The determination of financial structure: the incentive-signalling approach

Stephen A. Ross
Professor of Economics and Finance
University of Pennsylvania

The Modigliani-Miller theorem on the irrelevancy of financial structure implicitly assumes that the market possesses full information about the activities of firms. If managers possess inside information, however, then the choice of a managerial incentive schedule and of a financial structure signals information to the market, and in competitive equilibrium the inferences drawn from the signals will be validated. One empirical implication of this theory is that in a cross section, the values of firms will rise with leverage, since increasing leverage increases the market's perception of value.

The central results of modern corporate finance, the Modigliani-Miller irrelevancy propositions, are summarized nicely in the following quotations:

... the market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate $\rho_k$ appropriate to its class (Modigliani and Miller, 1958);

and

... the current valuation is unaffected by differences in dividend payments in any future period and thus ... dividend policy is irrelevant for the determination of market prices, given investment policy (Miller and Modigliani, 1961).

The concept of a "risk class" has passed out of fashion with the subsequent refinement of these propositions, and they are now generally viewed as consequences of perfection and competition in financial markets. Perhaps the simplest proof of the proposition that the value of the firm is unaffected by financial structure is that if such changes, say in the debt-equity composition, lowered value, then by purchasing the firm (or a proportion of it) and reissuing the value maximizing financial package on personal account (or as a reformed corporate structure) individuals could realize an arbitrage profit. Since such profits are inconsistent with equilibrium, the value of the firm must be constant across all financial packages, or, to put it somewhat differently, in the "inferior" situation value would be bid up to the maximum. To make the point in an alternative fashion, if individuals can issue securities in the market just as firms—they have equal
access to the capital market—then they can "undo" any financial package issued by the firm to restore a given general equilibrium. In this sense nothing fundamental, e.g., firm value, can be altered by the firm's financing decisions. (See Fama for an elegant treatment of these approaches to the irrelevancy propositions, their derivation from perfect competition in financial markets, and their limitations.)

An unfortunate consequence of the Modigliani-Miller insights has been the discarding (and denial) of theories for determining the financial structure. If the theory is complete and thought to be correct, then structure is indeterminate or random in actuality, and this is a somewhat inhibiting basis on which to develop an explanation of financial structure.¹

One possible approach to the problem is to modify the Modigliani-Miller theory to take account of the structural features of the "real world." These form the basis of the traditional view of corporate finance. Since debt payments are excluded from income in computing corporate income tax, the value of the firm should increase with the substitution of debt for equity financing.² Unless, however, there are offsetting costs, this has the awkward implication that the firm should be wholly debt financed. If there are true bankruptcy costs, e.g., wastage from mismanagement, or direct costs associated with reorganization and discounts in secondary markets, then such costs would act to mitigate the amount of debt financing. An optimum ratio of debt to equity trades off the tax benefits of increased debt against the costs of an increased probability of ruin.³

But this is not an entirely satisfactory resolution of the matter either. On the one hand, it is difficult to specify exactly what the costs of bankruptcy are, particularly when it is in the interest of all parties to simply reorganize the firm.⁴ Even granting that such costs might be significant, this puts a large burden on the effect of the tax benefits. Furthermore, it is rebutted by the evidence. As Jensen and Meckling (1975) observe,

Since we know debt was commonly used prior to the existence of the current tax subsidies on interest payments this theory does not capture what must be some important determinants of the corporate financial structure.⁵

Another possibility, of course, is to take a closer look at the

¹ Of course, as the Modigliani and Miller theory argues, there may simply not be a significant role for finance, but then some explanation is required of the apparently irrational effort that corporations put into the financial decisionmaking process. One of the more sobering experiences for a student of finance is to explain the irrelevancy propositions to a corporate treasurer.

² It is sometimes said that the irrelevancy propositions are "untrue" in a world with taxation benefits, but there is a more symmetric view that can be taken. With taxation, the government becomes a claimant to the firm's returns. Altering the private financing package now leaves the total (nonmarket) value of the firm unaltered but can increase the private market value of the firm at the expense of the public share. We are, of course, ignoring the offsetting effects of differential rates of personal taxation on capital gains and dividends in this analysis.

³ If bankruptcy costs accrue to a third party then note 2 can be applied. Also, we should note that the Internal Revenue's view of an all debt firm is somewhat more complicated than a simple linear tax schedule would suggest. As the debt component is raised, there is an increasing chance that the debt service will lose its exemption.

⁴ Except, of course, lawyers who would be claimants in bankruptcy and, if they held debt, might oppose reorganization.

⁵ These authors develop a detailed theory of financial structure that emphasizes the costs of management. We take a view that essentially ignores such costs.
underpinnings of the Modigliani-Miller theory itself. If changes in the financial structure of the firm affect the consumption and investment opportunity sets open to economic agents, then the pivotal role played by value maximization in arbitrage arguments may have to be rejected. Leland, Ekern and Wilson, Radner and others have examined this possibility for activity choice in models with incomplete (marginal) spanning and suggested alternative behavioral rules such as the requirement of stockholder unanimity. This is equivalent, though, to assuming that firms have monopoly power in financial markets, and it is difficult to see a definitive theory emerging from such an inherently game theoretic and strategic situation. If pricing is complete and value maximization is the goal, then we will have to look elsewhere for a theory of the financial structure.

