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Managing Posterior Price Matching: The Role of Customer Boundedly Rational Expectations

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1. Introduction

Markdowns are commonly observed and have become a decisive issue in retailing (Lai et al. 2010). It is reported that price discounts due to markdowns by U.S. retailers amount to $200 billion a year (Rice et al. 2014). Why should sellers mark down? A markdown strategy provides a seller with the flexibility to reach buyers with different valuations, i.e., to price discriminate, and is a means of progressively reducing its inventory. Hence, one would expect that in a market where consumers are homogeneous (i.e., no room for price discrimination) and there is no inventory to reduce, a monopolist seller should be recommended not to mark down since markdown is costly (Rice et al. 2014). In this paper, we propose a novel mechanism based on customer boundedly rational expectations, in which the firm can intentionally create markdowns and use posterior price matching to make a profit, which we coin as “probabilistic markdown,” even in the absence of all the contributing factors studied in the extensive literature. Moreover, we provide practical guidelines on how to dynamically manage probabilistic markdowns in repeated selling seasons when customers are boundedly rational.

This widespread markdown phenomenon has trained consumers to be “strategic” and take advantage of markdowns (Lai et al. 2010). They are forward-looking in determining the best time to buy. They have to trade off between buying now at a high price versus waiting for the potential markdown at a low price. This is arguably a very difficult decision to make, although recent advanced information technology presumably makes it easier. Firms have been constantly trying to “perfect the science of the markdown” (Merrick 2001) by using sophisticated new software programs. Consumers, however, typically interact less frequently with the market and have a more limited understanding of the market than firms. In particular, it is extremely challenging (if not impossible) for consumers to know

Marketing, economics, and operations management literature. Moreover, we provide practical guidelines on how to dynamically manage probabilistic markdowns in repeated selling seasons when customers are boundedly rational.

Abstract. The posterior price-matching policy, whereby a firm promises to reimburse the price difference to a customer who purchases a product before the firm marks it down, has been used in practice. The extensive literature has offered the following explanations for why posterior price matching is adopted: to reduce inventory, to soften competition, to price discriminate consumers, and to eliminate consumer strategic waiting incentives. In this paper, we provide a novel explanation and investigate the role of consumer bounded rationality in the sense of anecdotal reasoning. We adopt a simple model that allows us to isolate the role of customer bounded rationality on using posterior price matching. We demonstrate that while it is never optimal to adopt posterior price matching when consumers have rational expectations, it can be optimal when they have boundedly rational expectations. We show when and how a seller can intentionally mark down with some probability and adopt price matching to make a profit. Ignoring customer bounded rationality can result in a significant profit loss. Then, we build a dynamic programming model to investigate how the firm should dynamically manage its markdowns over the long run. We show that a cyclic policy switching between a high and low markdown probability is typically optimal for exploiting customer bounded rationality. We characterize the nature of the cyclic policy and the range in which it is optimal. Our findings underscore the importance of consumer bounded rationality and provide managerial and practical guidelines on how to manage price matching when customers are boundedly rational.

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exactly whether or how likely markdowns will occur. There are numerous anecdotes of consumers who purchased a product at regular price but regretted it when the product was later marked down (Özer and Zheng 2016), or of consumers who waited a long time for a markdown that never happened.

The game of firm markdown and consumer strategic waiting has recently received considerable attention from both practitioners and researchers. As summarized by Lai et al. (2010), firms have been recommended to use a variety of strategies to mitigate consumer strategic waiting (for example, committing to a fixed price, strategic rationing, or adjusting in-store display formats). In particular, Lai et al. (2010) investigate a marketing mechanism, posterior price-matching policies, for dealing with this strategic waiting problem. Under posterior price-matching policies, the seller will fully refund a consumer the difference between what the consumer paid (in the regular period) and the latest price (in the markdown period) if the latter is lower, provided that the consumer submits the claim for the refund to the seller. Such a policy is widely used in practice by numerous retailers (Target, Staples, Best Buy, Gap, Office Depot, Walmart, etc.).

A common notable feature in the existing and extensive price-matching literature is that, “rational expectations” are commonly assumed in studying the game between the firm and its consumers. This assumption dictates that both the firm and its consumers have “common knowledge” of the model, its parameters, and the opponents’ strategies. In particular, consumers can perfectly anticipate the firm’s markdown strategy (i.e., how likely a markdown occurs) in equilibrium. This assumption can be plausible in settings where adaptive learning in an infinite-time horizon is feasible. However, in some settings in practice, such infinite-learning opportunities might be rarely available to consumers (Spiegler 2006b). In those settings, consumers have only scarce or limited opportunities to learn the firm’s markdown strategy, and hence have to make decisions based on scarce information and anecdotal reasoning. In this paper, we are interested in investigating such settings. We conduct a complementary study to the existing price-matching literature by assuming that consumers may not have sufficient information or enough opportunities to learn the likelihood of this markdown opportunity.

