earlier in a wave is higher than that of a firm that goes public later in that same wave. Therefore, if these two firms have the same level of
post-IPO capital stock and market share, the post-IPO operating performance for the early issuer will be better than that of the late one.

Q.E.D.

**Proposition 6 (Industry characteristics and IPO waves)**

(i) Industries whose productivity shocks have large magnitudes ($\Delta A > A_1 - A_2$) tend to have more IPOs and more IPO waves than industries whose shocks have smaller magnitudes ($\Delta A < A_1 - A_2$).

(ii) Industries with more innovation opportunities (large $p$) will have more IPOs and more IPO waves than industries with fewer or no innovations ($p \approx 0$).

(iii) IPO waves in industries whose higher-productivity firms have a smaller market share will last longer than IPO waves in industries whose higher-productivity firms also have a larger market share.

(iv) IPO waves in industries whose firms have larger gaps in productivity levels will last longer than IPO waves in industries whose firms have more homogeneous productivity levels.

**Proof:** Part (i) of the proposition compares industries with a small magnitude of productivity shocks ($\Delta A < A_1 - A_2$) and those with a large magnitude ($\Delta A > A_1 - A_2$). When $\Delta A$ is small so that $\pi_{2H} < \pi_1$, we have the situation as in Proposition 3 where the only equilibria (Eq4 and Eq5) involve at most one IPO (by firm 1) and no IPO waves can occur. However, when $\Delta A$ is sufficiently large so that $\pi_{2H} > \pi_1$, the same industry may witness more IPOs and IPO waves due to the going-public efforts by firm 2 such as those in Eq1, Eq2, and Eq3.

Even within an industry with either $\Delta A < A_1 - A_2$ or $\Delta A > A_1 - A_2$, a greater $\Delta A$ will contribute to stronger incentives to go public. For example, in an industry with $\Delta A < A_1 - A_2$, we may observe either Eq4 or Eq5. Eq5 yields stronger going-public incentives because in this equilibrium firm 1 will go public at time 1 before the realization of any productivity shocks. On the contrary, in Eq4 firm 1 will wait until time 2 to do its IPO when the industry-wide productivity shock actually occurs. Thus we may compare the likelihood of Eq4 and that of Eq5 to see the effect of $\Delta A$ on going-public concerns. As we show in the proof of Proposition 1, 2, and 3, the conditions for Eq5 to occur rather than Eq4 is:

$$(*3) > 0 \text{ or } \begin{align*}
(*2) > 0 \\
 m > \frac{2 + x - \sqrt{x^2 + 4}}{2x}
\end{align*} \quad \text{(A11)}$$

In the case of $0 < m < \frac{2 + x - \sqrt{x^2 + 4}}{2x}$, we have shown that $(*3) > 0$ is equivalent to

$$x_2 - x_4 = sm_b(1 - m_b)(p\pi_{1H}^{PO} + (1 - p)\pi_1) + (1 - m)sm(1 - p)\pi_1 - (1 - p)B > 0, \quad \text{(A12)}$$

the left hand side of which is clearly increasing in $\Delta A$ (because $\pi_{1H}^{PO}$ increases with $\Delta A$). In fact, as $\Delta A$ decreases to 0, $(*3) < 0$ for sure. Similarly, $(*2) > 0$ is equivalent to

$$y_3 - y_4 = (p\pi_{2H} + (1 - p)\pi_2)[(1 + p)(1 - m)sm + (1 - p)(1 - m_c)sm_c] - B > 0, \quad \text{(A13)}$$
whose left handside also increases with $\Delta A$ as $\pi_{2H}$ rises with $\Delta A$. As $\Delta A$ decreases to 0, $\pi_2 < 0$ for sure. Combining the above results, we can see that a larger $\Delta A$ tends to increase the likelihood that Eq5 occurs, even if it is smaller than $A_1 - A_2$. We have repeated the analysis in all other possible cases and shown that the conclusion is true for all of them. (For brevity the detailed proofs are not listed here.)

Part (ii) of Proposition 6 also follows from Proposition 1, 2, and 3. The conditions (4) and (5) ensure that without productivity shocks ($p \simeq 0$), firms will optimally choose to remain private. Hence, an industry with large enough probability of productivity shocks will tend to witness more IPOs and IPO waves. Similar to the proof for part (i), we can show that within an industry with either $\Delta A < \Delta A_L$ or $\Delta A > \Delta A_H$, a greater $p$ will always contribute to stronger incentives to go public. For example, in an industry with $\Delta A < A_1 - A_2$, we may observe either Eq4 or Eq5. Eq5 yields stronger going-public incentives thus we may compare the likelihood of Eq4 and that of Eq5 to see the effect of $p$ on going-public concerns. As we show in the proof of part (i) of Proposition 6, the condition for Eq5 to occur rather than Eq4 is $(??)$. Since $x_2 - x_4$ is linear in $p$, it’s easy to show that $x_2 - x_4$ increases with $p$ by comparing its value when $p$ is 0 and 1. When $p = 0$, $x_2 - x_4 < 0$ for sure and vice versa when $p = 1$. Hence, $p$ increases $x_2 - x_4$. Similarly, it is straightforward to show that $y_3 - y_4$ increases with $p$ as well. Combining the above results, we can see that a larger $p$ tends to increase the likelihood that Eq5 occurs, even if $\Delta A$ is smaller than $A_1 - A_2$. We can repeat the analysis in other possible cases when either $\Delta A < \Delta A_L$ or $\Delta A > \Delta A_H$. However, when $\Delta A$ is moderate, $p$’s effect on going-public incentives may be ambiguous (with intuition given in the main text). Hence, part (ii) of Proposition 6 only compares industries with very high $p$ and those with $p \simeq 0$. Then regardless of the magnitude of $\Delta A$, industries with higher $p$ will unambiguously have more IPOs and IPO waves.

Part (iii) of this proposition follows Proposition 1 and 2. As we can see in Figure 2 in the main text, among the three IPO-wave equilibria, the occurrence of the Eq1 is not affected by the market share distribution while Eq2 and Eq3 do. When the higher-productivity firm, firm 1, has small enough existing market share ($m < \frac{2 + \sqrt{2^2 + 4}}{2s}$), only Eq3 can occur, resulting in a longer wave period (both time 1 and time 2) than when firm 1’s market share is larger than $\frac{2 + \sqrt{2^2 + 4}}{2s}$, in which case only Eq2, whose wave takes place at time 2, may occur.

Part (iv) follows Proposition 1 and 2 as well. When the existing productivity level of firm 1 is high while that of firm 2 is low, Eq3 is more likely to occur than when both firms’ productivity levels are similar (either both high as in Eq1, or both low as in Eq2). In Eq3, the IPO wave will span the whole period of time 1 and 2, while in the latter two cases any IPO waves will only take place simultaneously, either at time 1 or 2.

Q.E.D.