Implicit in the irrelevancy proposition is the assumption that the market knows the (random) return stream of the firm and values this stream to set the value of the firm. What is valued in the marketplace, however, is the perceived stream of returns for the firm. Putting the issue this way raises the possibility that changes in the financial structure can alter the market’s perception. In the old terminology of Modigliani and Miller, by changing its financial structure the firm alters its perceived risk class, even though the actual risk class remains unchanged.

In Section 2 we shall show in a simple example how this phenomenon can be linked with the managerial incentive structure to provide a theory that determines the financial structure and is consistent with the Modigliani-Miller framework. Section 3 describes the features of a general theory and Section 4 develops a somewhat more realistic model than that of Section 2. Section 5 summarizes the paper and considers some possible extensions and generalizations.

This section constructs a simple example that illustrates the relationships between signalling and the managerial incentive structure in the financial market. Throughout the paper we will make the usual perfect market assumptions.

Assumption 0: Financial markets are competitive and perfect with no transaction costs or tax effects.

As a consequence, no individual or firm exercises monopoly power in the financial markets and each participant acts as if demand were infinitely elastic at the quoted prices.

Suppose that the market, or the relevant corner of the market, consists of two types of firms, A and B. It is currently time 0 and at time 1, A firms will have a total return (value) of \( a \) and B firms will...
return $b$ with

$$a > b.$$ 

For additional simplicity, we will also assume that pricing in the market is risk neutral. Hence, if riskless bonds are traded, then assets will be valued at their expected discounted value. Risk neutrality can be justified at a more primitive level by simply assuming that investors are risk neutral, but it is also possible to base it on the assumption that this sector of the market is small and independent of the overall financial market. Alternatively, we could argue that there are sufficiently large number of firms of each type as to enable individuals to diversify away firm risk. The assumption of risk neutrality, though, is not essential; it is made largely to avoid complicating the model with the additional effects of risk-sensitive pricing.

☐ **A certain world.** If there is no uncertainty in the market and investors can identify the $A$ and the $B$ firms, then their respective values at time 0 will be given simply by

$$V_0^A = \frac{a}{1 + r},$$

(1a)

and

$$V_0^B = \frac{b}{1 + r} < V_0^A,$$ 

(1b)

where $r$ is the sure rate of interest.

There is little more that needs to be said about this case. It should be clear that the valuations in (1) are unaffected by the mode of financing chosen by the firm. For example, suppose the type $A$ firm is financed by debt, $D$, with a face value of $F$, and equity, $E$. The debt is the senior claimant to the firm's returns and will have a value of

$$\min \{F, a\},$$

at time 1 and the equity will claim the residual

$$\max \{a - F, 0\}.$$ 

The time 0 values respectively will be

$$E = \frac{\max \{a - F, 0\}}{1 + r}$$

and

$$D = \frac{\min \{a, F\}}{1 + r},$$

and, therefore

$$E + D = \frac{a}{1 + r} = V_0^A.$$ 

In this simple world, the Modigliani-Miller theorem is really just a restatement of Fisher's separation theorem.

☐ **Uncertainty and signalling—an irrelevance result.** Suppose, now, that investors cannot distinguish $A$ firms from $B$ firms. If $q$ is the proportion of $A$ firms, suppose, too, that investors all act as though
any firm has a $q$ chance of being an $A$ firm. The returns, $a$ and $b$, are conditional on the current exogenous information. Given the available information at time 0, firms in the model have a $q$ chance of being type $A$ and a $(1 - q)$ chance of being type $B$. It follows that all firms will have the same value,

$$V_0 = \frac{qa + (1 - q)b}{1 + r},$$

with

$$V_0^A > V_0 > V_0^B.$$  

This result is quite robust, and it follows directly from the Modigliani-Miller propositions that valuation will be unaffected by the mode of financing.

For example, it would be ineffective for $A$ firms to attempt to inform the market, or signal that they were of type $A$ rather than $B$. The difficulty is the moral hazard that $B$ firms would give the same false signal, once again leaving the equilibrium one where firms cannot be discriminated. Suppose that there were some activity, $X^A$, perhaps a financial package, that $A$ firms could engage in and that investors would observe and use to infer a value $V_0(X^A)$ for the firm. If $V_0(X^A) > V_0$, then (by the initial symmetry) $B$ firms would also engage in $X^A$ and realize the initial value $V_0(X^A)$. In equilibrium, we must have $V_0(X^A) = V_0$. Equivalently, if a $B$ firm were to follow a policy $X^B$ for which $V_0(X^B) > V_0(X^A)$, then by purchasing the firm for $V_0(X^B)$ and using the activity $X^A$ (e.g., refinancing it), a financier would realize a riskless capital gain of $V_0(X^A) - V_0(X^B)$.