In this paper, we intend to model the settings where the firm has perfect information while consumers only have scarce opportunities to learn the firm’s markdown strategy. To that end, we need a new modeling framework that captures consumer bounded rationality and includes rational expectations as a special case. We adopt the recent $S(1)$ model in the economics literature to capture consumer bounded rationality in the sense of anecdotal reasoning (see, e.g., Spiegler 2006b). In this model, consumers make their decisions based on the sampled outcome (i.e., anecdote). Instead of reasoning probabilistically with respect to a correct probability distribution, consumers reason 

We purposely adopt a simple model where the existing literature based on rational expectations would recommend not to use posterior price matching. In contrast with the literature, this simple model enables us to isolate and focus on the impact of customer boundedly rational expectations. Our first research question is the following: What is the impact of consumer boundedly rational expectations on the firm’s posterior price matching decisions? We find that bounded rationality alone can justify the use of posterior price matching (coupled with probabilistic markdowns). In other words, in the absence of all the reasons or motivations provided in the literature for adopting posterior price matching, the firm may still want to use this policy if consumers have boundedly rational expectations. Intuitively, price matching with probabilistic markdowns creates price obfuscation and can fool some customers into paying a higher price upfront because they falsely expect to be compensated later when customers are boundedly rational (which brings higher profit margins). However, customers who obtained unfavorable anecdotes do not purchase (which causes lost sales). Priori, it remains unclear whether the firm can make a profit through price matching. Indeed, this trade-off is nontrivial. We find that, under certain conditions, the benefit of higher profit margins can outweigh the cost of lost sales, so that the firm makes a profit from offering posterior price matching. We provide such conditions and analytically characterize the optimal markdown and pricing strategies. We demonstrate that consumer bounded rationality (commonly overlooked in the extant literature) is actually critical for firms who are considering adopting posterior price matching, and ignoring consumer bounded rationality can result in a significant profit loss.

After demonstrating the important role of customer boundedly rational expectations and how to implement price-matching policies when the firm commits
to a fixed markdown probability, we then proceed to investigate another practically relevant problem in which how a firm should dynamically manage its markdowns if it can adjust the markdown probability in each selling season. We thus build a dynamic programming model, and characterize its optimal policy. We show that a cyclic policy is typically optimal: it is profit maximizing to mark down with probability one followed with a small probability cyclically to exploit customer bounded rationality. We characterize the nature of the cyclic policy and the range in which it is optimal (e.g., as we vary the discount factor and the sample size or the level of customer rationality). The optimality of the cyclic policy comes from the submodularity of the firm’s profit function with respect to the markdown probabilities in any neighboring seasons. We prove that it is profit maximizing to set the neighboring markdown probabilities as far apart from each other as possible due to the substitutability between them.

The remainder of the paper is organized as follows. We review relevant literature in Section 2. Section 3 presents the basic model and its extensions, and Section 4 focuses on dynamic markdown. We conclude the paper and provide discussions in Section 5. All proofs and some technical lemmas are relegated to the online supplement.

2. Literature Review

Our study is in the setting of markdown pricing, which has received considerable attention in practice and in the academic literature. We are interested in the fundamental question of why markdown sales happen. Disposing leftover inventory is a common explanation (see, e.g., Pashigian 1988 and Rice et al. 2014). In this paper, we purposely rule out this explanation by focusing on the setting where inventory is not a concern. Price discrimination to heterogeneous consumers is another well-accepted explanation. For example, Conlisk et al. (1984) show that the heterogeneity of consumer tastes may lead a firm to vary its price periodically over time, which is also called cyclic pricing when selling durable goods in the extant literature (Heidhues and Kőszi 2014). In our model, we assume that all the consumers are homogeneous (in their valuations). Hence, there is no incentive for the firm to price discriminate consumers.

There is an extensive body of economics literature on price-matching policies that investigates the impact on competition. The literature starts from the early works of Salop (1986), Holt and Scheffman (1987), Png and Hirshleifer (1987), and Simons (1989). They show that price matching can soften the competition between sellers. Although there are several studies (e.g., Corts (1997) and Chen et al. (2001)) showing that price-matching policies can decrease sellers’ profits if there are heterogeneous consumers with different search and price-matching hassle costs, a general consensus is that price-matching policies soften competition, as also discussed in Sargent (1993) and Edlin (1997). We refer the reader to Hvild and Shaffer (1999), Coughlan and Shaffer (2009), and Hvild and Shaffer (2010) for this stream of literature. Price matching is also viewed as a means of price discriminating among consumers (see, e.g., Holt and Scheffman 1987, Corts 1997, Lee et al. 2015). The argument is that firms offering price-matching guarantees provide discounts selectively to consumers who are aware of lower prices in the market while charging a high list price for nonsearchers.

