Corporate Control Contests and the Disciplining Effect of Spin-offs: A Theory of Performance and Value Improvements in Spin-offs

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Current Version: January 2002

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For helpful comments or discussions, we thank Richard Arnott, Sris Chatterjee, Chinmoy Ghosh, Gautam Goswami, Jayant Kale, George Kanatas, Frank Lichtenberg, David Nachman, Tom Noe, Jun Qian, Steve Raymar, Fabio Schiantarelli, Susan Shu, Venkat Subramaniam, Bob Taggart, Hassan Tehranian, as well as participants at the August 2001 EFA meetings, and seminars at Boston College, Boston University, Brandeis University, University of Connecticut, Fordham University, Georgia State University, McGill University, and Tulane University. We alone are responsible for any errors or omissions.
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Abstract

We develop a new rationale for the performance and value improvements of firms following corporate spin-offs. We consider a situation of a firm with multiple divisions, where incumbent management may have differing abilities for managing various divisions. Giving up control to a rival with better ability in managing the firm, while it benefits equity holders (including incumbent management) by increasing the firm’s equity market value, also involves losing the incumbent’s benefits from control. Due to this trade-off, the incumbent, while willing to relinquish control to extremely high ability rivals, may not wish to do so for rivals who have only moderately higher management ability relative to him. Spin-offs increase the chance of loss of control to potential rivals in two ways: First, it reduces the ability of the incumbent to use firm size strategically against the rival in a control contest (after the spin-off, the rival can invest to the full extent of his wealth in the equity of the firm more vulnerable to a takeover). Second, it increases the probability that passive investors will vote with the rival in a contest for control for at least one division (in a joint firm, the superior management ability of any rival with respect to one division may be neutralized by inferior ability with respect to another one). This increased chance of loss of control following a spin-off, in turn, motivates the incumbent to work harder (despite his disutility for effort) in equilibrium in an attempt to maintain control. Thus, the increase in equity market value of the firm upon spin-off announcements arises not only from market participants incorporating in their valuations the increased probability of a takeover by a more able rival for control, but also from their anticipating the increase in managerial efficiency arising from the disciplining effect of the spin-off on firm management. Our analysis predicts that, in addition to positive announcement effects, the equity of a sample of spun-off firms will also exhibit long-term positive abnormal returns under certain conditions. Our model also explains a wide variety of other recently documented empirical regularities, and provides hypotheses for further empirical work.
Corporate Control Contests and the Disciplining Effect of Spin-offs: A Theory of Performance and Value Improvements in Spin-offs

1 Introduction

In recent years, the number of corporate spin-offs has accelerated. While the motivation often given for such spin-offs is corporate “focussing” (or “re-focussing”), little is known about the precise source of any benefits from such corporate restructuring. The empirical literature has repeatedly documented that parent company stockholders gain and bondholders are unaffected by spin-off announcements (see, e.g., Hite and Owers (1983), Miles and Rosenfeld (1983), or Schipper and Smith (1983)). However, the precise source of such value gains is still a matter of considerable debate.

Recent empirical evidence, however, has gone beyond documenting the positive announcement effects of spin-offs on stock price. Cusatis, Miles, and Woolridge (1993) document that, in addition to the positive abnormal stock returns documented for parent firms on the announcement date, both spin-offs and their parents experience significantly positive abnormal returns for up to three years beyond the spin-offs announcement date. They further document that both spin-offs and their parents experience significantly more takeovers than do control groups of similar firms. Finally, they document that spin-offs-parent combinations not reporting takeover activity within three years do not have positive long-term abnormal returns.

This paper develops a new rationale for the performance and value improvements arising from spin-offs which is consistent with this recent (as well as earlier) empirical evidence. We develop a theoretical analysis which demonstrates how spin-offs can increase the probability of a takeover by the “right kind” of (value-improving) management team. We show how such spin-offs can enhance the level of firm performance even in the absence of such a value-improving takeover by serving to discipline firm management. Finally, our analysis demonstrates that while a spin-off will lead to positive abnormal stock-price returns on the announcement day, it will also lead to abnormal stock price performance (on average) in the period following
the spin-off for certain categories of firms as well.

We study a setting where, while management would like to increase equity value, incumbent firm management also derives private benefits from control. The firm has two divisions; current management may have the same or differing abilities for managing these two divisions. Giving up control to a rival management team, while it may benefit equity holders (including the current management) by increasing firm value, is costly to the incumbent in that it involves loss of control (and hence the private benefits the incumbent derives from control). A spin-off increases the chance of loss of control to a potential rival in two ways. First, it reduces the ability of the incumbent to use firm size strategically against the rival in a control contest (the rival can invest to the full extent of his wealth in the more vulnerable firm subsequent to a spin-off). Second, it increases the probability that passive investors will vote with the rival in a contest for control for at least one division (in a joint firm, the superior management ability of any rival with respect to one division may be neutralized by his inferior ability with respect to the other one). This increased chance of loss of control of at least one division following a spin-off, in turn, motivates the incumbent to work harder in running the firm in equilibrium (even when doing so involves a personal cost: we assume that incumbent management has disutility for effort), in an attempt to minimize the probability of his losing control in equilibrium.

Other rationales for the positive announcement effects of spin-offs have been proposed in the literature. Aron (1991) argue that spin-offs benefit the firm since, after the spin-off, the equity values of the securities traded provide a much “cleaner” signal of managerial productivity than when the two divisions were part of a combined firm. The argument is that this enables the firm to provide better incentives for firm management based on the stock price of the individual firms. Habib, Johnsen and Naik (1997) argue that spin-offs improve the quality of the information managers and uninformed investors can infer from the prices of the firm’s traded securities, therefore leading to an increase in the expected price of the firm’s equity. Nanda and Narayanan (1999) suggest that the firm may be undervalued if the market cannot observe the cash flows of each individual division in that firm. Therefore the firm that needs external financing may resort

\[1\] We will analyze the first effect of spin-offs in detail in the basic model, and the second effect of spin-offs only in section 6.1 where we extend the basic model to examine the difference between related and unrelated spin-offs. Even though it is possible to introduce these two effects together in the basic model, we choose not to do so to minimize modelling complexity and to simplify exposition.
to divestures such as spin-offs so that it can raise capital at a fair market price after the divesture.\textsuperscript{2} \textsuperscript{3}

These information based rationales, while complementary to ours, clearly do not incorporate the important role that corporate control contests seem to play in the efficiency improvements from spin-offs. Neither are they able to convincingly explain the longer-term performance and value gains from spin-offs, nor the wide variety of other regularities that have been recently documented in the empirical literature.

There is a large empirical literature on spin-offs. A lot of studies have documented a positive equity market reaction to spin-off announcements.\textsuperscript{4} Two recent papers, Desai and Jain (1999) and Daley, Mehrotra and Sivakumar (1997), document that both the market reaction to spin-off announcements and the long-term abnormal returns and operating performance are significantly greater in unrelated spin-offs (where the spun-off subsidiary operates in an industry unrelated to the parent firm) than in related spin-offs.\textsuperscript{5} Other studies document that the magnitude of the market reaction to spin-off announcements is increasing in the size of the spun-off division as a fraction of the combined firm existing prior to the spin-off. In contrast to the existing theoretical literature, our model is able to explain much of this evidence, while providing many other hypotheses, as yet untested, for further empirical work.

The rest of the paper is organized as follows. Section 2 describes the basic model. Section 3 to 5 characterize the equilibria in the basic model and develop results. Section 6 studies three different extensions of our basic model: In section 6.1, we allow the incumbent’s management ability to differ across divisions, thus analyzing the differences between related and unrelated spin-offs. In section 6.2, we study the case where

\textsuperscript{2} This argument requires the somewhat strong assumption that equivalent incentive contracts cannot be written based on the profitability of the individual divisions when they are part of a combined firm.

\textsuperscript{3} Chemmanur and John (1996) study the optimal financial and corporate structure chosen by an incumbent with access to multiple projects when the incumbent always wishes to maintain control; see John (1993) for another model of spin-offs. The literature on internal capital markets (see, e.g., Rajan, Servaes, and Zingales (2000) and Scharfstein and Stein (2000)) is also indirectly related to our paper.

\textsuperscript{4} See, e.g., Hite and Owers (1983), Miles and Rosenfeld (1983), Schipper and Smith (1983), and Allen et al (1985). Krishnaswamy and Subramaniam (1999) test the hypothesis that such positive market reactions to spin-offs are due to a reduction in the information asymmetry existing in the market for the equity of the parent firm.

\textsuperscript{5} These empirical papers attribute this difference in value creation between related and unrelated spin-offs to the increase in “corporate focus” that accompanies unrelated spin-offs. However, “increase in corporate focus” is more a description of what happens as a result of an unrelated spin-off (namely, the firm’s business becomes more focused, since a subsidiary operating in an unrelated industry is spun off) rather than a coherent theory explaining where such value increments come from, see, e.g., Comment and Jarrell (1995). In contrast, our paper provides such a coherent theory; our model analyzes how such performance and value increments arise following spin-offs, and explains why the value increments following unrelated spin-offs are greater than those following related spin-offs.
the parent firm has available to it an executive capable of managing the spun-off firm (so that incumbent has the option to relinquish control of the spun-off firm even before a control contest takes place). In section 6.3, we allow for asymmetric information between the board and the incumbent regarding the incumbent’s capacity to resist a takeover attempt (in addition to the asymmetric information between the incumbent and the outside investors assumed in the basic model). Section 7 describes the testable implications of the model. Section 8 concludes. All proofs are confined to the appendix.

2 The Model

The model has four dates \((t = 0, 1, 2, 3)\) and four kinds of decision-making agents: the board, the incumbent, the rival, and the atomistic passive investors, all of whom are risk-neutral. Consider a firm with two divisions (division 1 and 2 hereafter) initially set up by an entrepreneur (the incumbent management, \(I\), hereafter) as an all-equity firm. The incumbent invests all his wealth, \(W_I\), in the equity of the firm, with the remaining equity held by the passive investors.

At time 0, the firm’s board decides whether or not to spin off one of the divisions (also referred to as the “subsidiary” hereafter). The incumbent comes to know this decision privately from the board, based on which he may choose to re-allocate his wealth between the equity of the two firms created by the spin-off; he may also choose to adjust the capital structure of the firm (these decisions are made strategically by the incumbent; we will discuss these in more detail below). At \(t = 1\), the decision to spin off (or not) is made public. Along with this, the spin-off plan is also announced in case of spin-off. The spin-off plan specifies the incumbent’s equity holdings in the two new firms and the capital structures of both firms. At \(t = 2\), a rival appears (with a certain probability) or may not appear. The rival may have different ability levels, wealth levels, etc., which become publicly known at this date right after his arrival. The rival invests strategically in the equity of one of or both the firms created in the spin-off (or in the equity of the joint firm if there

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\(^6\) Wruck and Wruck (2001) document the following regarding the top management of the spun-off firms subsequent to a spin-off. In a substantial fraction of their spin-off sample (33.7%), the top management (defined as the Chairman or the CEO/President or both) of the parent firm continues to be part of the top management of the spun-off firm (in 98.2% of these cases, both firms continued to have the same Chairman). This is consistent with the assumption we make in the basic model. They also document that, in another large fraction of their spin-off sample (56.4%), a division or lower level executive of the parent firm assumes the top management position of the spun-off firm. We allow for this possibility in section 6.2.
is no spin-off), in an attempt to take over control of firm(s) from the incumbent. At $t = 3$, the control contest takes place, with the incumbent, the rival, and the passive investors voting in the proxy contest. The outcome of this control contest becomes public immediately thereafter. The sequence of the events described above is depicted in figure 1.

2.1 The Incumbent

We assume that the incumbent, who initially runs both divisions of the firm, obtains both security benefits and private benefits of control from managing the firm(s) under his control. The security benefits arise from the cash flows accruing to the firm’s equity held by the incumbent, and are captured by the market value of this equity. Clearly, such security benefits accrue to all equity holders. In contrast, the control benefits, which are non-contractible, accrue only to the management team in control, and are not reflected in the market value of any of the securities issued by the firm.

We use $F_i^j$, $i \in \{1, 2, q\}$ to denote the expected value of the security benefits accruing to the incumbent (I) from firm 1 or firm 2 (in the case of a spin-off) or from the joint firm, $q$ (when there is no spin-off). Further, we assume that the expected value of the control benefits from a firm is decreasing in the face value of the debt against that firm. This is the consequence of the increase in the probability of bankruptcy, the reduction in managerial discretion due to the restrictiveness of additional debt covenants, the increase in the intensity of monitoring, and the decrease in the amount of free cash flow, which accompany a higher level of debt. We use $P_i^j(D_i)$, $i \in \{1, 2, q\}$ to denote the expected value of the control benefits accruing to the management from a firm which is supporting a promised payment $D_i$ of debt against the firm’s cash flows, where $P_i^j(0) = P_i^j$, $\frac{\partial P_i^j(D_i)}{\partial D_i} < 0$ and $\frac{\partial^2 P_i^j(D_i)}{\partial D_i^2} < 0$. Also $P_i^q = P_i^1 + P_i^2$.

The incumbent can exert two possible effort levels ($e_i$) in firm $i$: normal ($n_i$) or diligent ($d_i$). If he works diligently, the incumbent can increase his managerial efficiency, and thus the market value of the firm, which we will discuss about in detail later. However, the incumbent incurs a personal cost $C(e_i)$ for his effort in working. $C(e_i)$ is increasing in the level of managerial effort. For simplicity, we assume $C(e_i) = 0$

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7 This assumption has now become standard in the corporate control literature. See, for instance, Grossman and Hart (1982) or Harris and Raviv (1988).
when $e_i = n_i$, where $i \in \{1, 2\}$; $C(e_i) = c$ when $e_i = d_i$; and $C(e_q) = C(e_1) + C(e_2)$. We assume that the incumbent’s effort level is not observable to the outsiders and is thus non-contractible.

In summary, the objective of the incumbent in making various choices after he learns about the board’s spin-off decision is to maximize the expected value of the sum of his long-term ($t = 3$) security and control benefits, net of any effort costs incurred by him. This is given by:

$$\max_{x_i, e_i, D_i} \sum_i (F_i(x_i, e_i, D_i) + P_i^d(D_i) - C(e_i)),$$

(1)

where the summation in (1) includes all firms $i$ controlled by the incumbent, and $x_i$ refers to the fraction of the incumbent’s wealth invested in the equity of firm $i$. The detailed version of the incumbent’s problem will be introduced in section 3.2.

### 2.2 The Rival

We assume that the rival may belong to one of two possible types with different ability levels ($k$) in managing firm $i$: low ($l$) or high ($h$). The incumbent and the board are uncertain about the existence of the rival and also do not observe the ability level of the potential rival before the rival appears. However, they observe a prior probability distribution about the rival: they believe with probability $\phi_1$, no rival appears; with probability $\phi_2$, a high ability rival appears; and with probability $\phi_3$, a low ability rival appears, where $\phi_1 + \phi_2 + \phi_3 = 1$.