This concept of signalling was first studied in the context of job and product markets by Akerlof and Arrow, and was developed into an equilibrium theory by Spence. It has been subsequently examined (with emphasis on the possible lack of equilibrium) in different problems by Rothschild and Stiglitz and by Riley. The joining of Modigliani-Miller arguments with moral hazard, however, does not seem to leave much room for signalling in financial markets. If the chosen mode of signalling is the financial structure, then since finance is costless, the market valuation function $V_0(\cdot)$ will be the same for $A$ and $B$ firms and, as we have seen, the only equilibrium will be where

$$V_0^A = V_0^B.$$  

This result will hold in a very general setting and is not dependent on the special assumptions of the simple model. Even when the implications of financing decisions differ for the $A$ and $B$ firms, any such consequences, e.g., true bankruptcy costs, are realized at time 1. A financier who can buy a $B$ firm more cheaply than an $A$ firm would simply reissue the $A$ financial package and avoid any of the time 1 consequences. The only time 0 equilibrium, then, is one where (4) holds and firms have identical values.

**A managerial incentive-signalling equilibrium.** One way to break out of the constraint that binds the value of $A$ and $B$ firms is to assume a significant role for the manager. If the manager of a firm is accountable for time 0 decisions, then there is a means of validating financial signals and avoiding the moral hazard problem. Of course, as well as being accountable the manager must also be assumed to have special
or inside information about the firm's type. It would do little good to make managers bear the consequences of their decisions if they had the same information as ordinary investors. The following assumption summarizes these points.

Assumption 1: Manager-insiders are identified with firms as possessors of inside information. Furthermore, refinancing by outsiders conveys no information to the market.

In the simple model of this section we will assume that managers know their own firm's type, but have no inside information about firms other than their own. Refinancing by outsiders, including other managers, will be assumed to convey no information, i.e., it will not alter the market's perception of the firm's type. In addition to identifying a role for managers, we also have to specify exactly how they share in the consequences of their decisions.

Assumption 2: Manager-insiders are compensated by a known incentive schedule, i.e., a given rule which investors know.

Suppose that $F$ denotes the face value of the debt issued by a firm at time 0. In our model we shall assume that managers receive the following compensation

\[ M = (1 + r) \gamma_0 V_0 + \gamma_1 \left\{ \begin{array}{ll}
V_1 & \text{if } V_1 \geq F \\
V_1 - L & \text{if } V_1 < F
\end{array} \right\}, \tag{5} \]

where $V_0$ and $V_1$ are the respective values of the firm at time 0 and at time 1, and $L$ is a penalty assessed on the manager if the firm is bankrupt at time 1, i.e., if $V_1 < F$ and the value cannot cover the debt repayment. The constants $\gamma_0$ and $\gamma_1$ are fixed nonnegative weights.

We shall also suppose that manager-insiders actually act to maximize their incentive compensation, $M$, in (5). This implies, of course, that they will set a level of debt financing, $F$, at time 0 so as to maximize $M$. Since in the example there is no productive activity choice available, $F$ is, in fact, the only decision variable at the manager's discretion. The penalty, $L$, associated with bankruptcy is a penalty imposed on the manager and does not necessarily represent any true bankruptcy cost to the firm, but if there were any such costs, then they would fit into the incentive schedule through the penalty in a very natural fashion.

Given (5) and Assumption (2), the activities of the manager are circumscribed in a number of ways. In particular, the manager cannot trade in the financial instruments issued by his own firm. If the manager were to do so, then the incentive schedule would not be given by (5). Legal rules designed to prevent managers from trading in their own firm's liabilities are generally motivated by the desire to avoid moral hazard problems, but one consequence of such avoidance is a clearer specification of the managerial incentive structure. Disclosure rules on insider trading also serve the function of clarifying the managerial incentive schedule and make it easier for investors to "read" the financial signals. (See Ross, 1976b.)

We can use (5) to establish a signalling equilibrium, in the sense introduced by Spence, in the financial market. Suppose that investors use $F$, the face value of the debt, as a signal of the firm's type. Let $F^*$ be a critical level of financing, with

\[ b \leq F^* < a. \]
If 

\[ F > F^*, \]

then we shall assume that the market perceives the firm to be of type A, and if 

\[ F \leq F^*, \]

the market perceives the firm to be of type B. For this to be an equilibrium we must show that investors’ perceptions are accurate, i.e., all firms of type A must actually issue debt with \( F_A > F^* \) and all the type B firms must set \( F_B \leq F^* \).

If a firm signals itself to be of type A and if it also sets \( F_A \leq a \), so that it does not risk bankruptcy unnecessarily, then

\[
V_0 = V_0(F_A) = \frac{a}{1 + r}. \tag{6}
\]

Similarly a firm that gives a type B signal by setting \( F_B \leq b \) will have an initial value of

\[
V_0(F_B) = \frac{b}{1 + r}. \tag{7}
\]

The compensation of the manager of a type A firm, then, will be given by

\[
M^A(F) = \begin{cases} 
(\gamma_0 + \gamma_1)a & \text{if } F^* < F_A \leq a, \\
\gamma_0b + \gamma_1a & \text{if } F_A \leq F^*. 
\end{cases} \tag{8}
\]

The compensation of the manager of a type B firm is given by

\[
M^B(F) = \begin{cases} 
\gamma_0a + \gamma_1(b - L) & \text{if } F_B > F^*, \\
\gamma_0b + \gamma_1b & \text{if } F_B \leq b \leq F^*. 
\end{cases} \tag{9}
\]

where we have assumed that the manager will not expose himself to bankruptcy cost unnecessarily.