In monopoly settings, there are also studies exploring the motivation and impact of price matching. Butz (1990) studies the impact of a best-price policy on a durable goods seller who produces and sells in a long horizon. However, Butz (1990) assumes exogenous demand functions and completely ignores (strategic, rational, or boundedly rational) consumer behavior. Levin et al. (2007) study a practical revenue-management problem where a monopolist seller can sell a price guarantee option along with the product. Png (1991) and recently Xu (2011) study price-matching policies with forward-looking consumers. Png (1991) considers a scenario where a monopolist seller sells a fixed capacity in two periods. In his model, consumers have different and private valuations. Xu (2011) characterizes the optimal best-price policy as when the seller can control both the policy length (when the promise expires) and the refund scale (what portion of the price difference is refunded). Unlike this stream of literature that focuses on full price-matching policies, Lee et al. (2015) examine the implications of the fractional price-matching policy in the ocean freight service industry. However, none of these studies investigate boundedly rational consumer behavior.

Recently, Lai et al. (2010) study a posterior price-matching policy, where there are forward-looking consumers whose volume is random and valuation declines over time. They address the inventory/capacity decision in addition to the pricing issue. In addressing these issues, they assume that consumers have rational expectations. This paper is close to Lai et al. (2010). However, we deliberately build a simpler model where forward-looking consumers are homogeneous (with respect to their valuations of the product), and there is no demand uncertainty or capacity constraint. This model allows us to isolate the impact of consumer bounded rationality on the adoption of the price-matching policy. Our study is complementary to Lai et al. (2010), since we explore some settings where consumers may only have scarce opportunities to learn, which prevents them from forming rational expectations. Our model of boundedly rational consumers based on the anecdotal reasoning framework
includes the rational-expectations model as a special case. Hence, our study extends the existing literature by explicitly modeling consumer rationality and the lack thereof.

This paper belongs to a recent body of economics and marketing literature that studies market interactions between rational firms and boundedly rational customers (see Eliaz and Spiegler 2006, Huang and Yu 2014 and references therein). The common argument for this stream of literature is that the firm and customers typically differ in their capability to understand the market. In particular, our anecdotal reasoning framework follows the recent economics literature on modeling boundedly rational expectations and anecdotal reasoning that originates from Osborne and Rubinstein (1998). They formulate the S(1) procedure in the context of strategic-form games, in which all players behave according to this procedure. Their focus is developing a new equilibrium concept. In contrast, in our study, the firm is fully rational, and only the customers employ the S(1) or S(m) procedure. This procedure prescribes that each agent chooses the action after randomly sampling each alternative from the strategy set, picking the action that yields the highest payoff. It has been applied in a variety of settings. For instance, Spiegler (2006b) studies a setting where patients do not have enough opportunities to learn doctors’ “quality.” He finds that the element of boundedly rational expectations has significant implications, e.g., the “market for quacks” is active and consumers suffer from a welfare loss. (See Chapters 6–7 of Spiegler (2011a) for more applications of the S(1) procedure.) Recent studies have applied the anecdotal reasoning framework to marketing settings. In particular, Huang and Yu (2014) show that anecdotal reasoning may provide an explanation for why the novel marketing practice opaque selling is adopted. The posterior price matching studied in this paper is a different business strategy from that setting. Moreover, we also aim to provide practical guidelines for how to manage markdowns in a dynamic setting in practice when customers are boundedly rational.

Anecdotal reasoning is also related to the economics and psychology literature on sampling biases. Tversky and Kahneman (1971) demonstrate experimentally that people have erroneous intuitions about the laws of chance. In particular, they regard a sample randomly drawn from a population as highly representative of the population. This phenomenon is dubbed “the law of small numbers” and they explain it as a consequence of the representativeness heuristic. Tversky and Kahneman (1973) propose that when faced with the difficult task of judging probability, people evaluate the probability of events by availability, i.e., by the ease with which relevant instances come to mind. Another related decision bias is the overconfidence bias studied in the economics literature, see, e.g., Benoit and Dubra (2011) and references therein. In the operations management literature, Ren and Croson (2013) study overconfidence bias in newsvendor settings. They present two controlled experiments to test the impact of overconfidence on order bias.