The rival’s objective in investing his wealth $W_R$ in the equity of the firm(s) is to maximize the sum of his own security benefits and control benefits. $W_R = f W_I$, where $f$ is the wealth ratio between the rival and the incumbent, which is publicly known. Since the investors get to know all the features of the rival immediately after his arrival, the rival has to pay a fair price for the equity he buys from the passive investors. This fair price, which is based on the expected outcome in equilibrium, depends on the wealth of the rival.
the rival, the ability of the rival, and the outcome of the control contest.

In summary, the rival’s objective in deciding on the allocation of his own wealth between the equity of the firm(s) is given by:

\[ \text{Max} \sum P^j_R, \]

where \( P^j_R \) represents the expected value of the rival’s control benefits in firm \( j \), and the summation above is taken over all firms \( j \) that the rival succeeds in taking over from the incumbent in equilibrium.\(^{10}\)

### 2.3 Firm Valuation and Capital Structure

Initially, the firm is set up as an all equity firm. Thus if no new debt is issued, the equity value equals the market value of the firm. In the case of spin-off, the equity value of firm \( i \) under the incumbent’s control, \( V^i_I, i \in \{1, 2\}, \) is \( n_i v_i \) when the incumbent works normally, and is \( d_i v_i \) when the incumbent works diligently.

On the other hand, in the case of no spin-off, the equity value of the joint firm \( V^q_I = V^1_I + V^2_I + S \) under the incumbent’s management, where \( S \) represents the operating synergy arising from the conglomeration of two divisions. Note that similar to \( V^1_I, V^q_I \) is also a function of the incumbent’s effort.

In addition, when firm \( i \) after spin-off is taken over and managed by a rival, its equity value becomes \( V^i_R = lv_i \) if the rival is of low ability, and \( V^i_R = hv_i \) if the rival is of high ability. The equity value of the joint firm taken over and managed by the rival is \( V^q_R = V^1_R + V^2_R + S \).

We assume that when the incumbent is diligent, he is a better manager than the low ability rival, but worse than the high ability rival. We also assume that when the incumbent works normally, he is worse than either type of rival. Thus, \( h > d_i > l > n_i \).

If debt is issued against either firm as part of the spin-off plan or against the joint firm (in the absence of a spin-off), we denote by \( D_i \) the face value of debt with claim to the cash flows of the \( i^{th} \) corporation,\(^{10}\)

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\(^{10}\) The assumption that the rival obtains some private benefits from control does not play any important role in generating our results; all we require is that the rival has some desire (for whatever reason) to wrest control of the firm from the incumbent. In our model, all securities are priced in equilibrium, and the equilibrium price of equity incorporates the equilibrium inferences of all agents about the outcome of the control contest (if a rival appears). Given that all agents have symmetric information at \( t = 2 \) about the features of the rival and the incumbent, the price the rival has to pay for equity at \( t = 2 \) would already reflect the potential increase in firm value under his management, thus rendering him powerless to extract any of the security benefits associated with his taking over control (the argument here is similar in spirit to that underlying the free-rider problem studied by Grossman and Hart(1980)). We have therefore assumed that, like the incumbent, the rival also obtains some private benefits from control, in order to model the rival’s motivation to attempt a takeover in the simplest possible manner.
\(i \in \{1, 2, q\}\). Thus when debt is issued, the equity values become: 
\(V_i^f(e_i = n_i, D_i) = n_i v_i - D_i; V_i^f(e_i = d_i, D_i) = d_i v_i - D_i; V_i^h(k = h, D_i) = h v_i - D_i; \) and 
\(V_i^h(k = l, D_i) = l v_i - D_i.\)

In our model, debt may be issued by the incumbent as part of the strategy to maintain control of one of or both firms. In particular, using debt in any firm allows the incumbent to control a larger fraction of the firm’s equity, thus improves his position relative to potential rivals in a control contest. We assume that debt is introduced into the capital structure with an exchange offer (i.e., any amount raised by issuing debt is used to buy back equity).\(^{11}\) Thus, in our setting, the change in capital structure has no effect on the firm’s investment policy. Neither has it any direct effect on firm value, though it can have an indirect effect by affecting the probability of a takeover, with the resulting effects on firm value (either by affecting the identity of the management team in control, or through affecting the effort level of the incumbent).\(^ {12}\)

Clearly, the incumbent’s security benefits \(F_i^f\) are affected directly by the equity value of the firm(s) in which he has invested his wealth and the allocation of his wealth between the two firms (in a spin-off), \(x_i\). Note that the equity value of the firm is directly affected by the incumbent’s effort level, \(e_i\), and the capital structure of the firm, \(D_i\); it is also indirectly affected by \(x_i\) and \(D_i\) (through the impact on the identity of the management team, either incumbent or rival, in control). For example, suppose that the incumbent owns 15% of the total equity in firm \(i\). Then his security benefit \((F_i^f)\) is \(0.15V_i^f(e_i = n_i, D_i)\) if he retains control of firm \(i\) and works normally; it is \(0.15V_i^f(e_i = d_i, D_i)\) if he works diligently. On the other hand, if firm \(i\) he invests in is controlled by a low ability rival, the incumbent’s security benefit is \(0.15V_i^h(k = l, D_i)\); and it is \(0.15V_i^h(k = h, D_i)\) if firm \(i\) is controlled by a high ability rival. The incumbent’s percentage ownership comes from his investment of wealth \(W_i\) in firm \(i\). For instance, if the incumbent works normally, then in the case of a joint firm, his equity ownership is \(\frac{W_i}{n_1 v_1 + n_2 v_2 + S - D_1}\); and in the spin-off case, his equity ownership is \(\frac{x_1 W_i}{n_1 v_1 - D_1}\) and \(\frac{x_2 W_i}{n_2 v_2 - D_2}\) respectively in firm 1 and firm 2, where \(x_1\) and \(x_2\) represent the optimal split of his total investment between the equities of the two firms specified in the spin-off plan (i.e., a fraction \(x_1\) of his

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\(^{11}\) We assume an exchange offer rather than a new issue of debt to prevent us from getting into issues related to raising new capital, which is clearly not the focus of this paper.

\(^{12}\) We assume that the incumbent can credibly commit to implementing the capital structure specified in the spin-off plan. Given that this is the case, the exact time at which this exchange offer is implemented is irrelevant to our results (as long as this takes place prior to the arrival of the rival at \(t=2\)). However, to be specific, we assume that the exchange offer is implemented between \(t=1\) and \(t=2\), right after the spin-off announcement.
total wealth will be invested in the equity of firm 1, and the remaining fraction \( x_2 = 1 - x_1 \) invested in that of firm 2).

2.4 The Board

The board’s objective in choosing whether to spin off or not is to maximize the combined long-term (i.e., \( t = 3 \)) equity value of the firm(s).\(^{13}\) We assume that asymmetric information exists between the board and the equity market about the total amount of the incumbent’s wealth \( (W_I) \).\(^{14}\) More specifically, while the board (as well as the incumbent) knows the exact amount of \( W_I \) before the announcement of a spin-off, the market observes only the prior probability distribution over the incumbent’s wealth: at this date, with probability \( \gamma \), the market believes that the wealth is large and equals to \( W_H \); with probability \( 1 - \gamma \), it believes that the wealth is small and equal to \( W_L \).\(^{15}\)\(^{16}\) We assume that the actual amount of the incumbent’s wealth becomes publicly known at \( t = 2 \) prior to the control contest.

2.5 Passive Investors and the Control Contest

In this section, we will characterize the voting behavior of the passive investors in a control contest. We assume that in a control contest for the \( i^{th} \) firm where \( i \in \{1, 2, q\} \), the fraction of the passive investors who

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\(^{13}\) We recognize that, in many cases, the board’s decision to spin off or not may be partially influenced by the CEO and the other members of the incumbent management team. This means that the board’s action may not be solely driven by considerations of equity value maximization. Our results go through even in this case: all we require here is that the board places more weight on equity value maximization than the incumbent. Consistent with this, many spin-off decisions are made at the urging of independent board members, institutional investors or other large shareholders.

\(^{14}\) Note that it is not crucial that the asymmetric information between the board and the market be about the incumbent’s wealth. The key assumption we require is that the board has better information relative to the equity market regarding some variables which affect either the incumbent’s incentive to resist takeover by a rival or his ability to prevail in a control contest against such a rival. Two examples of variables which affect the incumbent’s incentive to resist a takeover are (1) the extent of the incumbent’s private benefits from maintaining control of one or both firms following a spin-off, and (2) his cost of working diligently. In contrast, the wealth level of the incumbent is a variable which affects the ability of the incumbent to prevail in the control contest. While, in practice, the board may have better information than the market regarding some, or all, of these variables, we have chosen to assume asymmetric information about the incumbent’s wealth alone to maintain analytical simplicity.

\(^{15}\) We will introduce asymmetric information between the board and the incumbent as well in section 6.3.

\(^{16}\) We use this assumption to capture the notion that the equity market usually has less information compared to the board regarding the ability of the incumbent to prevail in a control contest. For example, a considerable number of the passive investors may be totally committed to voting for the incumbent regardless of his management ability. Thus, the incumbent’s wealth \( W_I \) in our model can be thought of as including not only the personal wealth of the incumbent, but also the collective wealth of such passive investors who are committed to voting for the incumbent (e.g., personal friends of the members of the current management team, and those tied to the members of the team through their financial or business interests outside the firm). Therefore, even when the direct personal wealth of the incumbent is known to the market, the board may have private information about the extent of equity held by the incumbent’s committed supporters, resulting in the asymmetric information about \( W_I \) between the board and the equity market.
vote for the incumbent is a function of the relative managerial effectiveness of the incumbent and the rival in running that firm. We denote this fraction by \( \pi_i = \pi_i(e_i, k) \). In particular, we assume the following about the passive investors’ voting behavior:

(i) A majority of the passive investors vote for the better manager in any control contest. Given our earlier assumption that \( h > d > l > n \), this implies that the incumbent will be unable to obtain a majority of the passive investors’ votes against a high ability rival, regardless of how hard he works; and that against a low ability rival, the incumbent can obtain a majority of votes if he works diligently, but not if he works normally. Therefore, we have \( \pi_i(e_i = n, k = h) < \pi_i(n, l) < \pi_i(d, h) < 0.5 < \pi_i(d, l) \).

(ii) If the ratio between the incumbent’s managerial effectiveness and that of the rival is the same for any two firms, then the fraction of the votes obtained by the incumbent will also be the same in these firms. Thus, for a given rival type, \( \pi_1 = \pi_2 = \pi_q \) if \( e_1 = e_2 = e_q \).

### 2.6 Parametric Restrictions

We assume:

\[ v_1 > v_2, \quad (3) \]

\[ n_1 = n_2 = 1, \quad d_1 = d_2 = d, \quad (4) \]

\[ c > (d - 1)W_I, \quad (5) \]

\[ P_I^1 > Max[c + (h - 1)v_1, P_I^2 + (h - 1)(v_1 - v_2)], \quad (6) \]

\[ \left| \frac{\partial P_I^i}{\partial D_i} \right| > (d - 1)\frac{W_I}{v_i}, \quad (7) \]

\[ W_H > \frac{1}{2}(v_1 + v_2) \text{ and } W_L < \frac{1}{2}(v_1 + v_2). \quad (8) \]

Assumption (3) implies that firm 1 is larger than firm 2 after spin-off (without loss of generality). Assumption (4) implies that the incumbent has the same ability in managing the two divisions. This assumption is for simplicity of exposition and will be relaxed in section 6.1. Assumption (5) implies that the incremental cost of effort to the incumbent if he works diligently is greater than the incremental security benefits he can derive from working diligently, and thus implies that the incumbent will not work diligently merely to
pursue the security benefits. Assumption (6) implies that, due to the substantially large control benefits from firm 1, the incumbent would never want to relinquish control of firm 1 unless all the means to defend it are exhausted. Assumption (7) implies that the incumbent will not adjust the capital structure merely to pursue the security benefits. Finally, assumption (8) implies that, when the incumbent’s wealth is large, he can maintain control of both firms by relying only on the votes of the shares of equity in his personal account. It also implies that, when his wealth is small, the incumbent will not be able to ensure control in the above manner, so that he has to rely on the passive investors’ votes to maintain control. This, in turn, implies that he may be induced to work diligently, or to strategically adjust the capital structure of the two firms currently managed by him, in order to maximize the votes he can obtain from the passive investors in the control contest.

3 Equilibrium

In this section, we will first analyze the model backwards, starting from the rival’s problem, followed by the incumbent’s problem, and finally the board’s problem. We will then characterize the different equilibria that prevail in the equity market as various model parameters are changed.

3.1 The Rival’s Problem

At time 2, given the spin-off, the rival determines whether or not to acquire one firm, and if he decides to do so, which firm to acquire. Note that since the rival management always pays the fair share price in the equity market, his security transactions are zero net present value transactions. Therefore acquiring a firm is always beneficial for the rival since it allows him to generate private benefits from controlling that firm without incurring any additional costs.

Denote by $a_k^i(e_i, x_i, D_i)$ the outcome of the control contest regarding firm $i$ when the rival is of ability level $k$ ($k = h$ stands for the high ability rival and $k = l$ for the low ability rival), given the incumbent’s effort level in firm $i$, $e_i$, his equity position, $x_i$, and the firm’s capital structure, $D_i$. Note that $a_k^i$ implicitly is a function of the votes of the incumbent $(b_k^i)$ at $t = 3$ as well. The incumbent votes for himself in firm 2
\[ \sum_{i=1}^{2} \left[ x_i \frac{W_I}{v_i - D_i} V_i^1 + P_i^e(D_i) \right] \geq x_1 \frac{W_I}{v_1 - D_1} V_1^1 + P_1^e(D_1) + x_2 \frac{W_I}{v_2 - D_2} V_2^1. \]  \hspace{1cm} (9)

Otherwise he votes for the type \( k \) rival \( (b_2^k = 1) \).\(^{17}\) Here in inequality (9), the left hand side represents the sum of the private benefits and security benefits when the incumbent controls both firms; and the right hand side represents the sum when he controls only firm 1 after the control contest.

We set \( a_i^k = 1 \) \((i = 1, 2)\) if the type \( k \) rival succeeds in acquiring firm \( i \), and \( a_i^k = 0 \) if he fails (consequently the incumbent maintains control). In particular, \( a_i^k = 1 \) when \( b_i^k = 1 \) or:

\[
\frac{W_R}{V_i^R} + (1 - \pi_i)(1 - \frac{x_i W_I}{v_i - D_i} - \frac{W_R}{V_i^R}) \geq \frac{1}{2},
\]  \hspace{1cm} (10)

The first term on the left hand side of inequality (10) represents the fraction of equity that may be purchased by the rival from passive investors under the fair market price, and the second term represents the fraction of the votes that the rival management can win from the passive investors in a control contest which depends on his ability level. The rival needs at least 50% of the votes in the control contest to take over control of firm \( i \) from the incumbent management. Similarly, \( a_i^k = 0 \) when \( \frac{x_i W_I}{v_i - D_i} + \pi_i(1 - \frac{x_i W_I}{v_i - D_i} - \frac{W_R}{V_i^R}) \geq \frac{1}{2} \) and \( b_i^k = 0 \).