Suppose, now, that the A managers choose financing levels, \( F_A, F^* < F_A \leq a \), \( (10) \), and the B managers choose \( F_B, \)

\[
F_B \leq b. \tag{11}
\]

This is a signalling equilibrium in the sense defined by Spence if neither type of manager has an incentive to change signals and if, in addition, the signals are valid, that is, the inference drawn from the signal by the market is correct. Now, an A manager will have no incentive to change since, from (10), for \( F' \leq F^* \),

\[
M^A(F') = (\gamma_0 + \gamma_1)a > M(F) = \gamma_0b + \gamma_1a. \tag{12}
\]

With no bankruptcy costs being incurred, the manager will obviously give a signal that the firm is of type A.

Less obviously, the B type manager may not have an incentive to falsely signal that his firm is of type A. This requires that for \( F' > F^* \)

\[
M^B(F') = \gamma_0a + \gamma_1(b - L) < M^B(F_B) \]

\[
= (\gamma_0 + \gamma_1)b,
\]

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This is a very sensible result. The $B$ manager signals truthfully if the marginal gain to a false signal $(a - b)$ weighted by the manager’s share, $\gamma_0$, is less than the bankruptcy costs incurred, $L$, again weighted by the manager’s share, $\gamma_1$.

This equilibrium is illustrated in Figure 1. There are, in fact, many equilibrium values $(F^A, F^B)$. Any pair with

$$F^B \leq b,$$

and

$$b < F^A \leq a,$$

will do, and there is no reason to expect that one pair will be chosen uniquely (or that all firms of the same type will choose the same level of debt). This result, however, is a feature of the simple structure of the model and will disappear in a more complex setting.

It is worth examining the criterion (13) with care. If $\gamma_1 = 0$, then managers, like investors, will maximize time 0 value. Since they share no consequences of next period’s performance, they will be unable to signal and the equilibrium will degenerate into that described in (b) above. If $\gamma_0 = 0$, then (13) is satisfied for $\gamma_1 L > 0$, but oddly we may not have an equilibrium. In this case, $B$ managers will not falsely signal, but since $A$ managers do not care about the firm’s valuation at time 0, they also have no incentive to signal correctly to differentiate their firms from firms of type $B$. What occurs if the inequality in (13) is reversed? Now matters become somewhat more complex and the possibility that there is no equilibrium opens up. This case can be treated, but as the next section emphasizes, the incentive schedule $M$ is itself determined in equilibrium, and as a consequence, this particular problem does not arise in the financial incentive-signalling model.

It should also be clear now why equilibrium requires that managers be precluded from trading in their own instruments. An $A$ manager, for example, could raise his compensation by falsely giving a $B$ type signal and then purchasing his own stock, or a $B$ type could give a false $A$ signal and short his own stock. (Disclosure rules,
though, enable the market to use such insider trading as a signal.)

Another difference between the financial incentive-signalling model and the job signalling models is that there is no implication of inefficiency in the resulting financial equilibrium. Unlike the job signalling models where the investment in a signal is costly and, therefore, inefficient if signals serve no productive function, financial signalling is relatively costless. If \( L \) is only an incentive cost and there are no true costs to bankruptcy, this result will be quite general. In this simple example, with certain returns, even if \( L \) represents true bankruptcy costs, it is the prohibitive potential cost that validates the signal and in equilibrium neither type of firm must inefficiently risk bankruptcy to signal validly.

\[ \text{Setting the managerial incentive schedule.} \] Who establishes (5) as the incentive schedule for managers? In one sense this is similar to (and about as embarrassing as) asking who sets the price in a competitive market, but consider the following argument.\(^7\) Suppose that the opportunity cost or wage of individuals who might supply their services as manager-insiders is \( w \). If the managerial supply is perfectly elastic at \( w \), and if managers are randomly assigned to firms with no \textit{a priori} knowledge of firm type, then they will demand an expected wage equal to \( w \). Once assigned to a firm, the manager learns its type and is locked to it.

Suppose, then, that managers are all given a perfectly proportional wage schedule

\[
(1+r) \gamma_0 V_0 + \gamma_1 V_1, \quad (15)
\]

where (positive) \( \gamma_0 \) and \( \gamma_1 \) are chosen so that

\[
\gamma_0 E[V_0] + \gamma_1 E[V_1] = w, \quad (16)
\]

i.e., the expected compensation equals the wage. Managers assigned to an \( A \) firm will, by (15), find it in their interest to signal the market that their firm is of type \( A \). To do so the manager can simply alter (15) to (5) by issuing debt of face value \( F > b \) and announcing that he will be liable for a penalty of \( L \), satisfying (13) if bankruptcy occurs.