Our study also contributes to the growing behavioral operations management literature. For example, Su (2008) studies the ordering decisions made by a boundedly rational newsvendor. Rudi and Drake (2014) provide managers with insight into how order adjustment and order level affect behavioral mismatch costs in a newsvendor setting. Özer and Zheng (2016) study a seller’s optimal pricing and inventory strategies when nonpecuniary motives affect consumer purchasing decisions. Huang et al. (2013) and Huang and Chen (2015) focus on customer incapability of perfectly estimating expected waiting time in service settings. We refer the readers to Cui and Veeraraghavan (2016), Li et al. (2016), Huang and Liu (2015), and Baron et al. (2015) for this stream of research. Our study contributes to this literature by studying the role of customers’ boundedly rational expectations on the firm’s price-matching decision.

3. Basic Model

We consider the simplest possible model to demonstrate and isolate the impact of consumer bounded rationality on the firm’s markdown and price-matching strategy. A monopolist firm sells a product to a population of homogeneous consumers/customers whose valuation (i.e., willingness to pay) is $v$. There are two selling periods indexed by $t \in \{1, 2\}$, corresponding to a regular selling period and a period for potential markdown (i.e., probabilistic markdown). Note that the firm has all the freedom in determining whether to mark down or not. This assumption eliminates all the exogenous factors (such as leftover inventories) that may induce markdowns.

There is no demand uncertainty. There is a continuum of consumers, and the number of them is normalized to 1. In other words, an individual consumer has a negligible mass compared with the total population; see Lai et al. (2010) for a similar assumption. The valuation of the consumers, $v$ in the first period, declines to $v_t$ in the second period, where $v_t < v$. The difference $v - v_t$ is the value loss to the consumers caused by delayed consumption of the product. This could measure “the loss of utility in the first period,” or, “the reduced appeal of ‘not being among the first consumers’” (Lai et al. 2010, p. 37). Each customer decides whether or not to purchase a product from the firm. A customer who does not purchase the product in any of these two periods can enjoy an outside option with surplus $v_0 \geq 0$. The case without the outside option (commonly studied in the literature) is a special case of
ours when \( v_0 = 0 \). Allowing \( v_0 \) to be positive is meant to capture the fact that the firm may not always be able to extract all consumer surplus in practice. The firm’s marginal cost for the product is \( c \geq 0 \), and we assume \( v - v_0 - c > 0 \) to make sure that the firm has a positive profit margin.

3.1. The Posterior Price-Matching Policy

The price-matching policy, if offered, is a 100% difference refund policy triggered by customers (Lai et al. 2010). In other words, the firm will fully refund a customer the difference between what the customer paid (in the regular period) and the most recent price (in the markdown period) if the latter is lower, provided that the customer submits the claim for the refund to the firm. Following Lai et al. (2010), we use the notation “PM” and “NP” to indicate the price-matching policy and the no-price-matching policy. The decision on the PM policy needs to be made before the selling season. Once it is offered, the firm cannot revoke it. To break the tie, we assume that if the firm is indifferent between offering price matching versus not offering, it will prefer not to offer price matching. The reason is that, in practice, there is typically a strictly positive cost for the firm to process the refund claim. In our model, we shall assume that such a cost is strictly positive yet (sufficiently) small.

The firm’s PM policy \( \Theta \in \{PM, NP\} \) is observable to customers. However, the chance or likelihood of markdown, \( \xi \in [0, 1] \), is unknown to customers. In other words, whether the event \( (p_2 < p_1) \) happens or not and the probability \( \xi \equiv P(p_2 < p_1) \) is unknown to a customer when he or she makes a purchasing decision in the regular selling period. All other parameters are common knowledge.

To model customer bounded rationality, we consider the setting where the firm sells the product to different generations of new customers.¹ There are infinitely many discrete time “seasons” indexed by \( t = 0, 1, 2, 3, \ldots \), with each season consists of a regular selling period and a (potential) markdown period. The firm first commits to a PM or NP strategy before all the selling seasons. If the firm chooses the NP policy, there is no price matching even if \( p_1 > p_2 \). If the firm offers price matching, then customers who purchased at price \( p_1 \) can obtain a refund if \( p_1 > p_2 \). Given that there is no availability or stock-out issue, the firm has full control over the markdown event. Recall that we denote \( \xi \equiv P(p_1 > p_2) \in [0, 1] \) as the firm’s markdown strategy. We shall show that the firm has no incentives to mark down if price matching is not offered when customers are either rational or boundedly rational. We also assume that the firm has the commitment power, i.e., the markdown probability \( \xi \) can be committed ex ante. This commitment can be justified for several reasons, e.g., reputation considerations or the presence of online discussion forums. Nevertheless, we will relax this assumption in Section 4 on dynamic markdown (where there is no commitment).