On the other hand, in the case of no spin-off, the rival can take over control of the joint firm \((i.e., a_q^k = 1)\) if and only if:

\[
\frac{W_R}{V_R^q} + (1 - \pi_i)(1 - \frac{W_I}{v_1 + v_2 + S - D_q} - \frac{W_R}{V_R^q}) \geq \frac{1}{2},
\]  \hspace{1cm} (11)

### 3.2 The Incumbent’s Problem

Between time 0 and time 1, the incumbent maximizes the sum of his expected long-term security benefits and private benefits net of the possible cost associated with his diligent effort, given his belief about the outcome of the control contest \( a_i^k \) that will take place at time 3.

In the case of spin-off, the incumbent management has three choices to make: (1) optimal investment allocation of his wealth between two firms, \( x_1 \) and \( x_2 \); (2) optimal debt level, \( D_1 \) and \( D_2 \), to be issued and allocated to each firm; and (3) his effort level, \( e_1 \) and \( e_2 \), contributing to each firm.

\(^{17}\) Note that according to assumption (3), the incumbent always votes for himself in a control contest for firm 1, i.e., \( b_1^k = 0 \).
Thus, the incumbent’s problem can be characterized as

\[
\begin{align*}
\max_{e_i, D_i, x_i} & \quad J(e_i, x_i, D_i; a^k_i, i = 1, 2, k = h, l) = \\
\sum_{i=1}^{2} & \quad [F_i^j(e_i, x_i, D_i; a^k_i(e_i, D_i, x_i, f, \pi_i)) + P_i^j(D_i; a^k_i(e_i, D_i, x_i, f, \pi_i)) - C(e_i)]
\end{align*}
\]

(12)

where

\[
\begin{align*}
P_i^j(D_i; a^k_i = 1) &= 0, \quad P_i^j(D_i; a^k_i = 0) = P_i^j(D_i); \\
C(e_i = n) &= 0, \quad C(e_i = d) = c; \quad \text{and} \\
F_i^j(e_i = n, x_i, D_i; a^k_i = 0) &= \frac{x_i W_i}{\nu - D_i} V_i^n(e_i = n, D_i), \\
F_i^j(e_i = d, x_i, D_i; a^k_i = 0) &= \frac{x_i W_i}{\nu - D_i} V_i^d(e_i = d, D_i), \\
F_i^j(e_i, x_i, D_i; a^k_i = 1) &= \frac{x_i W_i}{\nu - D_i} V_R^i(k, D_i),
\end{align*}
\]

(13)

\(i \in \{1, 2\}\) and \(k \in \{h, l\}\).

Wealth Allocation

Changing his wealth allocation does not involve any cost to the incumbent. In contrast, the incumbent would suffer a decline in his control benefits if he issues debt \(\left(\frac{\partial P_i^j}{\partial D_i} < 0\right)\) and a cost of effort \(C(e_i)\) when he works diligently. Therefore, he prefers wealth allocation to the other two choice variables when he has to fight a take-over.

However, wealth allocation alone may not be sufficient for the incumbent to deter the takeover and simultaneously win control of both firms when

\[
x_1(\pi_1, V_{i1}^j) + x_2(\pi_2, V_{i2}^j) > 1,
\]

(16)

where \(x_i(\pi_i, V_i^j) = \frac{\nu - D_i}{W_i} \frac{1 - \pi_i}{1 - \pi_i} + f \frac{\nu - D_i}{V_i^j} \frac{1 - \pi_i}{1 - \pi_i}, \quad i \in \{1, 2\}\). Here \(x_i(\pi_i, V_i^j)\) represents the portion of the incumbent’s wealth which can guarantee the incumbent 50% of the votes of firm \(i\) in a control contest against the type \(k\) rival.

To facilitate the exposition in the following, we define a function \(f_W(e_i, D_i; \pi_i, i = 1, 2)\) at which the inequality (16) holds as an equality. Thus, \(f_W(e_i, D_i; \pi_i, i = 1, 2)\) measures the maximum wealth of the type \(k\) rival that the incumbent can defend against by using wealth allocation alone. Therefore \(f \leq f_W(\cdot)\) implies that the wealth allocation alone can ensure the incumbent’s control of both firms after a spin-off.
Capital Structure Choice

When \( f \) is somewhat greater than \( f_W(\cdot) \), the incumbent has to either issue more debt or work harder to succeed in the control contest. Since the marginal cost of effort in this case is greater than the cost of issuing debt, the incumbent’s first choice is to adjust the capital structure by issuing debt and buying back an equivalent amount of shares. By doing so, he will incur a loss due to the decline of his control benefits, but can reduce the number of the passive investors voting against him in the control contest.\(^{18}\) Thus, given the effort levels, the incumbent solves the following problem when \( f > f_W(\cdot) \):

\[
\max_{D_i, x_i} J = \sum_{i=1}^{2} \left[ F_i(e_i, x_i, D_i; a^k_i, k = h, l) + P_i(D_i; a^k_i, k = h, l) - C(e_i) \right] \tag{17}
\]

In particular, if the incumbent wants to guarantee control of both firms \((a^h_i = 0, i = 1, 2)\), the above problem (17) can be specified in detail as:

\[
\max_{D_i, x_i} J = \sum_{i=1}^{2} \left[ \frac{x_i W_i}{v_i - D_i} V_i + P_i(D_i) \right] \\
\text{s.t.} \quad \frac{x_i W_i}{v_i - D_i} + \pi_i (1 - \frac{x_i W_i}{v_i - D_i} - \frac{W_p}{V_i}) \geq \frac{1}{2}, \quad i \in \{1, 2\} \tag{18}
\]

where the above inequality constraints imply that the incumbent can maintain control of both firms in the control contest. At the optimal debt level, the weak inequalities in (18) will hold as equalities since debt is costly to the incumbent and the incumbent will only issue a minimum amount of debt required to guarantee control. Therefore the constraints in (18) can be reorganized as:

\[
\left( \frac{1}{2} - \pi_1 \right) \left( \frac{v_1 - D_1}{W_i} \right) + f(v_1 - D_1) \left( \frac{\pi_1}{1 - \pi_1} \right) + \left( \frac{1}{2} - \pi_2 \right) \left( \frac{v_2 - D_2}{W_i} \right) + f(v_2 - D_2) \left( \frac{\pi_2}{1 - \pi_2} \right) = 1. \tag{19}
\]

Equation (19) implies that the incumbent issues debt against only those firms where the rival has a comparative advantage in management skill compared to the incumbent, i.e., \( \pi_i < \frac{1}{2} \).\(^{19}\) He does so since, when the incumbent’s relative management ability is poorer, reducing the size of the firm can reduce the amount of the votes from the passive investors voting against him in the control contest.

\(^{18}\) Two other effects arise as well when debt is issued: the incumbent increases his fraction of ownership and so does the rival. However, these two effects offset each other to some degree.

\(^{19}\) It is easy to show that the partial derivative of the left hand side of equation (19) with respect to \( D_i \) is positive when \( e_i = d \ (\pi_i > \frac{1}{2}) \) and negative when \( e_i = n \ (\pi_i < \frac{1}{2}) \).
**Effort Choice**

The incumbent will not issue an unlimited amount of debt. He will stop borrowing when the total cost of debt (i.e., decline in his control benefits) exceeds the cost of effort he would suffer from working diligently. In this case, the incumbent will choose to work diligently rather than issue more debt to ensure his control of the firm(s) currently managed by him. Denote:

\[ f_D(e_i; a^k_i, a^k_j) = \arg \max_f \{ \max_{D_1, D_2, x_1, x_2} J(e_i, e_j = n, D_1, D_2, x_1, x_2; f, a^k_i, a^k_j) \geq \max_{x_1, x_2} J_W(e_i, e_j = d, D_1 = D_2 = 0, x_1, x_2; a^k_i, a^k_j) \} \]

(20)

where \( i, j \in \{1, 2\}, i \neq j \) and \( k \in \{h, l\} \).

Here \( f_D(e_i; a^k_i, a^k_j) \) represents the threshold wealth-ratio at which, given the effort level on firm \( i \) (\( e_i \)), the incumbent is indifferent between working diligently in firm \( j \) and issuing debt. Thus, when \( f \geq f_D(\cdot) \), the incumbent will choose to work harder in firm \( j \) instead of adjusting the capital structure as the means to retain control of the firm(s). Of course, if the rival is too powerful in the sense that the wealth of the rival is sufficiently high or the skill of the rival is sufficiently superior, the incumbent has to work diligently and issue debt simultaneously in order to maintain control of both firms.

In the above discussion, we have analyzed the possible actions that the incumbent may take when he is willing to resist a takeover and retain control of both firms in a control contest. However, in some cases, the incumbent may choose to relinquish control of one or both firms to either the high ability rival only or to both types of the rival if doing so can generate him a greater value of total benefits (sum of his security and control benefits). The incumbent’s problem in such cases is similar to the one we discussed before (with the difference that the incumbent’s objective function does not include the private benefits of the firm of which he chooses to relinquish control). We will discuss the incumbent’s trade-offs in those cases in detail when we characterize the equilibria where such loss of control takes place.

On the other hand, in the absence of a spin-off, the incumbent’s problem involves only the joint firm, with the rival attempting to take over that firm. Therefore the incumbent makes no wealth allocation (since there is only one firm and therefore only one issue of equity), but he has to decide whether or not to attempt to defend the firm against a potential rival, and if so, his effort level and the amount of debt to issue against
the joint firm. Thus the incumbent’s problem is now:

$$\max_{e_q, D_q} J(e_q, D_q) = F^l_q(e_q, D_q; a^h_q) + P^l_q(D_q; a^h_q).$$

(21)

We will discuss this problem in detail when we characterize the equilibrium where no spin-off occurs.

### 3.3 The Board’s Problem

At time 0, the board chooses whether or not to spin off by comparing the cost and the benefit associated with the spin-off. The cost of the spin-off is the loss of operating synergies. The benefit of the spin-off is the potential increase in the long-term value of the combined firm equity. This potential value increase results from the increased threat of takeover that follows a spin-off. In particular, facing a credible takeover threat, the incumbent may take two possible actions: (1) he may be motivated to work harder to win sufficient support from the passive investors so that he can win control of the firm(s) in a control contest (the discipline effect); or (2) he may relinquish control of one firm (either voluntarily, in a friendly takeover, or involuntarily, through a hostile takeover) to a rival with superior management ability (the change of control effect). Both actions can increase the market value of the firm(s) currently managed by the incumbent, and contribute to this potential equity value increase.

A spin-off can help to increase the above takeover pressure on the incumbent, since a spin-off allows the rival to concentrate his wealth in the firm more vulnerable to an acquisition. Thus, when the board realizes that the takeover pressure which mitigates the incumbent’s opportunistic behavior is not strong enough in the case of a joint firm, it will choose to restructure the firm through a spin-off, provided that the loss of synergies from breaking up the joint firm is smaller than the gain from the discipline effect and the change of control effect.

### 3.4 Equilibrium Financial Structure When the Amount of the Incumbent’s Wealth is Small

In this section, we will analyze the case where the incumbent’s wealth is small (i.e., \(W = W_L\)). Six possible equilibria may emerge in this case, including spin-off followed by a friendly takeover, spin-off followed by a hostile takeover, no spin-off, etc. Here a friendly takeover is defined as any takeover where the incumbent
wants the rival to win the control contest (i.e., the total benefits the incumbent can receive if he maintains
control is lower than the total benefits that he can earn when the firm is managed by a rival), and thus votes
for the rival in the control contest. A hostile takeover is defined as any takeover where the incumbent would
like to maintain control of a firm (i.e., the total benefits that the incumbent can obtain from that firm under
his own management is larger than his total benefits when the firm is managed by a rival), and therefore
votes for himself in the control contest, but nevertheless loses control of that firm.

Definition of equilibrium: In equilibrium: (i) the corporate structure (joint firm or spin-off) chosen
by the board at time 0 maximizes the expected value of the combined long-term equity value \((t = 3)\), of the
firm(s) created as a result of this decision; (ii) the capital structure, the wealth allocation and the effort level
chosen by the incumbent maximizes his objective; (iii) the investments made by the rival in the equity of the
firm(s) maximize his objective; and (iv) the price of the equity in the equity market at various points in time
incorporates all the information available publicly (e.g., the beliefs about the incumbent’s wealth, the type
of the rival, the effort level of the incumbent, etc.) and also incorporates market participants’ equilibrium
inference about the behavior of various agents and the outcome of the control contest.

In the following propositions, we will characterize the spin-off decision taken by the board (i.e., spin-off
or no spin-off), as well as the outcome of the control contest (if a rival appears at \(t = 2\)). Clearly, if a rival
does not appear at \(t = 2\), there is no control contest, and the incumbent maintains control of both firms
at \(t = 3\). Depending on parameter values, the equity market will react to the public announcement of the
spin-off decision by the board at \(t = 1\); we will characterize these announcement effects in section 5. We will
define the threshold values mentioned in the various propositions below in the appendix, within the proofs

\(^{20}\) Based on the Perfect Bayesian Equilibrium (PBE) concept, formally defined for dynamic games with incomplete information

\(^{21}\) Thus, when the rival purchases equity from passive investors, the pricing of this equity is such that, wherever possible,
the present value of any increase in the firm’s cash flows due to improved firm performance after a takeover by the rival is
extracted by the passive investors. However, there is a range of parameter values where, if equity is priced giving the entire
value generated by a takeover to the passive investors, the price will be so high that the rival will in fact be unable to buy up
enough equity to succeed in the takeover. Our assumption here is that, in such situations, the price of equity will be set just
low enough to allow the takeover to succeed (so that, in such cases, the value created by the takeover is shared between the
passive investors and the rival). Such a pricing rule is reasonable, since, in the absence of this transfer of equity between them,
the takeover attempt will fail (so that both the passive investors and the rival are better off when equity is traded between
them on these terms compared to the case where no such trade takes place).
of these propositions.\footnote{Note that these threshold values are such that $f_1 < f_2 < f_3 < f_4$, $f_5$, $f_6 < f_7 < f_8$ and $p_1 < p_2 < \ldots < p_5$.}

**Proposition 1 (Spin-off Followed by a Friendly Takeover)** When (1) the private benefits that the incumbent can enjoy from firm 2 are substantially small such that $P^2_1 \leq p_1$; (2) the synergy between two divisions in the combined firm is substantially small such that $S \leq \bar{S}$; and (3) the capital available to the rival to finance a takeover is such that $f \leq f_6$, the equilibrium behavior of the four kinds of agents is as follows:

The board: It chooses to spin off at time 0.

The incumbent: He chooses to work normally in both firms at time 1, issuing no debt against both firms. If a rival appears, he votes for himself in firm 1, and votes for the rival in firm 2, losing control of firm 2. Otherwise he controls both firms.

The rival: If he appears, he takes over firm 2, regardless of ability.

The passive investors: A majority vote for the rival.