Managers of type \( B \) firms would then find their firms identified as such and, by the analysis above, with \( L \) sufficiently high they would have no incentive to follow the type \( A \) managers and signal falsely. Furthermore, since (5) is uniformly less than (15) all holders of equity in the firm would unanimously favor such a change. (In addition, the total compensation of the managerial class is unaltered in the new equilibrium.) Given (15), then, an incentive schedule like (5) (with similar implications) would arise in equilibrium. Or, to put the matter somewhat differently, (15) is not an equilibrium incentive schedule, and (5) is.

There are, of course, other alternatives to financial signalling. Without issuing any debt, for example, the \( A \) type manager could simply assert his liability of \( L \) if return falls below \( a \). I cannot claim to have eliminated such possibilities, but finance has at least one important advantage over the simple announcement. By using debt the manager creates an instrument which is priced in the market and returns on which are realized at time 1. The manager’s compensation

\[ ^7 \text{A colleague suggested, not entirely facetiously, that the financial analysts did it.} \]
schedule, (5), is now equivalent to a financial claim on the firm's returns, or, more precisely, a derivative security written on the equity or bonds of the firm (see Ross, 1976a). Since the manager's compensation is now directly dependent on the value of the firm's financial claims in the market and equivalent to a package of marketed claims, it is relatively simple to monitor and enforce (5) as the manager's true compensation. Implicit in the argument, of course, is the assumption that this is cheaper than monitoring and bonding an *ad hoc* assertion of liability.

In addition, while the analysis above provides a more detailed justification for the emergence of an incentive structure such as (5) in equilibrium, it remains to explain why one such as (15) was established *a priori*. What is at issue here is not the uniqueness of (5) or (15). Obviously these are not unique and are important only in terms of the incentives they give; any penalty structure on bankruptcy satisfying a condition analogous to (13) would do. A schedule like (5) would, I think, emerge from some additional features that are missing from the simple model. If the manager makes activity choices, then some incentive scheme will be required irrespective of signalling needs. Furthermore, the two-period model misses the ongoing nature of the relationship which would act to stabilize the incentive structure over time. There is no reason to believe that the resulting schedule would be linear, but, except for discount factors, it should not change form over time.

To summarize the basic argument of this section, given an *a priori* incentive structure, the type A manager has a further incentive to modify it in such a fashion as to permit him to differentiate his firm through the financial package. With more types than two, type B managers then respond in such a way as to separate their firms from type C firms, given the constraints imposed by the financing-incentive schedule adopted by the A managers, and so on down the hierarchy. We shall consider a more complex example of this below, but first it might be valuable to examine some of these issues in a more general setting.

### 3. A general structure

One value of trying to put the analysis of Section 2 into a more general framework is that it forces us to think carefully about the essential features of the example. We shall stay within a two-period context, although by thinking of second period returns as discounted future values this limitation is less restrictive than might appear. (This does, however, impose a myopia condition on investors.)

The compensation of the manager, \( M \), consists of two components, a time 0 compensation,

\[
M_0(f),
\]

which is a function solely of the financial claims, \( f \), which he holds against the firm, and the time 1 returns on the claims themselves. Generally, the current compensation, \( M_0 \), is actually a composite function of the form

\[
M_0(a(f), f),
\]

where \( a(f) \) is the market's subjective perception of firm type based
upon the financial package issued by the firm. Total compensation for a type $t$ firm is then given by

$$M' = M_0(f) + E[f(x_t)], \quad (17)$$

where $x_t$ is the random time 1 return of the type $t$ firm.

By valuing the incentive claim according to its expectation, we have ignored the preference structure of the manager and considerations of his reaction to uncertainty. This is an important omission since Assumption 2 effectively precludes the manager from participating in the market in the firm’s claims. If the manager’s risk aversion affects his evaluation of the incentive schedule, this will alter the manager’s evaluation of differing financial structures, and without a complete knowledge of the preference structure, the effect of a particular incentive schedule on financial choice will be indeterminate.

There is a large literature on such incentive problems, under various headings, but it generally assumes a knowledge of the manager’s von Neumann-Morgenstern utility function in setting the motivational schedule. We could consider incentive schedules designed to induce the manager to reveal his utility function, but such procedures are difficult to implement. Alternatively, it might be possible to assume a probabilistic knowledge and to develop a theory on this basis, perhaps allowing the manager to signal preferences as well as firm characteristics, but this would take us well beyond the scope of the present paper. We shall use (17), then, and justify it on the grounds that the variable compensation given by $M$ is small relative to the manager’s wealth.8

The final requirement is that of specifying the feasible set, $A$: of admissible financial packages and incentive schedules. One way to do this was suggested in the discussion at the end of Section 2. For example, $A$ could denote the set of incentive schedules satisfying a constraint of the form of (16) on the ex ante expected return of the manager. The manager would now be free to choose any financial package $f$ that satisfied such a constraint. The term feasible thus refers either to $f$ or $M$, and as it is always possible to scale the equilibrium schedules in such a way that the manager’s actual compensation is arbitrary, generally feasibility will be implicit in the limited financial instruments we permit.

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8 Suppose that the incentive compensation considered above is a small portion of the manager’s total compensation. If $W$ denotes the manager’s total wealth and $U$ his utility function, then

$$E[\{W + M\}] \equiv U(W) + U'(W)E[M], \quad (i)$$

and the manager acts to maximize expected compensation. Among its other virtues, this simple procedure requires no knowledge of the manager’s risk preferences.