Boundedly rational customers do not exactly know \( \xi \), if price matching is offered. We use the following dynamics to model the anecdotal learning and reasoning process (Huang and Yu 2014): in season 0, the generation-0 customers enter into the market and make their purchasing decisions based on their prior beliefs. After their purchasing decisions, each customer may obtain an individual markdown realization. Then, they leave the market. In season 1, the generation-1 customers enter the market. Before making their purchasing decisions, each customer has the opportunity to talk to one of the generation-0 customers so that each customer can obtain a sample/anecdote of the markdown realization in stage 0. In general, in season \( t = 1, 2, 3, \ldots \), each generation-\( t \) customer can sample from one of the generation-\((t')\) customers with equal chance about the realized price before making a purchasing decision in season \( t \), for all \( t' < t \). Each generation-\( t \) customer decides to purchase the product or not at the regular selling period based on the sample and prior belief (the mechanism of combining them will be specified later). However, the customer’s own price offered by the firm at season \( t \) is an independent realization from \( \xi \). Figure 1 depicts the sequence of events for \( t = 0, 1, 2 \). Our model focuses on the long-run steady state, i.e., for \( t = \infty \). Hence, our multisellon model reduces to a single-season model where this single-season dynamics represent the boundedly rational expectations we intend to capture.

Our model is consistent with the extant literature (see, e.g., the recent work of Lai et al. 2010). However, there are several notable differences between our model in this section and the existing literature. First, we assume that customers are homogeneous with deterministic valuation \( v \). Second, the market size, i.e., the demand, is deterministic (and normalized to one for convenience), which is intentionally assumed to rule out demand uncertainty and supply-demand mismatches. Third, capacity is ample so that the firm does not make the quantity decision. Fourth, and importantly, while all the papers in the price-matching literature assume rational expectations, we focus on boundedly rational expectations. These model differences allow us to separate and focus on the role of boundedly rational expectations on the firm’s PM policy.

3.2. Benchmark

Before analyzing the model of boundedly rational customers, let us first discuss the benchmark case where customers are (fully) rational as studied in Lai et al. (2010), meaning that they perfectly know the firm’s strategy \( \xi \). Note that, “strategic customers” in the recent operations literature (e.g., Lai et al. 2010)
are those who are both rational and forward-looking (i.e., they compare the utility of purchasing now versus postponing to the future period). In this setting, we show that the firm does not have any incentives to use price matching in Proposition 1.

**Proposition 1.** If customers have rational expectations, the firm does not mark down or use the posterior PM policy in equilibrium.

Proposition 1 is completely consistent with the literature (e.g., Lai et al. 2010): without demand uncertainty, capacity issues, and heterogeneous customers, the PM policy is not recommended. In our model, customers are also implicitly assumed to be forward-looking, yet they can be boundedly rational. We shall show that customer bounded rationality alone can provide a new rationale for adopting the PM policy, and characterize the optimal policy in the presence of customer boundedly rational expectations. Before analyzing the model, we first discuss the specifics of the anecdotal-reasoning framework.

### 3.3. The S(1) Framework

Each customer obtains a single anecdote/sample about a price pair \( p_1 \) and \( p_2 \) that occurred in some previous periods (in the same season). Denote the indicator random variable \( I_i = I_{(p_1 < p_2)} \) for customer \( i \). It is clear that \( \mathbb{E} I_i = \xi \) since we focus on the steady-state system dynamics (depicted in Figure 1). Hence, the sample tells the customer whether or not a markdown occurred. Each customer’s sample is totally independent of other customers’, and thus customer samples are heterogeneous. We assume that each customer has the same prior belief of the probability of markdown \( \xi_i \in [0, 1] \) before observing any sample. After observing a sample, each consumer estimates the probability of markdown \( \xi_i \) by simply taking the weighted average between \( \xi_0 \) and \( I_i, \xi_i = \lambda \xi_0 + (1 - \lambda)I_i \), where \( \lambda \in [0, 1] \) is the weight for the prior, and \( 1 - \lambda \) is the weight for the sample information. Hence, \( 1 - \lambda \) measures the extent to which each customer relies on his or her sample/anecdote to make a purchasing decision. This approach generalizes the S(1) model used in the economics literature (e.g., Spiegler 2006a) where there is no prior, and the marketing literature (e.g., Huang and Yu 2014) where the prior is assumed to be rational.