In this equilibrium, the board chooses to spin off one division since it knows that, after the spin-off, the incumbent has no incentive to fight a takeover in firm 2 due to his substantially small private benefits from controlling firm 2. The incumbent decides whether to resist a takeover or not by comparing the costs and the benefits from doing so. By resisting an attempt to take over any firm under his management, the incumbent would incur the cost of diligent effort, but would enjoy the private benefits from controlling that firm. Thus, when the incumbent’s private benefits are sufficiently low or when the cost of his diligent effort is sufficiently great, the incumbent prefers to strategically relinquish control of firm 2 to either type of rival. By doing so, he can also maximize his security benefits by allocating more of his equity investment to firm 2 to take advantage of the better management of firm 2 by the rival. As a consequence, a friendly takeover takes place at time 3, when the incumbent votes for the rival in firm 2 and the rival takes over control of firm 2 in the control contest.\footnote{\textit{Numerical Example 1}: Set $W_L = 100$, $W_H = 200$, $v_1 = 180$, $v_2 = 120$, $S = 8$, $c = 32$, $d = 1.3$, $l = 1.1$, $h = 1.5$, $\phi_1 = 0.4$, $\phi_2 = 0.3$, $\phi_3 = 0.3$, $\gamma = 0.5$, $\pi(n,h) = 0.38$, $\pi(n,l) = 0.42$, $\pi(d,h) = 0.45$, $\pi(d,l) = 0.6$, $P^1_I = 160$ and $P^2_I = 3$, $f = 0.42$. Also for the purpose of the numerical examples only, we assume that $\frac{\partial P^2_I}{\partial \gamma} = P^2_I$, which implies that the incumbent does not issue any debt in equilibrium. This assumption is only meant to avoid the computational complexity caused by the nonlinearity of the private benefit functions in these numerical examples. In this case, the incumbent chooses to work normally in both firms and allocate his wealth in such a way that he can maintain control of firm 1. His expected total benefits by doing so are at least 263.9, greater than what he can earn by voting for himself and controlling both firms by himself (263). As a consequence, the rival takes over control of firm 2 in a friendly takeover.} However, if the private benefits from control are larger, the incumbent may not want to lose control to the low ability rival, which may result in a hostile takeover. We characterize this situation in the next proposition.
Proposition 2 (Spin-off Followed by a Friendly or a Hostile Takeover) When $S \leq \bar{s}$, $f \in [f_1, f_6]$, and when the private benefits of the incumbent from firm 2 are such that $P^2_I \in [p_1, p_2]$, the equilibrium behavior of the four kinds of agents is as follows:

**The board:** It chooses to spin off at time 0.

**The incumbent:** He chooses to work normally in both firms at time 1, issuing no debt against both firms. If a high ability rival appears, he votes for himself in firm 1 and votes for the rival in firm 2; if the low ability rival appears, he votes for himself in both firms. In both cases, he loses control of firm 2.

**The rival:** If the high ability rival appears, he takes over firm 2 in a friendly takeover; if the low ability rival appears, he takes over firm 2 in a hostile takeover.

**The passive investors:** A majority vote for the rival.

In this equilibrium, the incumbent’s private benefits in firm 2 are lower than the increase in his security benefits when firm 2 is controlled by a high ability rival, but greater than the increase of the security benefits when firm 2 is controlled by a low ability rival. In this case, the incumbent prefers to relinquish control only to a high ability rival but not to a low ability rival. However, since his cost of working diligently is substantially large, the incumbent chooses to work normally at $t = 1$ instead of working diligently to fight the possible acquisition from either type of rival. Therefore, although the incumbent votes for himself instead of the low ability rival in the control contest, he can not keep the low ability rival from acquiring firm 2, and a hostile takeover occurs. On the other hand, if a high ability rival appears, the incumbent votes for the rival and a friendly takeover occurs.²⁴

Proposition 3 (Spin-off Followed by a Hostile Takeover by Either Type of Rival) When $S \leq \bar{s}$, $f \in [f_1, f_6]$, and when the private benefits of the incumbent from firm 2 are such that $P^2_I \in [p_2, p_3]$, the equilibrium behavior of the four kinds of agents is as follows:

**The board:** It chooses to spin off at time 0.

**The incumbent:** He chooses to work normally in both firms at time 1, issuing no debt against both firms. If a rival appears, he votes for himself in both firm 1 and firm 2, regardless of the rival’s type, and loses control of firm 2.

**The rival:** If he appears, he takes over firm 2, regardless of ability.

**The passive investors:** A majority vote for the rival.

If the incumbent’s private benefits are even larger than in the situation characterized in Proposition 2, so that they outweigh any increase in his security benefits arising from an acquisition by either type of rival, then the incumbent always votes for himself in the control contest, regardless of the rival’s type. However, he cannot succeed in maintaining control given his low level of effort in equilibrium and the resulting lack

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²⁴ For a numerical illustration of the ranges of values for which the equilibria discussed in Proposition 2 to 6 hold, please refer to figure 2.
of support from the passive investors. Note that the incumbent optimally chooses to work normally in equilibrium because he has to incur a significantly large cost of effort if he works diligently.

**Proposition 4 (Spin-off Followed by No Take-over)** When (1) the private benefits of the incumbent from firm 2 are such that $P_2^I \geq p_4$, and (2) the capital available to a potential rival is such that $f \in [f_1, f_3]$, and (3) the synergy $S \leq \tilde{S}$, then the equilibrium behavior of the four kinds of agents is as follows:

**The board:** It chooses to spin off at time 0.

**The incumbent:** He chooses to work diligently in firm 1 and normally in firm 2 when $f \leq f_2$. In this case, he issues debt and distributes it to both firms when $f > f_W(e_1 = d, e_2 = n; \pi_1 = \pi(d, h), \pi_2 = \pi(n, h))$. When $f \geq f_2$, he works diligently in both firms. In this case, he issues debt, allocating it strategically to both firms when $f > f_W(e_1 = d; \pi_1 = \pi(d, h))$.

**The rival:** Even if he appears, he is unable to take over either firm, regardless of his ability.

**The passive investors:** If the rival is of low ability and the incumbent works diligently, a majority of the passive investors vote for the incumbent. If the rival is of high ability, or if the rival is of low ability and the incumbent works normally, a majority vote for the rival.

In this equilibrium, the board chooses to spin off since it knows that the rival has the financial capacity to implement an acquisition and that the incumbent cannot deter an acquisition merely by strategically allocating his wealth and adjusting the capital structure of the firms currently run by him.

Given the spin-off, at $t = 1$, the incumbent faces two choices: First, he can relinquish control of firm 2 to a rival with better management ability, which can generate him larger security benefits. Alternatively, he can work harder to maintain control of both firms and enjoy the private benefits from controlling these firms, in which case he also incurs a higher cost of effort. When the incumbent’s private benefits are large, or the cost of diligent effort is small, or when the potential increase in his security benefits under the rival’s management are small (either because the management ability of the rival is close to that of the incumbent or because of the small probability of the rival’s arrival), the incumbent favors the second choice, i.e., he chooses to work diligently and maintain control of both firms. On the other hand, if the cost of working diligently is sufficiently large, then the incumbent prefers to relinquish control of one firm. This situation is characterized in the following proposition.

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25 In this proposition and also in the following propositions, $i \in \{1, 2\}$, $k \in \{h, l\}$, $D_i = 0$ unless otherwise specified.

26 Our assumption is that the incumbent’s effort level, once chosen, remains unchanged, even in the absence of a rival appearing at $t=2$. This is realistic since, in practice, the incumbent would face the takeover threat into the indefinite future so that any reduction in his effort level would again make him more vulnerable to a takeover.
Proposition 5 (Spin-off Followed by a Hostile Takeover Only by a High Ability Rival) When (1) the private benefits of the incumbent from firm 2 are substantially large such that $P_2^2 \geq p_5$, (2) the capital available to the rival to finance a takeover is such that $f \in [f_4, f_5]$, and (3) the synergy $S \leq \tilde{s}$, the equilibrium behavior of the four kinds of agents is as follows:

**The board:** It chooses to spin off at time 0.

**The incumbent:** He chooses to work diligently in both firm 1 and firm 2, issuing debt against both firms. If the high ability rival appears, he votes for himself in both firms, and loses control of firm 2. If the low ability rival appears, he maintains control of both firms.

**The rival:** If he is of high ability, he takes over firm 2; if he is of low ability, he does not take over either firm.

**The passive investors:** If the rival is of high ability, a majority of the passive investors vote for the rival; if the rival is of low ability, a majority vote for the incumbent.

This proposition characterizes another situation where a hostile takeover occurs after a spin-off. In this case, the market power of the high ability rival management is substantially strong either because the rival possesses a large amount of wealth or because the rival’s ability is sufficiently superior compared to the incumbent (in which case the rival can win a large proportion of votes from the passive investors). Therefore, the incumbent cannot win a control contest against such a rival even with all the antitakeover strategies at his disposal. However, the incumbent still chooses to work diligently when it is possible for him to defeat the low ability rival, and when his control benefits are greater than the cost of effort he would incur. As a consequence, he will retain control of both firms if the low ability rival appears, but lose control of one firm in a hostile takeover if the high ability rival appears.

In the above propositions, we characterized several equilibria involving a spin-off. In the following proposition, we will discuss the circumstances under which the board may decide not to spin off a division at all.

Proposition 6 (Equilibrium with No Spin-off) When any one of the following three conditions is satisfied: (1) $f \leq f_1$ and $P_1^2 \geq p_1$; (2) $f \in [f_4, f_6]$ and $S \geq \tilde{s}$; or (3) $f \geq f_6$ and $S \geq s'$, then the equilibrium behavior of the four kinds of agents is as follows:

**The board:** It chooses not to spin off at time 0.

**The incumbent:** He chooses to work normally in the joint firm when $f \leq f_1$; he chooses to work diligently when $f \in [f_6, f_8]$.

**The rival:** If he is of the high ability, he takes over the joint firm when $f \geq f_7$; if he is of the low ability, he takes over the joint firm when $f \geq f_8$.

**The passive investors:** If the rival is of low ability and the incumbent works diligently, a majority of the passive investors vote for the incumbent; if the incumbent works normally, or if the incumbent works diligently and the rival is of high ability, a majority vote for the rival.
As we discussed before, the spin-off decision made by the board at time 0 is determined by the trade-off between the loss of synergy to the firm and the potential increase in the firm’s long-term equity value. The increase in the long-term equity value emerges only when the spin-off can induce either a discipline effect (i.e., the incumbent works harder after the spin-off) or a change of control effect (i.e., there is a significant probability that the firm will be acquired by a rival with superior management ability). Therefore, if the financial capacity of the rival is very weak, there will be no spin-off, since, in this case, the board knows that the rival cannot challenge the incumbent successfully even after a spin-off. At the other extreme, if the rival is so financially capable that he is able to exert sufficient takeover pressure on the incumbent even in the absence of a spin-off, then the board will decide not to spin off, since a spin-off merely leads to the loss of synergy between the two divisions without any incremental benefits over keeping those two divisions under a joint corporate umbrella. Finally, if the synergy lost from a spin-off is so large that it is greater than any long-term equity value increase from the discipline or the change of control effect, then the board will again decide not to spin off.

### 3.5 Equilibrium Financial Structure When the Amount of the Incumbent’s Wealth is Large

When the incumbent’s wealth is large, i.e., $W_I = W_H$, the incumbent can retain control of both firms after a spin-off against both types of rival by allocating his own wealth strategically and working normally. Thus he will not use other costly methods such as adjusting capital structure and working harder to deter a potential take-over.

**Proposition 7 (Equilibrium When The Incumbent’s Wealth is Large)** When the incumbent’s wealth is large, the equilibrium is as follows:

(i) When the private benefits of the incumbent from firm 2 are substantially large such that $P_2^I \geq p_2$ or when the synergy between the two divisions in the combined firm is substantially large such that $S \geq \overline{s}$, the board chooses not to spin off; the incumbent works normally in the joint firm and maintains control always.

(ii) When the private benefits of the incumbent from firm 2 are such that $P_2^I \in [p_1, p_2]$ and when the synergy is substantially small such that $S \leq \underline{s}$, the board chooses to spin off at time 0. The incumbent works normally in both firms and votes for the high ability rival but not for the low ability rival in the control contest. The high ability rival takes over firm 2 if he appears.
(iii) When the private benefits of the incumbent from firm 2 are substantially small such that $P_2^2 < p_1$ and when the synergy is substantially small such that $S < \frac{1}{2}$, the board chooses to spin off at time 0. The incumbent works normally in both firms and votes for either type of rival when he appears. The rival takes over firm 2 regardless of his type.

In this case, since the incumbent can maintain control only by allocating his wealth between the two firms, a takeover happens only if the incumbent is willing to vote for the rival in the control contest. He will do so if his private benefits are smaller than the increase in the security benefits that he could earn by relinquishing control to the rival. Thus, when the incumbent’s private benefits are large enough, he will vote for himself in the control contest. Then, realizing this, the board will choose not to spin-off at time 0 since otherwise it would lose the synergy without gaining any increase in the share price of the firm from either the discipline or the change of control effect.27

4 Debt Allocation in a Spin-off

In this section, we will investigate some features of the adjustment of the capital structure by the incumbent following a spin-off.

Proposition 8 (Debt Allocation in a Spin-off)

(i) If the rate of decline in private benefits with debt is the same for either firm, the incumbent allocates a greater proportion of the total debt issued to firm 2 and less to firm 1.

(ii) Assume that, for any given debt level, the private benefits associated with firm 1 decline faster with debt than those associated with firm 2, and the sizes of two firms are the same. Then the proportion of debt allocated to firm 2 by the incumbent is greater than that allocated to firm 1.

The first part of this proposition demonstrates that the incumbent would use more debt in the smaller firm. This is because the usage of debt in fighting a takeover is more effective in a smaller firm where a marginal decline of the firm size through stock repurchase can cause a larger decrease of the relative size of the equity held by the passive investors.

27 Numerical Example 2: Keep the parameter values the same as in numerical example 1. Then, when the incumbent’s wealth is large, the board chooses not to spin off when $P_2^2 > 70.5$, since it knows that the incumbent will not vote for the rival with such a large private benefit. When $P_2^2 \in [14.1, 70.5]$, the board chooses to spin off only if $S \leq 18$. This is because, in this case, the incumbent will vote for the high ability rival but not for the low ability rival. When $P_2^2 < 14.1$, the board then chooses to spin off if $S \leq 21.6$, since the incumbent now will vote for both types of rival.
The second part of this proposition demonstrates that when his private benefits decay faster in firm 1 than in firm 2, the incumbent would distribute a greater percentage of debt to firm 2. Recall that debt and wealth allocation can be used by the incumbent as substitutes in fighting a takeover and that issuing debt is costly to the incumbent since it reduces the expected value of his private benefits. Thus, by relying on debt to maintain control of firm 2, the incumbent is able to allocate most of his wealth to firm 1, which allows him to maintain control of both firms with the least dissipation in his private benefits.

5 Market Reaction to the Spin-off Decision

5.1 Stock Returns Upon Spin-off Announcements

In this section, we will investigate the impact of a spin-off announcement on the equity market value of the combined firm. We will refer to firm 1 as the parent firm after the spin-off and firm 2 as the subsidiary, and define the announcement effect as the stock return to an investor who buys the stock on the day before the spin-off announcement and sells right after the announcement.

Proposition 9 (Announcement Effect)

(i) When the incumbent can derive enough private benefits from controlling firm 2 \( (P^2_I \geq p_2) \), the announcement of a spin-off results in a positive abnormal equity return for the combined firm equity.