In practice managerial bonuses are generally small compared to total wealth, but even to the extent that incentive schemes provide a large portion of the manager’s compensation, what is relevant in the approximation of (i) is the variable portion that is influenced by the financial decision. A major executive in the automobile industry might receive a yearly bonus amounting to half of his total compensation, but of the total bonus, the principal portion is determined by the influence of industry effects and overall economic conditions, and the variable portion influenced by the manager’s financial decisions is probably fairly small. In the jargon of capital market theory, the bonus is primarily determined by the firm’s betas with the market and the industry and only secondarily by the manager’s financial choices. (For those who feel that executive salaries strain the credulity of managerial productivity theory, this provides an alternate—equally plausible—explanation. Executives receive large salaries to eliminate the need for evaluating their risk preferences.)
The following definition of an equilibrium coincides with that used in the example of Section 2.

**Definition:** A financial incentive-signalling equilibrium is a vector $(M_0, a, f^t, \ldots, f^t)$ such that for all types $t$, $M'(f^t)$ is feasible,

\[(i) \quad M'(f^t) \geq M'(f), \quad \text{for all feasible } f, \quad (18)\]

and

\[(ii) \quad a(f^t) = t. \quad (19)\]

Part (i) of the definition specifies $f^t$ as the financial package chosen by firm $t$, given the incentive function $M$ and the market signalling function, $a(f)$, and part (ii) is Spence's rationality criterion that the signal be valid, i.e., that a type $t$ firm give a type $t$ signal. Notice that in contrast with job market signalling models, the incentive function, $M$, as well as the signalling mechanism, $a$, is an object of equilibrium. Also, as argued in Section 2, it will be assumed that $f^t$ is formed from the marketed instruments of the firm to facilitate valuation and therefore monitoring and enforcement. This will link the equilibrium compensation schedule with the firm's financial structure.

To prove the existence of an incentive-signalling equilibrium is not a straightforward task and we shall not take it up here. In fact, though, as the previous example illustrates, there may be a multiplicity of equilibria. Given the incentive schedule, however, the possible distinct modes of financing are limited. While there are a plethora of different financial instruments, what matters is the set of incentive returns they yield. To be concrete about this, suppose that $M$ is given as in the example of Section 2 by (5). By altering the package of financial instruments that are issued, a firm can arbitrarily affect the probability of bankruptcy, but by simply issuing debt the firm can achieve the same range of possible incentive returns, i.e., values for (5). In other words, given the incentive schedule of (5), debt and equity constitute a sufficient financial package and the incentive effects of any other package will be equivalent to those obtained by some package of debt and equity. More generally, still, in the example the signalling implications of any incentive-financial structure pair $(a, f^t)$ will be matched by a particular pair in (5) using debt and equity. (This phenomenon of redundancy is identical to that studied by Ross, 1976a.)

In conclusion, the manager of a firm maximizes his incentive return by choosing a financial package that trades off the current value of the signal given to the market against the incentive consequences on that return. In equilibrium, firms are correctly distinguished by their financial choices. What matters, though, is not the particular package chosen, but rather the essential characteristics of the financial package, i.e., its implications for incentives. Many seemingly distinct financial packages may actually have the same incentive properties.

### 4. A model with random returns

**The model.** The two-firm model of Section 2 can be generalized by simply adding types, but one feature that makes it particularly unacceptable is the assumption that managers know the returns of their own firms with certainty. By making returns uncertain, we shall be able to create a natural incentive, analogous to bankruptcy risk, for
managers to limit their debt financing to as low a value as is consistent with signalling. This will provide a clearer example of the tradeoff between signalling and incentive consequences described in Section 3.

Once again we shall assume that only expected values matter for valuation. Suppose that firms have random returns, \( X \), uniformly distributed on \([0, t]\) where manager-insiders know their own firm’s \( t \) value and there are a continuum of types \( t \in [c, d] \). If we adopt the incentive schedule

\[
M = (1+r) \gamma_0 V_0 + \gamma_1 E \left\{ \begin{array}{ll}
X & \text{if } X \geq F \\
X-L & \text{if } X < F
\end{array} \right.,
\]

where \( F \) is, again, the face value of debt, then debt and equity will be sufficient instruments in the sense of Section 3.

If the type, \( t \), of a firm is known for sure, then its current value will be given by

\[
V_0 = \frac{t}{2(1+r)},
\]

hence if the signal of type is \( a(F) \), from (20)

\[
M = \frac{1}{2} \gamma_0 a(F) + \gamma_1 \left[ \frac{1}{2} t - L \frac{F}{t} \right],
\]

where we have assumed that \( F \leq t \). To find the optimum financial level, we assume the maximum is internal, and differentiating (22) with respect to \( F \) yields

\[
\frac{1}{2} \gamma_0 a'(F) = \gamma_1 \frac{L}{t}.
\]

Condition (19) for signal validity requires that

\[
a(F) = t.
\]