Under anecdotal reasoning, customer \( i \) purchases in period 1 if and only if

\[
 v - p_1 + \xi_i (p_1 - p_2) \geq (1 - \xi_i) \max\{v_L - p_1, v_0\} + \xi_i \max\{v_L - p_2, v_0\},
\]

where the left-hand side is the net payoff the customer obtains from purchasing in the regular period, and the right-hand side corresponds to the surplus from purchasing in the (potential) markdown period or from the customer’s outside option. The variable \( p_1 \) denotes the regular price and \( p_2 \) the markdown price.

To simplify the consumer decision rule in Equation (1), we first assume that the production cost \( c \) is greater than \( v_L - v_0 \) which is the maximum price consumers are willing to pay in the markdown period. This assumption simplifies our analysis and will be maintained in this section and Section 3.4. We shall continue to analyze the setting when this assumption does not hold (i.e., \( c \leq v_L - v_0 \)) in Section 3.5 and show how this assumption affects the firm’s optimal pricing and markdown strategies.

To simplify Equation (1), we discuss three cases:

1. Suppose \( v_L - p_1 \geq v_0 \) and \( v_L - p_2 \geq v_0 \), i.e., \( p_1 \leq v_L - v_0 \). Then, (1 - \( \xi_i \))\( \max\{v_L - p_1, v_0\} + \xi_i \max\{v_L - p_2, v_0\} = (1 - \xi_i)(v_L - p_1) + \xi_i(v_L - p_2).
2. Suppose \( v_L - p_1 \leq v_0 \) and \( v_L - p_2 \leq v_0 \), i.e., \( p_1 \geq v_L - v_0 \).
\( v_1 - v_0 \) and \( p_2 \geq v_1 - v_0 \). Then, \( (1 - \xi_i) \max \{v_1 - p_1, v_0\} + \xi_i \max \{v_1 - p_2, v_0\} = v_0 \). (3) Suppose \( v_1 - p_1 \leq v_0 \) and \( v_1 - p_2 \geq v_0 \), i.e., \( p_1 \geq v_1 - v_0 \) and \( p_2 \leq v_1 - v_0 \). If we assume that \( c > v_1 - v_0 \), then this case cannot be true since we require \( p_1 \geq p_2 \geq c \). Given this assumption, only cases (1) and (2) are relevant, where Equation (1) can be simplified to

\[
v - p_1 + \xi_i(p_1 - p_2) \geq \max\{(1 - \xi_i)(v_1 - p_1) + \xi_i(v_1 - p_2), v_0\}. \tag{2}
\]

First, it is useful to note that if the firm does not use the PM policy, then no markdown is optimal. We formally state this simple fact as a lemma below.

**Lemma 1.** If the firm chooses NP, then \( \xi_{iNP} = 0 \) and \( p_i^* = p_2^* = v - v_0 \) is the optimal strategy for the firm.

The rationale behind Lemma 1 is simple: without price matching, the firm can never charge a price higher than \( v - v_0 \) to get a positive demand. Therefore, under the NP policy, each customer receives a sample that uniformly suggests \( p_1 = p_2 \), and it is as if customers know \( p_2 \) for sure when making a purchasing decision. In what follows, we shall focus on the subgame where the firm uses the PM policy and identify the operating regimes in which using price matching is optimal.

We now discuss how a consumer makes purchasing decisions based on the realized markdown outcome \( I_i \) in the sample/anecdote. If \( I_i = 1 \), i.e., the sample indicates a markdown, \( \xi_i = \lambda \xi_0 + 1 - \lambda \). We have \( v - p_1 + (\lambda \xi_0 + 1 - \lambda)(p_1 - p_2) \geq \max\{v_1 - (\lambda \xi_0)p_1 - (\lambda \xi_0 + 1 - \lambda)p_2, v_0\} \), thus, if \( v - p_1 + (\lambda \xi_0 + 1 - \lambda)(p_1 - p_2) \geq v_0 \), a consumer purchases in period 1; otherwise, the consumer leaves the market. If \( I_i = 0 \), \( \xi_i = \lambda \xi_0 \). We have \( v - p_1 + \lambda \xi_0(p_1 - p_2) \geq \max\{(1 - \lambda \xi_0)(v_1 - p_1) + \lambda \xi_0(v_1 - p_2), v_0\} \). Thus, if \( v - p_1 + \lambda \xi_0(p_1 - p_2) \leq v_0 \), a consumer leaves the market.

We then have the following three subcases: (i) If \( p_1 - (\lambda \xi_0 + 1 - \lambda)(p_1 - p_2) \leq v - v_0 \), each consumer purchases in period 1. (ii) If \( p_1 - (\lambda \xi_0 + 1 - \lambda)(p_1 - p_2) \leq v - v_0 < p_1 - \lambda \xi_0(p_1 - p_2) \), consumer i purchases in period 1 if \( I_i = 1 \) and leaves the market if \( I_i = 0 \). (iii) If \( v - v_0 < p_1 - (\lambda \xi_0 + 1 - \lambda)(p_1 - p_2) \leq p_1 - \lambda \xi_0(p_1 - p_2) \), each consumer leaves the market.