(ii) Keeping the combined firm size constant, when \( P^2_I \in [p_2, p_3] \), the announcement effect on the combined firm equity is positively related to the size of the subsidiary as a fraction of the combined firm.

(iii) The size of the announcement effect on the combined firm equity is increasing in the wealth ratio of the rival to the incumbent, \( f \), when \( P^2_I \geq p_4 \) and \( f \in [f_1, f_3] \).

(iv) When \( P^2_I \in [p_2, p_3] \), the size of the announcement effect on the combined firm equity is increasing in the potential rival’s abilities, \( h \) and \( l \); it is also increasing in the probabilities of the arrival of the rival, \( \phi_2 \) and \( \phi_3 \).

(v) When \( P^2_I < p_1 \), the announcement of a spin-off leads to a zero announcement effect.

The intuition of the above proposition is as follows. When the private benefits to the incumbent are sufficiently large, the board would take different corporate structuring decisions facing different incumbent’s wealth. In particular, when the incumbent’s wealth is large, the board would choose not to spin-off; and when the incumbent’s wealth is small, the board would choose to spin-off. Since the market is uncertain about the wealth of the incumbent, it would value the firm before time 0 in such a way that incorporates
the average equilibrium outcomes over these two cases. Therefore, when a spin-off is announced, the market can infer that the incumbent is of small wealth. It would then adjust its valuation of the firm upward since the market expects that the firm(s) after the spin-off would be managed better either due to the discipline effect of the spin-off on the existing management or because of a change of control to a better management team. This adjustment leads to a positive announcement effect. However, when the private benefits to the incumbent are small, there is no uncertainty about the board’s corporate structuring decision: the board will always choose a spin-off. In this case, the spin-off announcement does not convey any information about the wealth of the incumbent, and thus leads to a zero announcement effect (e.g., the equity market return is zero), as shown in part (v) above.

Part (ii) states that the announcement effect is positively related to the relative size of the subsidiary when the incumbent is willing to maintain control, but chooses not to work diligently, thus losing control of the subsidiary to the rival. In this case, the announcement effect arises only from the change of control effect on the subsidiary, and thus increases in the relative size of the subsidiary. In part (iii), the size of the announcement effect is increasing in the wealth ratio because a larger amount of the rival’s wealth can either discipline the incumbent to work harder by intensifying the takeover threat, or make it easier for the rival to achieve the acquisition. Finally, under the conditions specified in part (iv), a takeover occurs following a spin-off if a rival appears. Thus, the announcement effect will be larger if it is more likely that the rival may appear, or if the management skill of the potential rival is much greater.

5.2 Long-term Firm Performance and Equity Value Changes Following Spin-offs

In this section, we study the impact of a spin-off on the long-term performance and the long-term equity value changes of the firm. For the purposes of measuring long-term equity value changes, we take the stock price right after the spin-off is announced as the base (t=1 in our model), and compare it to the equity value prevailing after the outcome of the control contest is publicly known (t=3 in our model). In other words,

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28 Numerical Example 3: Set \( f = 0.43, l = 1.4, h = 1.8, d = 1.6 \) and keep the other parameters values the same as in numerical example 1. The market value of the joint firm before time 0 is 325.6. It becomes 343.2 after a spin-off. Thus the announcement effect is 5.4% for the combined firm equity value.
we define long-term equity value changes as those occurring in addition to the changes associated with the spin-off announcement.\footnote{This definition of long-term equity value changes and returns is consistent with that adopted in the empirical literature.}

**Proposition 10 (Long-term Firm Performance and Equity Value Changes)**

(i) When the management ability of the low ability rival is such that \( l \geq \frac{q_1 + \phi h}{1 - \phi s} \), there is a long-term increase in the combined firm equity value, provided that any one of the two firms resulting from the spin-off experiences a takeover.

(ii) If neither firm resulting from a spin-off experiences a takeover, the long-term value changes of the combined firm equity are non-positive (i.e., zero or negative).

This proposition characterizes the long-term performance and equity value increments that follows a spin-off. The change of the long-term firm equity value following a spin-off arise from the uncertainty of the equity market about both the arrival of the rival and the potential rival’s management ability. Because of the uncertainty, the announcement effect only incorporates noisy information regarding the rival, which is also reflected in the equity value. After the spin-off, however, the market receives additional information which enables it to assess the firm’s future value more accurately. In particular, a takeover conveys an important piece of information by signaling the arrival of the rival and the success of the acquisition. Since the rival’s management ability is superior to that of the incumbent and thus the rival will enhance the firm’s value to the greatest extent, the equity market responds positively to those firms experiencing a takeover. On the other hand, the market responds negatively or has no response to those firms not experiencing a takeover, since the absence of a takeover arises from either the absence of a rival (leading to a negative long-term value change) or from the incumbent working hard and thereby successfully defending control of both firms against the rival (leading to a zero long-term value change).\footnote{Numerical Example 4: Keep the same parameter values as in numerical example 3. The long-term equity value increment of the combined firm is 15.4\% if any one of the firms created in the spin-off is taken over by a high ability rival and 1.4\% if any firm is taken over by a low ability rival.}

6 Extensions of the Basic Model

In this section, we study three different extensions of the basic model. In each case, we will modify only one assumption of the basic model, leaving the other assumptions unchanged.
6.1 Related versus Unrelated Spin-offs

In this section, we relax the assumption we make in the basic model that the incumbent’s ability is the same across divisions. We now assume that, in the case of an unrelated spin-off (defined as a spin-off where the parent firm and the spun-off division operate in unrelated businesses), the incumbent has different abilities in managing the two firms resulting from the spin-off. We will continue to maintain the assumption that the incumbent has the same management ability for the two firms in the case of a related spin-off (defined as a spin-off where the spun-off division operates in the same or in a closely related business as the parent firm).

Thus in an unrelated spin-off, we assume:

\[ d_1 > d > d_2, \quad n_1 > n > n_2; \]  \hspace{1cm} (22)

\[ d_1 - n_1 > d_2 - n_2, \]  \hspace{1cm} (23)

where \( d_i \) and \( n_i \) (\( i = 1, 2 \)) refer only to the case of the unrelated spin-off, and \( d \) and \( n \) to the related spin-off. Assumption (22) states that the incumbent is able to manage division 1 better than he is able to manage division 2 in the case of unrelated spin-offs. Assumption (23) implies that, when the incumbent works diligently, the incremental firm value (relative to the case where he works normally) is greater in division 1 than in division 2.\(^{31}\) Further, in order to isolate the effect of the unevenness in the management ability of the incumbent across divisions, we assume that the weighted average of the incumbent’s abilities is constant across the two kinds of spin-offs, i.e.,

\[ d = \frac{d_1 v_1 + d_2 v_2}{v_1 + v_2}, \quad \text{and} \quad n = \frac{n_1 v_1 + n_2 v_2}{v_1 + v_2}. \]  \hspace{1cm} (24)

The above assumption also implies that the value of the joint firm under the management of the incumbent would be the same for the unrelated spin-off as in the case of the related spin-off. All other assumptions remain the same as in the basic model.

\(^{31}\) Given our assumption that the incumbent is better at managing division 1 than division 2, it is natural to assume that any increase in effort by the incumbent will also create greater incremental value in division 1 than in division 2.
Incorporating these assumptions for the case of the unrelated spin-off into the expressions for the equity values of the two divisions, we get: \( V_i(\epsilon_i = d_i, D_i) = d_i v_i - D_i \) and \( V_i(\epsilon_i = n_i, D_i) = n_i v_i - D_i \) for \( i = 1, 2 \). On the other hand, for the related spin-off, these equity values are given by: \( V_i(\epsilon_i = d, D_i) = d v_i - D_i \), and \( V_i(\epsilon_i = n, D_i) = n v_i - D_i \) for \( i = 1, 2 \).

As a consequence, the control contest structure would change accordingly. For the related spin-off, all our previous assumptions about the atomistic investors’ voting behavior continue to hold. For the unrelated spin-off, following assumption (22), we have:

\[
\pi_2 (\epsilon_2 = d_2, k) < \pi (d, k) < \pi_1 (d_1, k), \ k = h, l; \tag{25}
\]

\[
\pi_2 (\epsilon_2 = n_2, k) < \pi (n, k) < \pi_1 (n_1, k), \ k = h, l, \tag{26}
\]

where \( \pi \) now refers only to the fraction of the passive investors’ votes obtained by the incumbent in a related spin-off (which is the same for both firms), and \( \pi_1 \) and \( \pi_2 \) only to the same fraction in an unrelated spin-off for firm 1 and firm 2 respectively.

**Proposition 11 (Related vs. Unrelated Spin-off)** When (1) the private benefits of the incumbent from controlling firm 2 are such that \( P_2^I < p_6 \); (2) the management ability of the incumbent in firm 2 is such that \( n_2 < n \); (3) the combined firm value under the incumbent’s management is no greater than the expected combined firm value under the rival’s management, i.e., \( d_1 v_1 + n_2 v_2 \leq n_1 v_1 + (\phi_1 n_2 + \phi_2 h + \phi_3 l) v_2 \); and (4) the incumbent’s wealth is small, then:

(i) The announcement effect is greater in an unrelated spin-off than in a related spin-off.

(ii) Given the other parameter values, the difference of the announcement effects between an unrelated spin-off and a related spin-off is greater when the difference in the incumbent’s management abilities across the two divisions, \( n_1 - n_2 \) or \( d_1 - d_2 \), is greater.

(iii) If any one of the two firms resulting from a spin-off is taken over, then the long-term value increase of the combined firm equity is greater in an unrelated spin-off than in a related spin-off.

Clearly, in an unrelated spin-off, the incumbent is better at managing one of the firms resulting from the spin-off compared to the other. This means that the chances of a successful takeover of at least one of the firms resulting from the spin-off is greater in an unrelated spin-off than in a related spin-off, since the firm which the incumbent is poorer at managing is more vulnerable to a takeover. This also implies that, even in the absence of a successful takeover, the takeover pressure (and therefore the discipline effect) is greater on
average in an unrelated spin-off than in a related spin-off. Thus, the announcement effect is greater in an unrelated spin-off than in a related spin-off since both the discipline effect and the change of control effect are greater in the unrelated spin-off. Further, the long-term value increase will also be greater in the case of an unrelated spin-off, since the increase in the firm performance created by a successful takeover will always be greater in the case of an unrelated spin-off than in a related spin-off.\footnote{Numerical Example 5: Set $d_1 = 1.48, d_2 = 1.28, n_1 = 1.05, n_2 = 0.925$ for the unrelated spin-off and $d = 1.4, n = 1$ for the related spin-off. Also set $h = 1.55$ and $l = 1.25$, and keep the values of the other parameters the same as in numerical example 1. In this case, the announcement effect on the combined firm equity is 3.9\% in the related spin-off, and 4.9\% in the unrelated spin-off. The long-term value increment for the combined firm equity is 12.2\% if any one of two firms resulting from an unrelated spin-off is taken over by a high ability rival; it is 11.3\% following a related spin-off.}

### 6.2 Change of Management Team Before a Control Contest

In this section, we extend the basic model by changing one assumption: we now assume that, with probability $\beta$, an executive capable of becoming the manager of firm 2 (after a spin-off) is available within the joint firm.\footnote{This contrasts with the basic model, where the implicit assumption was that no such executive was available.}\footnote{Our results go through even when the executive capable of managing firm 2 comes from outside the joint firm.} This implies that, in addition to the various incumbent’s choices that we have analyzed in the basic model, the incumbent now has one additional choice to make after the spin-off is announced by the board: he can choose to quit the management of one firm and let the executive take control of that firm. If the incumbent chooses to do so, this decision will be announced at $t = 1$ in the spin-off plan along with the decisions on wealth allocation and the capital structure of both firms.

At the time when the spin-off decision is made ($t = 0$), the incumbent and the board know for sure whether or not such an executive capable of managing firm 2 is available. However, before the public announcement of the spin-off (and the spin-off plan) at $t = 1$, outsiders know only the prior probability $\beta$ of the existence of such an executive.

For simplicity, we assume that the executive has no wealth. Thus, after the spin-off, she owns no equity in both firms. We also assume that the executive derives significant private benefits from managing the firm, so that she is always willing to take control of firm 2 if the position is offered to her. Further, we assume that the executive is of the same management ability as the low ability rival.\footnote{Allowing the executive to have other ability levels would simply complicate the analysis without changing our results.} Therefore if the executive
faces a low-ability rival in a control contest at \( t = 3 \), half of the passive investors will vote for her, with the other half voting for the rival. On the other hand, if the rival is of high ability, the executive can only win a minority of votes (denoted by \( \pi' \)) from the passive investors in the control contest (i.e., \( \pi' < \frac{1}{2} \)).

**Proposition 12  (Spin-off Followed by a Change of Management Before the Control Contest)**

(i) Let the private benefits of the incumbent from controlling firm 2 be low enough such that \( P^2_I \leq p_1 \), and \( S \leq \frac{1}{2} \) and \( f \leq f_0 \). Then if a suitable executive exists, the equilibrium behavior of the five kinds of agents is as follows:

**The board:** It chooses to spin off at time 0.

**The incumbent:** He relinquishes control of firm 2 to the executive and remains control of firm 1. He works normally in firm 1 and issues no debt against firm 1. If a low ability rival appears, he votes for himself in firm 1 and votes for the executive in firm 2; if a high ability rival appears, he votes for himself in firm 1 and votes for the rival in firm 2.

**The rival:** If the high ability rival appears, he takes over firm 2; if the low ability rival appears, he does not succeed in taking over either firm.

**The executive:** She controls firm 2 subsequent to the spin-off. At \( t = 3 \), she loses control of firm 2 in the control contest if the high ability rival appears; otherwise she maintains control of firm 2.

**The passive investors:** If the rival is of high ability, a majority of them vote for the rival; if the rival is of low ability, half of them vote for the rival and the other half vote for the executive.

(ii) When the private benefits of the incumbent in firm 2 are low enough such that \( P^2_I \leq p_1 \), but no executive exists, the equilibrium behavior of the other four kinds of agents is the same as that characterized in Proposition 1 (i.e., a friendly take-over occurs following the spin-off upon the arrival of a rival).

(iii) When the private benefits of the incumbent in firm 2 are such that \( P^2_I \geq p_1 \), then the incumbent does not relinquish control of either firm even when a suitable executive exists, and the equilibrium will be similar to those characterized by Propositions 2 to 6 (when the incumbent’s wealth is small) and by Proposition 7 (when the incumbent’s wealth is large).