Conditions (23) and (24) permit us to solve for the equilibrium structure. Recalling that \( F \) is a function of \( t \) in equilibrium, we can differentiate (24) and combine it with (23) to obtain the differential equation

\[
F' = \frac{1}{2} \frac{\gamma_0}{\gamma_1} \frac{t}{L},
\]

whose solution is

\[
F_t = \frac{\gamma_0}{4 \gamma_1} \frac{t^2}{L} + b,
\]

where \( b \) is a constant of integration. From (24), then

\[
a(F) = 2 \sqrt{\frac{\gamma_1 L}{\gamma_0}} [F - b]^{1/2}.
\]

Since the lowest firm, with \( t = c \), will clearly gain no advantage from signalling, it will set

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9 The distribution, \( Q(t) \), on the continuum is the probability distribution of types, \( t \), given the exogenous information at time 0. For each firm in this sample, \( Q(t) \) would be the probability that the firm has a type less than or equal to \( t \). For example, if time 1 returns are based on an econometric forecast, \( Q(t) \) would be that forecast for all firms in the model. Firms with different forecasts would be lumped into different samples.

10 Conditions (23) and (24) are, in fact, formally identical to those of a model that Spence has studied. We include the solution for completeness only.
\[ F_e = 0, \]

which implies that
\[ F_t = \frac{\gamma_0}{4\gamma_1 L} [t^2 - c^2]. \tag{28} \]

We must have
\[ d^2 - c^2 \leq \frac{4\gamma_1 L}{\gamma_0} d, \tag{29} \]

to insure that the required financing level will not exceed \( t \). As in the simple example, (28) implies that no manager will wish to give a false signal as long as the probability of bankruptcy is not one. By direct comparison, we also set
\[ \frac{1}{2} \gamma_0 (d - c) \leq \gamma_1 L, \tag{30} \]

which is precisely the form of (13), and implies (29).

Now, if the set of incentive schedules available to managers is given by (20) with \( L \) as a parameter, a manager of a type \( d \) firm will choose a particular incentive structure by setting \( L \) at \( L^* \) where (30) is an equality. This will just permit full discrimination and any greater \( L \) would needlessly increase bankruptcy incentive costs.\(^{11}\) This completes the conditions for an equilibrium incentive-signalling pair.

As with the previous examples, the Modigliani-Miller irrelevancy theorem holds within a risk class, i.e., given \( t \), value is determined by (21) independent of the financial structure. But, by changing \( F \) the manager-insider alters the market’s perception of the firm’s risk class (or type) and therefore its current value changes with \( F \) according to (27) and, by (28), there is a unique optimum level of financing for each firm type.

□ **Some empirical implications.** Let us look at some variables whose values are often cited as tests of the Modigliani-Miller theory.

\(^{11}\) We are assuming that managers do not falsely signal when there is no positive incentive to do so and, hence, there is still discrimination when (30) is set on equality. We are also assuming that \( d \) is sufficiently great so that a \( c \) type manager must bankrupt with certainty to signal type \( d \). Note that there is nothing inherent in the problem to assure us that (28) is a unique equilibrium.

If the density of low value firms in the sample is sufficiently small, then there might be another equilibrium with no firms issuing debt. In this case the difference between the average value and the highest value is small, and the cost to a manager of differentiating a firm would be prohibitive. Nevertheless, (28) still describes an equilibrium. While it is to the advantage of the managers of the high return firms, acting as a group, to cut their financing level, any single manager doing so would find that his firm value dilutes down to the lower value. (Of course, if low value firms were in elastic supply, with free entry, this problem would disappear.)

This is the same sort of structure that arises in the nonequilibrium examples of Rothschild and Stiglitz, and the financial-signalling problem would be subject to the same difficulties if a mutual fund or financial intermediary could make discriminating offers to the market as a whole, pooling intragroup risks. These matters are still unresolved and are further complicated in our setting by the endogenous nature of the incentive schedule itself, and while they are of interest to us, we shall not pursue them here. As a conjecture, though, I suspect that the problems of nonequilibrium in these models are identical to those that arise in the two-person game with nonconvex strategy spaces. By augmenting the strategy spaces available to the firms, specifically by considering randomized strategies, definitive equilibria might be found.
The cost of capital. One way to define the (average) cost of capital is by the ratio

$$\rho = \frac{E\{X\}}{V_0}.$$  \hfill (31)

Now, irrespective of financing,

$$E\{X\} = \frac{1}{2} t,$$  \hfill (32)

and if we are in an incentive signalling equilibrium, the financial package, $F$, will signal type $t$ correctly and the relevant value, $V_0$, for (31) will be

$$V_0 = \frac{\frac{1}{2} t}{1 + r}.$$  \hfill (33)

This implies, of course, that

$$\rho = 1 + r,$$  \hfill (34)

for all firms.

In cross section, then, the "cost of capital" will be unaffected by the financing decision, even though the level of debt is uniquely determined.