From this analysis, we find that a consumer should purchase in the first period if that individual’s belief based on the obtained sample is favorable. Hence, the firm has only one-period effective sales (OES). This phenomenon also appears in Lai et al. (2010, Lemma 4, p. 45), where strategic consumers purchase immediately rather than delaying to the markdown period.3

In addition, we can show that the PM policy can be potentially effective for the firm only in case (ii). Define the set \( \Omega \equiv \{p_1, p_2 | p_1 - (\lambda \xi_0 + 1 - \lambda)(p_1 - p_2) \leq v - v_0 < p_1 - \lambda \xi_0(p_1 - p_2), p_1 \geq p_2 \geq c\} \), the pricing region as illustrated in Figure A-1 (relegated to the online supplement).

### 3.4. Optimal Price-Matching Policies

In this section, we characterize the firm’s optimal pricing strategies and the probability of offering markdowns. Through analytical comparative statics and systematic numerical analysis, we investigate directional properties of optimal prices and markdown probabilities and provide useful managerial guidelines.

The firm’s optimization problem can be expressed as

\[
\max \quad \Pi(\xi, p_1, p_2) \equiv \xi[p_1 - c - \xi(p_1 - p_2)]
\]

\[
s.t. \quad \xi \in [0, 1], \quad (p_1, p_2) \in \Omega.
\]

We first characterize the optimal pricing decisions for an arbitrarily given markdown probability \( \xi \in [0, 1] \). From Lemma A-1 (in the online supplement), we know that if a markdown happens, the price in the second period should be equal to the marginal cost \( c \). Based on Lemma A-1, we can fully derive the optimal probabilistic markdown policy, and characterize under what conditions the PM policy is more profitable than the NP policy.

**Proposition 2.** Suppose \( c > v_1 - v_0 \). (i) The optimal PM policy satisfies \( p_i^* = (v - v_0 - (\lambda \xi_0 + 1 - \lambda)c)/(\lambda - \lambda \xi_0) \), \( p_2^* = c \), and \( \xi^* \xi = 1/4 \). The firm profit under the optimal PM policy is \( 1/4((v - v_0 - c)/(\lambda - \lambda \xi_0)) \). (ii) If \( \lambda - \lambda \xi_0 < 1/4 \), the PM policy is better than the NP policy.

Proposition 2 shows that the PM policy is better than the NP policy if \( \lambda - \lambda \xi_0 < 1/4 \). Why can the firm make a profit by markdown and posterior price matching when customers have boundedly rational expectations (in the sense of anecdotal reasoning)? Compared to the NP policy, first, the firm suffers from lost sales because only a fraction of the customers purchase (while they all purchase under the NP policy). Second, the firm loses profit from the customers who are able to obtain the markdown deal \( p_2^* \leq v - v_0 \). However, the firm makes a profit from those who got samples indicating markdowns but did not obtain the markdown deal since \( p_i^* > v - v_0 \). The trade-off is intricate, and it appears unclear whether and when the benefit from a higher profit margin would dominate the lost sales. The condition in Proposition 2 suggests that if customers put more weight on their samples (a lower \( \lambda \)) and/or have a higher prior belief of markdowns (a larger \( \xi_0 \)), then the PM policy is optimal.

Proposition 2 provides the unique optimal PM policy. However, under the condition \( \lambda - \lambda \xi_0 < 1/4 \), there are certainly many other markdown strategies that strictly dominate the NP policy with \( p_1 = p_2 = v - v_0 \). We have the following corollary.

**Corollary 1.** Suppose \( c > v_1 - v_0 \). If \( \lambda - \lambda \xi_0 < 1/4 \), any PM policies that satisfy \( (p_1, p_2) \in S_1 \) and \( \xi \in S_2 \) yields a strictly
**Figure 2.** (Color online) The Region for PM Policies Better Than the NP Policy Under Different Profit Margins

![Graphs showing the region for PM policies better than the NP policy under different profit margins.](image)

Notes. Parameters are as follows: \( v = 14, 2, v_0 = 12, c = 11; \) for each subfigure, left panel \( \xi_0 = 0.3 \) and right panel \( \lambda = 0.3. \)