When an executive with suitable management ability (discussed above) is available, the incumbent can choose to relinquish control of firm 2 to the executive and remain control of firm 1. By doing this, he will lose the control benefits that he could have enjoyed from managing firm 2. But he avoids incurring the higher personal cost arising from working diligently in order to fight a potential take-over, and at the same time, he can enjoy the greater security benefits arising from the increase in the equity value of firm 2 under the management of the executive. Thus, when the incumbent’s private benefits from controlling firm 2 are

\[ \text{qualitatively, as long as the executive’s ability is above that of the incumbent working normally and below that of the high ability rival.} \]
substantially low, or his cost of working diligently in that firm is large, this trade-off induces the incumbent to relinquish control of firm 2 to the executive.\footnote{Notice that, under the above conditions, it is always better for the incumbent to give up control of firm 2 to the executive right after the spin-off rather than wait for a friendly takeover in the future. By letting the executive take control of one firm, the incumbent can enjoy the increase in security benefit arising from the better management skills of the executive even if no rival appears in the future.} \footnote{Numerical Example 6: Set $\beta = 0.5$ and the other parameter values the same as in numerical example 1. In this case, the incumbent chooses to relinquish control of firm 2 to the executive after the board decides to spin-off. By doing so, the incumbent can derive an expected total benefit equalling 269.9, which is greater than the expected benefit he can derive from choosing not to relinquish control of firm 2 (268.5).}

If, however, the incumbent’s private benefits from firm 2 are large enough, he will choose not to relinquish control of that firm to the executive, maintaining control of both firms even after the spin-off. Of course, in this case, the incumbent may face a subsequent control contest if a rival appears at $t = 3$, with the outcome of such a contest determined according to the trade-offs discussed under Propositions 2 to 6 in the basic model.

Proposition 13 (Announcement Effect and Long-term Equity Returns) When the incumbent relinquishes control of firm 2 to a suitable executive as part of the spin-off plan, then:

(i) The announcement of a spin-off results in a positive abnormal equity return for the combined firm equity.

(ii) There is a long-term increase in the combined firm equity value, provided that any one of the two firms resulting from the spin-off experiences a takeover.

When the incumbent chooses to relinquish control of firm 2 to the executive, the market expects that the firm’s future cash flows to increase due to the superior management skill of the executive in firm 2 compared to that of the incumbent exerting normal effort in the combined firm. In addition, there will be a “change of control effect” as well (as in the basic model), since the market expects a change of control of firm 2 from the executive to a high ability rival to occur, should such a rival appear at $t = 2$. Both effects together lead to a positive announcement effect following the spin-off. The intuition behind the positive long-term equity return following the spin-off is similar to that discussed under Proposition 10 in the basic model.\footnote{Numerical Example 7: Set $\beta = 0.5$ and keep the other parameter values the same as in numerical example 3. In this case, the announcement effect is 2.7% for the combined equity value. The long-term equity return is 9.3% if firm 2 is taken over by a high ability rival subsequent to the spin-off.}
6.3 Asymmetric Information between the Board and the Incumbent

In the basic model, we assumed asymmetric information between the market and the board about the incumbent’s wealth. In this section, we will extend the basic model by introducing asymmetric information between the incumbent and the board as well, and investigate the implications of this new assumption on the announcement effect and the long-term value changes following a spin-off.

In particular, we assume that while the incumbent knows the true magnitude of his wealth at \( t = 0 \), the board (as well as the investors in the equity market) is uncertain about this true value. Therefore, the board decides on whether or not to spin off the subsidiary based on its prior beliefs about \( W_I \). We assume that at \( t = 0 \), the board has the same prior beliefs as the passive investors about \( W_I \): i.e., it is large (\( W_H \)) with probability \( \gamma \) and small (\( W_L \)) with probability \( 1 - \gamma \). As in the basic model, we assume that \( W_I \) becomes publicly known at \( t = 2 \), before the control contest takes place.

**Proposition 14 (Equilibrium with Asymmetric Information between the Board and the Incumbent)** If the board’s prior about the incumbent’s wealth is such that \( \gamma < \gamma_1 \), and \( f \in [f_1, f_6] \), then the board spins off the subsidiary; there is no spin-off if \( \gamma \geq \gamma_2 \).

Since the board does not know the true value of the incumbent’s wealth, the board makes the spin-off decision based on its prior beliefs about the incumbent’s wealth. Thus, when the board’s prior \( \gamma \) about the incumbent’s wealth being large is low, it decides to spin off since its expectation of the firm’s equity is maximized under a spin-off. In this case, the expected benefit from a spin-off, (which arises from the disciplining effect and the change of control effect discussed before), is larger than the cost (arising from the loss of synergies). If, however, the board’s prior about the incumbent’s wealth being large is high, then the board will not spin off, since the expected benefits from doing so are low. This is because, given that his wealth is large, the incumbent will be successful in defending himself against a hostile takeover, and further, the rival’s ability is not high enough to induce the incumbent to voluntarily give up control.

**Proposition 15 (Announcement Effect and Long-term Equity Returns)** Consider the situation where there is a spin-off. In this case:

(i) If the true value of the incumbent’s wealth is small:
(a) The announcement effect on the combined firm equity is positive or zero.
(b) The long-term equity return on the combined firm equity is positive if either one of the two firms resulting from the spin-off experiences a takeover.

(ii) If, however, the true value of the incumbent’s wealth is large:
    (a) The announcement effect on the combined firm equity is negative or zero.
    (b) The long-term equity return on the combined firm equity is zero, provided that $P_2 > p_2$.

The intuition behind the above proposition is as follow. Upon announcement of the spin-off plan, the market is able to infer the incumbent’s true wealth. This is because the incumbent’s allocation of wealth between the two firms resulting from the spin-off, as well as the level of debt he issues against each firm (both are described in the spin-off plan), will convey the information about this true wealth to the market. When the market realizes that the incumbent’s true wealth is low, it will infer that there will be disciplining effect on the current firm’s management and a significant probability of change in control to the rival as discussed in the basic model, resulting in a positive announcement effect. Further, depending on the outcome of the control contest, there will also be a long-term increase in the combined equity value for the firm experiencing a takeover (due to the reasons discussed in the basic model).

If, however, the market realizes that the incumbent’s true wealth is large, there is a negative announcement effect. This is because the incumbent’s true wealth being large conveys to the market that there will be neither a disciplining effect on current firm management nor any probability of a change in control, so that there is a net loss in intrinsic firm value as the result of spin-off (arising from the loss of synergies between the two divisions).\(^{39}\)

### 7 Implications of the Model

1. **Implications for the likelihood of spin-offs**: Our model has several implications for the likelihood of spin-offs for a given firm. First, our results imply that spin-offs are more likely if a firm operates in an

\(^{39}\) **Numerical Example 8**: Use the same parameter values as in numerical example 1. In this case, the board chooses to spin off one division although it is uncertain about the amount of the wealth of the incumbent. This is because its expected benefits from doing so (323.2) are higher than that from keeping the joint firm unseparated (303.5). The announcement effect of the spin-off on the combined firm equity is 8.7% if the market realizes that the wealth of the incumbent is small, and −5.8% if it realizes that the wealth is large. The long-term value increment for the combined firm equity, in the case of the incumbent’s wealth being small, is 8.9% if any one of two firms resulting from the spin-off is taken over. On the other hand, in the case of the incumbent’s wealth being large, the long-term value increment for the combined firm equity is zero.
industry with a high degree of takeover activity. This is because, in our model, the benefits of spinning off arise from the pressure exerted on incumbent management by potential rivals for control. Second, our results imply that spin-offs are more likely in the case of divisions which under-perform other divisions in the same firm, or other firms in the same industry. This is because, if a particular division is under-performing other firms in the same industry, it is more likely that a takeover by another firm in the same industry can improve performance. Further, if the division is under-performing other divisions of the parent firm, then spinning off that division and letting that division be taken over by a rival firm which is able to manage it better is more likely to increase long-term equity value. Third, our model implies that divisions of a firm which are unrelated to the core business of the firm are more likely to be spun off, since (a) incumbent management is likely to have a lower management ability for these divisions (so that the long-term equity value increments from such a spin-off will be greater); (b) the synergy of such a division with the rest of the firm is likely to be low (so that the cost of the spin-off is low); and (c) incumbent management’s control benefits from such a division are, in general, lower than from a division related to the firm’s core business (so that incumbent management is less likely to fight a future takeover of such a spun-off firm, making the spin-off more long-term value improving to equity holders). Fourth, firms operating in industries characterized by rapid technological change are more likely to spin off divisions. When the firm experiences rapid technological change in one division, the incumbent will become less capable at managing that division, so that the incumbent’s management ability across divisions is likely to be more lopsided in such firms. Therefore the board will have more motivation to spin-off that division since the change of control effect and the discipline effect will be greater in this case. Finally, we predict a negative correlation between spin-off likelihood and financial constraints in the economy, such as the level of interest rates: an increase in interest rates will reduce the frequency of spin-offs by reducing the financial ability of potential rivals for control.

2. Implications for stock market reactions to spin-off announcements: Our model has a wide variety of implications for the market reaction to spin-off announcements (i.e., the “announcement effect”). First, our model predicts that, in case where the equity holding by the incumbent management or the parent firm do not have very significant equity holdings in the subsidiary that is spun off, the announcement effect on the
combined firm equity will be positive on average (i.e., equity holders in the joint firm will enjoy a positive abnormal return on the day of the announcement of the impending spin-off). Second, the magnitude of this announcement effect is increasing in the size of the subsidiary as a fraction of the size of the joint firm. Third, the magnitude of this announcement effect is larger as the extent of takeover activity in the industry the parent firm is operating in (or, in the case of an unrelated spin-off, the subsidiary is operating in) is greater. Fourth, the magnitude of the announcement effect will be greater in the case of unrelated spin-offs than in related spin-offs. Fifth, the announcement effect will be decreasing in the fraction of equity in the spun-off subsidiary held by the incumbent management of the parent firm (either on their personal account, or through the parent firm). In particular, if the incumbent management or the parent firm holds enough equity to essentially guarantee control, our model predicts that the announcement effect will be zero or negative.\footnote{While we are not aware of any empirical work which sheds light on this implication of our model, there is some anecdotal support for this prediction. Consider, for example, the split-up of ICN pharmaceuticals into three companies in 1998. When the spin-off plan (which involved the incumbent chairman, Milan Panic, remaining chairman of all three companies, and the parent company maintaining a controlling stake in the other two firms resulting from the spin-off) was announced, ICN’s share price declined by twenty percent on the announcement day.}

Some of the above predictions are supported by the existing empirical literature on spin-offs. However, one prediction regarding which there has been no empirical work so far is the fifth prediction above. Thus, this prediction can serve as a test of our model. An indirect way to test this prediction is to compare the announcement effects of spin-offs and equity carve-outs. The testable prediction here is that, on average, the announcement effect in equity carve-outs will be smaller than that in spin-offs (since the parent firm typically holds a large fraction of equity in the subsidiary in an equity carve-out, while this is not the case for spin-offs).

3. Implications for the long-term firm performance and value changes following spin-offs: Our model has several implications for the long-term performance and value changes following spin-offs. First, our model implies that when the incumbent management’s equity holding (either directly or through the parent firm) in the subsidiary is small, spin-offs will be followed by increases in the long-term performance (as measured by accounting numbers) of the subsidiary. Second, if one of the two firms resulting from a spin-off is taken
over subsequently, equity holders (those who hold equity in the two firms resulting from the spin-off) will enjoy long-term positive abnormal returns. These long-term abnormal returns will be greater in unrelated spin-offs than in related spin-offs. Third, there will be no such positive long-term returns for spin-offs-parent combinations not reporting takeover activity. Finally, if the incumbent management or the parent firm holds enough equity to essentially guarantee control (so that no takeovers take place), our model predicts that this long-term abnormal return will be zero. Thus, we would expect the long-term abnormal returns following spin-offs to be smaller than those in equity carve-outs, for the reasons we discussed under implication 3 above.

4. Implications for the debt allocation in spin-offs: Our model predicts that, of the two firms resulting from a spin-off, a smaller fraction of the debt originally associated with the joint firm will be allocated to the firm which has private benefits declining at a faster rate with an increase in debt.\(^{41}\) This, in turn, implies that, other things remaining the same, of the two firms resulting from a spin-off, the firm which has a larger base level of private benefits associated with it would be allocated a disproportionately small fraction of the joint firm debt in relation to its size.\(^{42}\)

8. Conclusion

This paper has developed a new rationale for the performance and value improvements arising from spin-offs. We studied a setting where, while the incumbent firm management would like to increase equity value, it also derives private benefits from control. The firm has two divisions; current management may

\(^{41}\) This implication of our model is consistent with the debt distribution in the 1993 Marriott spin-off discussed in Parrino (1997). Parrino documents that, in the spin-off plan, Marriott distributed a disproportionate fraction of debt to Host Marriott, the parent firm in the spin-off. He also points out that the growth rate of Host Marriott was lower than Marriott International, the firm being spun off, which implies that the rate of decline in private benefits with debt would be smaller for Host Marriott than for Marriott International. Thus, the debt allocation in the Marriott case seems to have minimized the loss in private benefits to the Marriott family, which maintained control of both firms even after the spin-off. However, the debt allocation in the above case may also have been driven by the desire of shareholders to transfer wealth from bondholders, which is not a driving factor in our model.

\(^{42}\) One can think of a “base-level” of private benefits as the level of private benefits accruing to top management with zero debt issued against the firm. Examples of situations where the base level of private benefits will be different between parent and spin-off subsidiary include cases where these two firms operate in different industries, or in different countries. Deriving the last testable implication requires us to place additional structure on the \(P_i(D_i)\) function. In particular, we need to assume that the decline in the expected private benefits from the base level with debt occurs only due to the increase in the bankruptcy-probability of the firm with an increase in the amount of debt issued against it. Given this, it is easy to show that the decline in private benefits with debt will be faster for the firm with a larger base level of private benefits.
have differing abilities for managing these two divisions. Giving up control to a rival management team, while it may benefit equity holders (including the incumbent management) by increasing firm value, is costly to the incumbent in that it involves loss of control (and hence the private benefits the current management derives from control). Spin-offs increase the chance of loss of control to a potential rival in two ways: First, it reduces the ability of the incumbent to use firm size strategically against the rival in a control contest (the rival can invest to the full extent of his wealth in the more vulnerable firm subsequent to a spin-off). Second, it increases the probability that passive investors will vote for the rival in a control contest for at least one division (in a joint firm, the superior management ability of any rival with respect to one division may be neutralized by inferior ability in managing the other one). This increased chance of loss of control of at least one division following a spin-off, in turn, motivates the incumbent to work harder in running the firm in an attempt to minimize the probability of his losing control.

Our model demonstrates that spin-offs can increase the probability of a takeover by the “right kind” of management team. We also show that such spin-offs can enhance the level of firm performance even in the absence of such a value-improving takeover by serving to discipline firm management. In addition, our analysis demonstrates that while a spin-off will lead to positive abnormal stock-price returns on the announcement day, it will also lead to abnormal stock price performance (on average) in the period following the spin-off as well for parent-spin-offs combinations reporting subsequent takeover activity. We also explained a wide variety of other recently documented empirical regularities, and provided hypotheses for further empirical work.
References


Appendix

Proof of Proposition 1. At \( t=3 \), a friendly takeover occurs if \( b_i^k = 1 \) (i.e., \( P_i^2 \leq p_1 \equiv [1 - x_1(\pi_1(n, h), v_1)](l - 1)W_i \)). At \( t=0 \), if \( f \leq f_0 \equiv \text{Argmax}_f \{\text{Max}_J(e_q = n, D_q; a_q^h = 0) \geq J(e_q = d, D_q = 0; a_q^h = 0)\} \), the rival has financial ability to acquire at least one of the firms resulting from a spin-off, but not the joint firm. In this case, the value of the combined firm equity is \( v_1 + (\phi_1 + \phi_2 h + \phi_3 l)v_2 \) in the case of a spin-off, and \( v_1 + v_2 + S \) in the case of no spin-off. Thus, when \( S \leq g \equiv \text{Min}\{\phi_2(h - 1)v_2, (d - 1)v_1\} \), and \( f \leq f_0 \), the board chooses to spin off.