Bankruptcy or incentive "L" risk. It is also possible to determine how the risk of bankruptcy or having an incentive loss, $L$, varies with the level of debt. Since $X$ is uniformly distributed, from (28)

$$P \Prob\{X < F|t\} = \frac{F}{t} = \frac{\gamma_0}{4\gamma_1 L} \left[ t - \frac{C^2}{t} \right].$$  \hfill (35)

The bankruptcy risk, $P$, is an increasing function of firm type, $t$, and equivalently of the debt level $F$. That the risk should increase with $F$ is in accord with traditional theory, but that it should also increase with $t$ may seem counterintuitive. In fact, though, it is because increasing debt brings greater risks that this can be taken as a valid signal of a more productive firm. The traditional view that higher debt lowers firm value may be correct with true dead weight bankruptcy costs for a given firm, but in equilibrium exactly the opposite is true in a cross section of firms.

The financial ratios. Defining the current value of debt and equity as $D$ and $E$, it is easily shown that

$$D = \frac{F}{1 + r} \left[ 1 - \frac{F}{2t} \right],$$

$$E = \frac{1}{1 + r} \left[ \frac{t}{2} - F + \frac{F^2}{2t} \right],$$  \hfill (36)

and

$$V_0 = D + E = \frac{1}{1 + r} \left[ \frac{t}{2} \right].$$

Differentiating (36) verifies that as $t$ and, equivalently, $V$ increase, $D/E$ increases as well. In other words, in a cross section, value increases as the debt-equity ratio rises.

Despite these somewhat paradoxical results, a great deal of care must be taken in actual empirical testing. It must be stressed, first,
that the firms in the model are all ex ante identical in that on the basis of exogenous information they each have the same probability of belonging to any given type. A corner garage cannot signal that it is General Motors simply by raising its debt-equity ratio. Furthermore, the above comparative statics hold at time 0. In a continuous time model it is certainly the case that for a given firm, with true bankruptcy costs, value will fall with increases in the debt-equity ratio; in fact $D/E$ must approach infinity as bankruptcy is approached (in a diffusion model). Even without such costs, $D/E$ and $V$ will move in opposite directions. These effects will tend to counter the initial incentive-signalling effects and may make empirical testing more difficult.

5. Generalizations, extensions and conclusions

Considerable work remains, though, before the incentive-signalling model is in a form suitable for empirical testing. First, the model should be generalized to incorporate the possibility of activity choice by the manager. In the examples, the returns of the firm were specified exogenously to the manager. More generally, the manager faces a production problem as well as a financial decision and must choose an optimal activity from a given production set according to an endogenously determined investment criterion. The interplay between the incentive schedule and activity choice produces some interesting results, but in a perfect market even with incentive-signalling phenomena some Fisherian separation results should hold as well.

The introduction of activity choice also imposes a need for a more general treatment of uncertainty in the model. If we retain a competitive financial market, then the valuation rule will remain linear, but in either a state-space framework or a $k$-parameter theory, the tradeoff between return and risk will influence value. This, in turn, will affect both the equilibrium incentive structure and the resulting activity and financing choices. A number of these extensions are studied in Ross (1976b).

A third requirement is to specify the model intertemporally, and in continuous time. This would not only provide a natural setting for empirical work, but it would also draw on the current work on option pricing theory. For example, an incentive schedule that gave the manager a stock holding could be priced directly, as a function of the current value of the firm, $V_0$, from the existing literature on pricing call options. Similarly, an option to buy stock could also be priced as a function of $V_0$ by considering the option as an option on the firm.  

Finally, of course, for empirical work the effects of taxation and true bankruptcy costs must be considered.  

12 I am grateful to John Cox and Mark Rubinstein for pointing out this latter possibility to me.  

13 The model should also be extended to consider problems in personal or small firm finance as well as corporate finance. In such problems the manager is an owner, and questions of managerial risk aversion become significant. Furthermore, we cannot assume that the manager's compensation is small relative to the firm's value. Much of our intuition about finance appears to be derived from analysis at this level, where the severity of the moral hazard problem is dominant and enforcement and monitoring costs become significant. In an independent paper, Leland and Pyle have
Nevertheless, even without these extensions, the simple incentive-signalling model developed in this paper provides a theory for the determination of the financial structure of the firm. The assumptions of perfection and competition in financial markets underlie not only the Modigliani-Miller irrelevancy theory, but also the capital asset pricing models and the option pricing literature. If we must drop these assumptions to build a more realistic theory of corporate finance, then we should also be prepared to develop pricing theories in imperfect markets. The incentive-signalling model, though, provides a role for corporate finance within the framework that supports both the pricing theories and the Modigliani-Miller theory.

References


analyzed some of these issues in a model where an entrepreneur is seeking to finance a project. It is also necessary to consider the role of financial intermediaries and structural changes in the incentive signalling model. For example, in the model of Section 4, managers, while not initially risk averse, may adopt a risk averse equilibrium schedule. Such a schedule would provide incentives for managers to merge firms, and it would also open up a role for mutual funds to pool managerial risk. (See note 11 for a glimpse of some of the problems with such analyses, though.)

Gerald Jaynes has also suggested that the incentive-signalling apparatus might have implications for modelling a wide variety of economic phenomena involving general producer warranties and guarantees (that are not intrinsically financial). In the realm of purely financial signals, dividend payments, warrant offerings, earnings retention rates and all manners of financial relations between the firm and the market have signalling content.