Proof of Proposition 2. In this proof, we will solve the equilibrium backward. Here and also in the following proofs, \( i = 1, 2 \), and \( k = h, l \), unless otherwise specified; and we will omit in the functional form the choice variables which equal zero.

At \( t=3 \), the incumbent votes for the high ability rival \( (b_i^h = 1) \) but not for the low ability rival \( (b_i^l = 0) \) if and only if \( P_i^2 \in [p_1, p_{a5} \equiv [1 - x(\pi_1(n, h), v_1)](h - 1)W_I] \).

At \( t=2 \), the rival decides to acquire firm \( i \) if \( \frac{W_R}{V_R} + (1 - \pi_i(e_i, k))(1 - \frac{x_i W_I}{v_i - D_i} - \frac{W_R}{V_R}) \geq \frac{1}{2} \), which incorporates the assumption that the passive investors extract the full value increase \( V_R^i - V_I^i \) associated with the takeover. Note that the incumbent maintains control of firm \( i \) if \( \frac{W_R}{V_R} + (1 - \pi_i(e_i, k))(1 - \frac{x_i W_I}{v_i - D_i} - \frac{W_R}{V_R}) < \frac{1}{2} \), which implies that in some range of parameters, neither the incumbent can maintain control nor can the rival acquire firm \( i \). To avoid the occurrence of this indeterminacy, we further assume that, in this case, the value created by the takeover is shared between the passive investors and the rival so that the exchange of shares between them can go through and the takeover can succeed. Mathematically, it means that the equity price charged by the passive investors to the rival in this case is such that \( \frac{W_R}{V_R} + (1 - \pi_i(e_i, k))(1 - \frac{x_i W_I}{v_i - D_i} - \frac{W_R}{V_R}) = \frac{1}{2} \), where \( V \) is the firm’s equity value under such a equity price. Therefore, the incumbent can maintain control of both firms if

\[
\sum_{i=1}^{2} \left[ \frac{x_i W_I}{v_i - D_i} + \pi_i(e_i, k)(1 - \frac{x_i W_I}{v_i - D_i} - \frac{W_R}{V_R}) \right] \leq 1. \tag{A1}
\]

Between time 0 and time 1, the incumbent chooses his effort, wealth allocation and the capital structure of the firms currently under his control to maximize his total benefits (12). To ease the exposition, we define
\( J_W(e_i, D_i; a_i^k) \equiv \text{Max}_{x_i} J(e_i, x_i, D_i; a_i^k) \). \( J_W(\cdot) \) is the total benefits to the incumbent after the incumbent allocates his wealth optimally, given his effort level and the capital structure in each firm. We further define \( J^*(e_i; a_i^k) = \text{Max}_{D_i, x_i} J(e_i, x_i, D_i; a_i^k) \) and \( \{D_i^*(e_i; a_i^k, b_i), x_i^*(e_i; a_i^k)\} = \text{Arg} \max_{D_i, x_i} J(e_i, x_i, D_i; a_i^k) \). Therefore, the objective of the incumbent is to choose his optimal effort level such that \( J^*(\cdot) \) is maximized. In particular, the incumbent has the following choices for his efforts in firm 1 and 2:

(a1) Working diligently in firm 1 and normally in firm 2, and choosing \( D_i, x_i \) so that \( a_i^k = 0 \);

(a2) Working diligently in firm 2 and normally in firm 1, and choosing \( D_i, x_i \) so that \( a_i^k = 0 \);

(a3) Working diligently in firm 1 and normally in firm 2, and choosing \( D_i, x_i \) so that \( a_i^h = a_i^l = 0 \);

(a4) Working diligently in firm 2 and normally in firm 1, and choosing \( D_i, x_i \) so that \( a_i^h = a_i^l = 0 \);

(a5) Working diligently in both firms, and strategically chooses \( D_i, x_i \) so that \( a_i^k = 0 \);

(a6) Working diligently in both firms, and strategically chooses \( D_i, x_i \) so that \( a_i^h = a_i^l = 0 \);

(a7) Working normally in both firms, and strategically chooses \( D_i, x_i \) so that \( a_i^k = 0 \).

It can be shown that alternative (a2) is dominated by (a1) since the total benefits \( J \) in the case of (a2) is lower than that in the case of (a1). So is (a4) by (a3).

Define \( f_{a1} \equiv \text{Max}\{0, f_D(e_2 = n; a_i^l = 0), f_D(e_2 = n; a_i^l = a_i^h = 0, a_i^k = 1)\} \), and \( f_{a2} \equiv \text{Arg} \max_f \{J^*(e_1 = d, e_2 = n; a_i^k = 0) \geq J_W\} \). If \( f \in [f_{a1}, f_{a2}] \), all the alternatives are feasible. However, (a5) is dominated by (a1), (a6) by (a3), and (a3) by (a1). Thus if (a7) dominates (a1), the incumbent prefers to working normally after a spin-off and ensuring control of firm 1 against both types of rival and control of firm 2 against only a low ability rival, i.e.,

\[
J_W'(e_i = n; a_i^h = 0, a_i^l = 1) \geq J_W(e_i = d, e_2 = n; a_i^k = 0).
\] (A2)

If \( f \in [f_{a2}, f_{a3}] \), where \( f_{a3} \equiv \text{Arg} \max_f \{J^*(f, e_1 = d, e_2 = n; a_i^h = a_i^l = a_i^k = 0, a_i^l = 1) \geq J_W'\} \), then alternative (a1) and (a2) become infeasible. (a4), (a5), and (a6) are dominated by (a3). In this case, the incumbent
chooses to work normally in both firms if:

\[ J'_W \geq J_W(e_1 = d; e_2 = n; a^h_1 = a^h_2 = 0, a^h_3 = 1). \]  

(A3)

If \( f \in [f_{a3}, f_{a4}] \), where \( f_{a4} \equiv \text{Arg} \max_f \{ J^*(f, e_i = d; a^k_i = 0) \geq J'_W \} \), then alternative (a1), (a2), (a3), and (a4) disappear. In this case, the incumbent chooses to work normally in both firms if

\[ J'_W \geq J_W(e_1 = d; a^k_i = 0). \]  

(A4)

Finally, if \( f \in [f_{a4}, f_{a5}] \), where \( f_{a5} \equiv \text{Arg} \max_f \{ J^*(f, e_i = d; a^h_1 = a^h_2 = 0, a^h_3 = 1) \geq J'_W \} \), then alternative (a1), (a2), (a3), (a4), and (a5) disappear. In this case, the incumbent chooses to work normally in both firms if

\[ J'_W \geq J_W(e_i = d; a^h_1 = a^h_2 = 0, a^h_3 = 1). \]  

(A5)

Define \( p_{a1} \equiv \text{Arg} \max_{P_2^2} \{ J_W(e_i = d; a^k_1 = 1, a^k_2 = a^k_3 = 0) \leq J'_W \} \), \( p_{a2} \equiv \text{Arg} \max_{P_2^2} \{ J_W(e_i = d; a^k_i = 0) \leq J'_W \} \), \( p_{a3} \equiv \text{Arg} \max_{P_2^2} \{ J_W(e_1 = d, e_2 = n; a^h_1 = a^h_2 = 0, a^h_3 = 1) \leq J'_W \} \) and \( p_{a4} \equiv \text{Arg} \max_{P_2^2} \{ J_W(e_1 = d, e_2 = n; a^h_1 = 0) \leq J'_W \} \). Also Define \( p_2 \) as \( p_{a5} \) when \( f \in [f_{a5}, f_{a6}] \); \( \text{Min}(p_{a1}, p_{a5}) \) when \( f \in [f_{a4}, f_{a5}] \); \( \text{Min}(p_{a2}, p_{a5}) \) when \( f \in [f_{a3}, f_{a4}] \); \( \text{Min}(p_{a3}, p_{a5}) \) when \( f \in [f_{a2}, f_{a3}] \); and \( \text{Min}(p_{a4}, p_{a5}) \) when \( f \in [f_{a1}, f_{a2}] \). Thus, if \( P_2^2 \in [p_1, p_2] \), \( S \leq s \), and \( f \in [f_1, f_6] \) where \( f_1 \equiv f_{a1} \), condition (A2), (A3), (A4), and (A5) are satisfied, which implies that a friendly takeover occurs following a spin-off if the rival is of high ability and a hostile takeover occurs if the rival is of low ability. ■

**Proof of Proposition 3.** Given \( b^k_i = 0 \) and \( f \in [f_1, f_6] \), the incumbent votes for himself in the control contest if \( P_2^2 \geq p_{a5} \). Set \( p_3 \) equalling \( \infty \) when \( f \in [f_{a5}, f_6] \); \( p_{a1} \) when \( f \in [f_{a4}, f_{a5}] \); \( p_{a2} \) when \( f \in [f_{a3}, f_{a4}] \); \( p_{a3} \) when \( f \in [f_{a2}, f_{a3}] \); and \( p_{a4} \) when \( f \in [f_1, f_{a2}] \). Thus, if \( P_2^2 \in [p_2, p_3] \), \( S \leq s \), and \( f \in [f_1, f_6] \), the incumbent chooses to work normally in both firms at \( t = 1 \), and a hostile takeover occurs regardless of the type of rival at \( t = 3 \). ■

**Proof of Proposition 4.** Follow the proof of Proposition 2. If \( f \in [f_{a1}, f_{a6}] \), where \( f_{a6} \equiv \text{Arg} \max_f \{ J^*(f, e_1 = d, e_2 = n; a^h_i = 0) \geq J_W(e_i = d; a^h_i = 0) \} \), then no takeover occurs provided the incumbent
chooses (a1) instead of (a7) and (a3), i.e.,

\[ J^*(e_1 = d, e_2 = n; a^h_1 = 0) \geq J_W^*, \text{ and} \]

\[ J^*(e_1 = d, e_2 = n; a^h_1 = 0) \geq J_W(f, e_1 = d, e_2 = n; a^h_1 = a^l_2 = 0, a^h_2 = 1). \quad (A6) \]

If \( f \in [f_{a6}, f_{a4}] \), no takeover occurs provided the incumbent chooses (a5) instead of (a7), (a6), and (a3), i.e.,

\[ J_W(e_1 = d; a^h_1 = 0) \geq J^*(e_1 = d, e_2 = n; a^h_1 = a^l_2 = 0, a^h_2 = 1), \]

\[ J_W(e_i = d; a^h_1 = 0) \geq J_W^*, \text{ and} \]

\[ J_W(e_i = d; a^h_1 = 0) \geq J_W(e_i = d; a^h_2 = a^h_1 = 0, a^h_2 = 1). \quad (A7) \]

Define \( f_1 \equiv f_{a1}, f_3 \equiv M_i n[f_{a4}, f_{a6}], f_2 \equiv f_{a6}, p_{a6} \equiv \text{Arg min}_{P_f^I} \{ J_W(e_1 = d, e_2 = n; a^h_1 = 0) \geq J_W(e_1 = d, e_2 = n; a^h_1 = a^h_2 = 1) \}, p_{a7} \equiv \text{Arg min}_{P_f^I} \{ J_W(e_i = d; a^h_1 = 0) \geq J_W(e_1 = d, e_2 = n; a^h_1 = a^h_2 = 1) \}, \text{and} \]

\( p_{a8} \equiv \text{Arg min}_{P_f^I} \{ J_W(e_i = d; a^h_1 = 0) \geq J_W(e_1 = d; a^h_2 = a^h_1 = 0, a^h_2 = 1) \}, \text{and} \)

\( p_4 \equiv \text{Max}(p_{a6}, p_{a4}) \) when \( f \in [f_{a1}, f_{a6}] \) and \( \text{Max}(p_{a7}, p_{a8}, p_{a2}) \) when \( f \in [f_{a6}, f_{a4}] \). Thus, when \( P_f^2 \geq p_4 \) and \( f \in [f_1, f_3] \), condition (A6) and (A7) are satisfied.

Back to time 0. When \( f \leq \text{Min}(f_{a4}, f_{a6}) \), the combined equity value at least equals \( dv_1 + v_2 \) in the case of a spin-off; it is \( v_1 + v_2 + S \) in the case of no spin-off. This implies that when \( S \leq (d - 1)v_1 \), the board prefers to spin-off at time 0. ■

**Proof of Proposition 5.** Define \( f_4 \equiv \text{Arg max}_{f} \{ J^*(f, e_i = d; a^h_1 = 0) \geq J_W(f, e_i = d; a^h_1 = 1, a^l_2 = a^h_1 = 0) \} \). When \( f \in [f_4, f_5 \equiv \text{Min}(f_{a5}, f_{a6})] \), given \( b^h_1 = 0 \), only alternative (a6) and (a7) are feasible for the incumbent. In this case, the incumbent chooses (a6) only if

\[ J_W(e_i = d; a^h_1 = a^l_2 = 0, a^h_2 = 1) \geq J_W^*, \quad (A8) \]

Note (A8) implies \( J^*(e_i = d; a^h_1 = a^l_2 = 0, a^h_2 = 1) \geq J_W^* \). Thus, when \( P_f^2 \geq p_5 \equiv p_{a1} \), (A8) is satisfied, i.e., the incumbent chooses to work diligently in both firms. At \( t = 0 \), the board chooses to spin off if \( S \leq (d - 1)v_1 + [\phi_2(h - 1) + (1 - \phi_2)(d - 1)]v_2 \). ■

**Proof of Proposition 6.** The board chooses to spin off only when the combined value of two firms following a spin-off is higher than the value of the joint firm in the case of no spin-off.
If \( f \leq f_1 \), and \( P_f^2 > p_1 \), the joint firm value in the absence of a spin-off is \( v_1 + v_2 + S \); the value of the combined firm equity is \( v_1 + v_2 \) following a spin-off. Thus, no spin-off occurs in this case. If \( f \in [f_1, f_6] \), the maximal value of the combined firm equity is \( v_1 + v_2 + \phi \) in the case of a spin-off, where \( \phi = (d-1)v_1 + [\phi_2(h-1) + (1-\phi_2)(d-1)]v_2 \). Thus, when \( S \geq \phi \), no spin-off occurs. If \( f > f_6 \), the maximal combined value is \( dv_1 + dv_2 + \phi_2(h-d)v_2 \) in the case of a spin-off, and \( d(v_1 + v_2) + S \) in the case of no spin-off. Thus, when \( S \geq \phi' \equiv \phi_2(h-d)v_2 \), no spin-off occurs.

When the board chooses not to spin off, at \( t=1 \), the incumbent faces the following alternatives:

(a1') Working normally in the joint firm, and strategically choosing \( D_i \) so that \( a_q^k = 0 \);

(a2') Working diligently in the joint firm, and strategically choosing \( D_i \) so that \( a_q^k = 0 \);

(a3') Working diligently in the joint firm, and strategically choosing \( D_i \) so that \( a_q^l = 0 \);

(a4') Working normally in both firms, and issuing no debt.

Define: \( f_7 \equiv \text{Arg max}_f \{ \text{Max}_{D_q} J(e_q = d, D_q; a_q^h = 1, a_q^l = 0) \geq F(e_q = n; a_q^l = 1) \} \), and \( f_8 \equiv \text{Arg max}_f \{ \text{Max}_{D_q} J(e_q = d, D_q; a_q^h = 0) \geq J(e_q = d; a_q^h = 1, a_q^l = 0) \} \). The incumbent works normally if \( f < f_1 \), and diligently if \( f \in [f_6, f_8] \). The derivation of such incumbent’s choices is similar to those presented in the proofs of Proposition 1 to 6. Thus, we omit the detailed proof for this part of the proposition.

**Proof of Proposition 7.** When the incumbent’s wealth is large, the incumbent can maintain control of the firm(s) by wealth allocation only. First, when the synergy is sufficiently high or when \( P_f^2 \geq p_2 \) so that the incumbent only votes for himself, no spin-off occurs. Second, when \( P_f^2 \in [p_1, p_2] \), the value of the combined firm equity will be \( (\phi_2h + \phi_1 + \phi_3)v_2 + v_1 \) if the board chooses to spin off since the incumbent will only vote for the high ability rival, but not for the low ability rival. On the other hand, if the board chooses not to spin-off, the joint firm value is \( v_1 + v_2 + S \). Thus when \( P_f^2 \in [p_1, p_2] \) and \( S < \phi_2(h-1)v_2 \), the board chooses to spin off. Third, when \( P_f^2 < p_1 \), if the board chooses to spin off, the incumbent votes for any appearing rival who will take over the control of firm 2 in the control contest. The value of the combined firm equity in this case is \( (\phi_1 + \phi_2h + \phi_3l)v_2 + v_1 \). Thus when \( P_f^2 < p_1 \) and \( S < [\phi_2(h-1) + \phi_3(l-1)]v_2 \), the board chooses to spin off.
Proof of Proposition 8. (i) The incumbent’s problem is:

\[
\begin{align*}
\text{Max}_{D_i, \pi} & \quad J = \frac{2}{\sum_{i=1}^{2} \left[ x_i W_i \left( d v_i - D_i \right) + P_i(D_i) \right]} \\
\text{S.t.} & \quad \frac{x_i W_i}{v_i - D_i} + \pi_d h \left( 1 - \frac{x_i W_i}{v_i - D_i} - \frac{W_i}{\pi D_i} \right) \geq \frac{1}{2}, \quad i = 1, 2 \\
& \quad x_1 + x_2 = 1
\end{align*}
\]

(A9)

The first order condition is \( \frac{\partial P_i(D_i)}{\partial D_i} = \frac{1}{2 - \pi} (1 - \lambda \pi) - \frac{\lambda}{\pi} (d - 1) \pi \), \( i = 1, 2 \). In equilibrium, \( \frac{\partial P_i}{\partial D_1} > \frac{\partial P_i}{\partial D_2} \) since \( v_1 > v_2 \), which implies \( D_1^* < D_2^* \). Here \( D_i^* \) represents the optimal debt in firm \( i \), and \( \lambda \) is the Lagrangian factor.

When \( v_1 = v_2 \), the first order condition implies \( \frac{\partial P_1}{\partial D_1} = \frac{\partial P_2}{\partial D_2} \) in equilibrium. Since \( \left| \frac{\partial P_1}{\partial D_1} \right| > \left| \frac{\partial P_2}{\partial D_2} \right| \) given any \( D, D_1^* < D_2^* \). ■

Proof of Proposition 9. (i) Conditional on the occurrence of a spin-off, when \( P_f^2 \geq p_2 \), the value of the joint firm equals \( \gamma(v_1 + v_2 + S) + (1 - \gamma)(v_1 + v_2 + M) \) before the announcement, where \( v_1 + v_2 + S \) represents the value of the joint firm in the absence of a spin-off; and \( v_1 + v_2 + M \) represents the value of the combined firm equity in the case of a spin-off. After a spin-off, the value changes to \( v_1 + v_2 + M \).

Thus, \( M \geq S \), since otherwise, the board does not choose to spin off. Therefore, the announcement effect is positive.

(ii) If the incumbent chooses alternative (a7) between time 0 and time 1, the combined value of the parent and the spin-off is \( [\phi_2 (h - 1) + \phi_3 (l - 1)] v_2 + v_1 + v_2 \), which is increasing in \( \frac{v_2}{v_1 + v_2} \).

(iii) If no takeover occurs following the arrival of the rival, then the incumbent chooses alternative (a1) if \( f \in [f_1, f_2] \), resulting in a combined value of \( (d - 1) v_1 + v_2 \); he chooses alternative (a5) if \( f \in [f_2, f_3] \), resulting in a combined value of \( (d - 1) (v_1 + v_2) \).

(iv) \( (\phi_1 + \phi_2 h + \phi_3 l) v_2 + v_1 \) is increasing in \( \phi_2, \phi_3, h, \) and \( l \).

(v) When \( P_f^2 < p_1 \), the combined value is \( \gamma(v_1 + v_2 + M) + (1 - \gamma)(v_1 + v_2 + M) \) before the spin-off; and it does not change after the spin-off. Thus, the announcement effect is zero. ■

Proof of Proposition 10. Consider the case where a takeover occurs following a spin-off. The combined value of the parent and the spin-off is \( (\phi_1 + \phi_2 h + \phi_3 l) v_2 + v_1, \phi_2 (h - 1) v_2 + v_2 + dv_1, \) or \( \phi_2 (h - 1) v_2 + dv_2 + dv_1 \).
upon the spin-off announcement. After a takeover, the above values change to $hv_2 + v_1 / lv_2 + v_1$, $hv_2 + dv_1$, or $hv_2 + dv_1$ correspondingly. Thus when $l \geq \frac{\phi h + \phi f}{1 - \phi h}$, the combined firm equity value increases in the long term when the firms experiences a takeover after a spin-off.

Consider the case where no takeover occurs following a spin-off. If no takeover is due to the diligent effort exerted by the incumbent, the combined value of the parent and the spin-off at $t = 3$ is the same as that at $t = 1$, i.e., $dv_1 + v_2$ or $d(v_1 + v_2)$, which implies a zero long-term effect. On the other hand, if no takeover is due to the absence of a rival, the combined value is $v_2 + v_1$, $v_2 + dv_1$, or $dv_2 + dv_1$ at $t=3$, which is lower than the corresponding value upon the spin-off announcement at $t = 1$. ■

**Proof of Proposition 11.** When $P_f^2 \in [p_{a1}, p_{a3}]$, the combined firm value may be:

**Firm Value in an Unrelated Spin-off**

<table>
<thead>
<tr>
<th>$f$</th>
<th>$t=1$ (Upon Announcement)</th>
<th>$t=3$ (after a takeover)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f \in [f_1, f']$</td>
<td>$d_1 v_1 + n_2 v_2$</td>
<td>$d_1 v_1 + n_2 v_2$</td>
</tr>
<tr>
<td>$f \in [f'_2, f_6]$</td>
<td>$n_1 v_1 + (\phi_1 n_2 + \phi_2 h + \phi_3 l)v_2$</td>
<td>$hv_2 + n_1 v_1$ or $lv_2 + n_1 v_1$</td>
</tr>
</tbody>
</table>

**Firm Value in a Related Spin-off**

<table>
<thead>
<tr>
<th>$f$</th>
<th>$t=1$ (Upon Announcement)</th>
<th>At $t=3$ (after a takeover)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f \in [f_1, f_2]$</td>
<td>$dv_1 + nv_2$</td>
<td>$dv_1 + nv_2$</td>
</tr>
<tr>
<td>$f \in [f_2, f_6]$</td>
<td>$n_1 v_1 + (n \phi_1 + \phi_2 h + \phi_3 l)v_2$</td>
<td>$hv_2 + n_1 v_1$ or $lv_2 + n_1 v_1$</td>
</tr>
</tbody>
</table>

where $f'_{a2} \equiv \arg \max \{ J^*(f, e_1 = d_1, e_2 = n_2; a^h_0 = 0) \geq J_W(e_i = n_i; a^h_0 = 0, a^h_2 = 1) \}$. When $n_2 < \bar{n}$, where $\bar{n} \equiv \arg \max_{n_2} [x_1(\pi_1(d_1, h), d_1 v_1 - D_2^*(d_1, n_2)) + x_2(\pi_2(n_2, h), n_2 v_2 - D_2^*(d_1, n_2)) = x_1(\pi(d, h), dv_1 - D_1^*(d, h)) + x_2(\pi(n, h), v_2 - D_2^*(d, h))]$, $f'_{a2} < f_{a2}$. This, together with the assumptions $d_1 - n_1 > d - n > d_2 - n_2$, $d_1 > d > d_2$, and $n_1 > n > n_2$, implies that, the combined value of the parent and the spin-off upon announcement in an unrelated spin-off is larger than that in a related spin-off when $d_1 v_1 + n_2 v_2 \leq n_1 v_1 + (\phi_1 n_2 + \phi_2 h + \phi_3 l)v_2$, and that the long-term equity return associated with a takeover in an unrelated spin-off is the same as or higher than that in a related spin-off. Similarly, when $P_f^2 < p_{a4}$, both the announcement effect and long-term equity return are larger in an unrelated spin-off than those in a related spin-off.

It is easy to show that the difference of the announcement effects between an unrelated spin-off and a
related spin-off is greater when \( n_1 - n_2 \) or \( d_1 - d_2 \) is greater.

**Proof of Proposition 12.** (i) Between time 0 and time 1, the incumbent chooses to relinquish control of firm 2 to the executive if \( P_f^2 \leq [1 - x_1(\pi_1(n, h), v_1)](l - 1)W_1 \). At \( t = 3 \), the low ability rival cannot win control of firm 2 in the control contest when \( f \leq [1 - x_1(\pi_1(n, h), v_1)] \). Thus when \( P_f^2 \leq p_1, S \leq 2 \), and \( f \leq f_9 \equiv \text{Min}[f_6, (1 - x_1(\pi_1(n, h), v_1))] \), the incumbent is willing to relinquish control of firm 2 to the executive who will control firm 2 unless a high ability rival appears. The proof of part (ii) and (iii) is the same to the proofs of Proposition 1 to 6.

**Proof of Proposition 13.** When \( P_f^2 \leq p_1 \), the market values the joint firm at \( \beta[(\phi_1 + \phi_2)(lv_2 + v_1) + \phi_3(hv_2 + v_1)] + (1 - \beta)[\phi_2(lv_2 + v_1) + \phi_1(v_2 + v_1) + \phi_3(hv_2 + v_1)] \). After the announcement of spin-off plan, the market upgrades the valuation to \( (\phi_1 + \phi_2)(lv_2 + v_1) + \phi_3(hv_2 + v_1) \). After the take-over by a high ability rival, the market upgrades the valuation further to \( hv_2 + v_1 \).

**Proof of Proposition 14.** When \( f \in [f_1, f_6] \), the joint firm value is \( v_1 + v_2 + S \) if the board chooses not to spin-off; if the board chooses to spin off, the expected value of the combined firm equity at \( t = 0 \) is \( v_1 + (\phi_1 + \phi_2 h + \phi_3 l)v_2 \), \( \gamma[v_1 + \phi_2(h - 1)v_2 + (1 - \phi_2)\gamma v_2 + (1 - \gamma)[v_1 + (\phi_1 + \phi_2 h + \phi_3 l)v_2]] \), \( \gamma(v_1 + v_2) + (1 - \gamma)(dv_1 + v_2) \), \( \gamma(v_1 + v_2) + (1 - \gamma)[v_1 + (\phi_1 + \phi_2 h + \phi_3 l)v_2] \), \( \gamma(v_1 + v_2) + (1 - \gamma)(dv_1 + dv_2) \), or \( \gamma(v_1 + v_2) + (1 - \gamma)[dv_1 + ((1 - \phi_2)d + \phi_2 h)v_2] \). Thus, the board chooses to spin off when \( \gamma \leq \gamma_1 \equiv 1 - \frac{S}{(d - 1)v_1 + ((1 - \phi_2)(d - 1) + \phi_2 h - 1)v_2} \), and not to spin-off when \( \gamma \geq \gamma_2 \equiv 1 - \frac{S}{\phi_2(d - 1) + \phi_3(t - 1)v_2} \).

**Proof of Proposition 15.** (i) When the incumbent’s wealth is small, the announcement effect is positive following the same rationale in the proof of Proposition 9; and the consequence on the long-term equity return follows the same rationale in the proof of Proposition 10.

(ii) When the incumbent’s wealth is large, the value of the combined firm equity after a spin-off is \( v_1 + v_2 \), \( v_1 + (\phi_1 + \phi_2 h + \phi_3 l)v_2 \), or \( v_1 + (\phi_1 + \phi_2 h + \phi_3 )v_2 \), which is the same as or smaller than such value at \( t = 0 \). The long-term equity return is zero when \( P_f^2 > p_2 \) since no takeover occurs.
Board decides to spin off or not.

The incumbent comes to know the board’s decision privately; based on that, he decides on his effort, capital structure of firm(s), and his wealth allocation between firms’ equity (in case of spin-off).

The board’s decision is announced publicly; along with the spin-off plan (in case of spin-off).

If a rival has appeared, a control contest takes place. The outcome of the control contest becomes public.

Rival appears/does not appear; In case he appears, all characteristics of the rival become known; the rival strategically invests in the equity of the firm(s) set up by the incumbent. All the incumbent’s characteristics also become publicly known.

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Figure 1: Sequence of Events
Figure 2: Existence of Various Equilibria. This figure illustrates the parameter regions for the existence of various equilibria when the incumbent's wealth is small, with variations in the private benefit to the incumbent from firm 2 and the amount of the rival's wealth. The following values of the model parameters are used: $W_i$, the incumbent's wealth, is 100; $v_1$, the market value of firm 1 under the incumbent's management with a normal effort, is 160; $v_2$, the market value of firm 2 under the incumbent's management with a normal effort, is 120; $S$, the synergy benefit, is 8; $c$, the cost of effort when the incumbent works diligently, is 20; $d$, the ratio of the increase of the firm value when the incumbent works diligently, is 1.3; $h$, the ratio of the increase of the firm value when a high ability rival manages the firm, is 1.5; $I$, the ratio of the increase of the firm value when a low ability rival manages the firm, is 1.1; $\phi$, the probability of no rival, is 0.4; $\phi_1$, the probability of the existence of a high ability rival, is 0.3; $\phi_2$, the probability of the existence of a low ability rival, is 0.3; $\pi_{nh}$, the percentage of the passive investors voting for the incumbent when the incumbent works normally and a high ability rival appears, is 0.38; $\pi_n$, the percentage of the passive investors voting for the incumbent when the incumbent works normally and a low ability rival appears, is 0.42; $\pi_{lh}$, the percentage of the passive investors voting for the incumbent when the incumbent works diligently and a high ability rival appears, is 0.45; and $\pi_l$, the percentage of the passive investors voting for the incumbent when the incumbent works diligently and a low ability rival appears, is 0.6.