**higher profit than the NP policy**, where \( S_1 \equiv \Omega \cap \{ p_1 > 2p_2 - c \} \cap \{ (p_1 - c)^2 - 4(v - v_0 - c)(p_1 - p_2) > 0 \} \) and

\[
S_2 \equiv \begin{cases} 
\frac{p_1 - c - \sqrt{(p_1 - c)^2 - 4(p_1 - p_2)(v - v_0 - c)}}{2(p_1 - p_2)}, \\
\frac{p_1 - c + \sqrt{(p_1 - c)^2 - 4(p_1 - p_2)(v - v_0 - c)}}{2(p_1 - p_2)}
\end{cases}
\]

We are now interested in how the PM policy can be effectively used in practice. In particular, we investigate: (i) how the range in which the PM policy strictly dominates the NP policy changes with respect to the model parameters, and (ii) how the optimal price discount magnitude \( p^*_2/p^*_1 \) varies with the parameters. Figure 2 presents a set of representative numerical examples, for each \( p_1 = 14, 16, 18, 20 \), representing different levels of profit margins. In the left panel, as \( \lambda \) decreases, i.e., the extent to which each customer relies on the sample/anecdote to make a purchasing decision increases, the range in which the PM policy strictly dominates the NP policy becomes larger. In the right panel, as \( \xi_0 \) increases, i.e., a customer’s prior belief of the markdown probability increases, the range in which the PM policy strictly dominates the NP policy becomes larger. Intuitively, as customers rely on anecdotes more or are a priori more optimistic about markdown opportunities, the firm has more leverage to charge a higher price in using the PM policy to make a profit. In addition, we find that the region in which the PM policy strictly dominates the NP policy becomes larger when the profit margin \( p_1 - c \) is intermediate (e.g., \( p_1 - c = 5 \) in our numerical examples). This suggests that there is more leverage to adopt the PM policy when the profit margin is at an intermediate level.

Figure 3 presents a set of representative numerical examples for the optimal price discount magnitude \( p^*_2/p^*_1 \). This figure shows that under the optimal PM policy, a deep markdown discount may not be necessary in order to make a profit (e.g., \( p^*_2/p^*_1 \) can be greater...
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Figure 3. (Color online) The Optimal Price Ratio \( p_2^* / p_1^* \) as \( \lambda \) and \( \xi_0 \) Changes Respectively

Notes. Parameters are as follows: \( \nu = 14.2, \nu_1 = 12, \nu_0 = 2, c = 12, \) left panel \( \xi_0 = 0.3, \) and right panel \( \lambda = 0.3. \)

3.4.1. Impact of Ignoring Customer Bounded Rationality. A key message from our analysis is that bounded rationality in the sense of anecdotal reasoning alone can provide strict incentives for the firm to use the PM policy. This is because customers overinfer from their limited information. By creating price uncertainty or obfuscating its pricing through possible markdowns, the firm is able to take advantage of customers. If the firm incorrectly assumes that customers are (fully) rational, it is expected to use suboptimal policies in some situations, and hence suffer from decreased expected profit. In this section, we focus our attention on the possible consequences of this kind of misperception about customer behavior on the firm’s expected profit (see, e.g., Aviv and Pazgal 2008 and Özer and Zheng 2016 for a similar approach). It turns out that we can analytically quantify the profit loss in Corollary 2.

Corollary 2 (Impact of Ignoring Customer Bounded Rationality). Suppose \( c > \nu_l - \nu_0. \) When \( \lambda - \lambda \xi_0 < \frac{1}{4}, \) if the firm incorrectly assumes that customers are rational, the profit loss \( \Delta \pi_{\ell} \equiv 1 - \Pi_{NP}/\Pi_{PM} = 1 - 4\lambda(1 - \xi_0), \) where \( \Pi_{NP} \) is the firm’s expected profit under the NP policy and \( \Pi_{PM} \) is that under the PM policy.

Corollary 2 demonstrates that the firm will suffer from the profit loss if it mistakenly assumes that customers have rational expectations in this situation. We can see that the profit loss is decreasing in the weight for the prior \( \lambda \) and increasing in the prior belief of the probability of markdown \( \xi_0. \) This is because the value of the PM policy is larger if customers rely more on the samples or have a larger prior belief on the markdown probability. Ignoring the impact of the customer bounded rationality behavior to use the NP policy can result in a significant profit loss. In fact, such a profit loss would approach 100% as \( \lambda \rightarrow 0 \) or \( \xi_0 \rightarrow 1. \) This result underscores the importance of considering consumer bounded rationality when the firm makes its markdown and posterior price-matching decisions.

3.5. Equilibrium Analysis When \( c \leq \nu_l - \nu_0 \)

In this section, we analyze the setting when the production cost \( c \) is smaller than \( \nu_l - \nu_0. \) Then, following the analysis in section 3.3, we have to analyze case (3) in addition to cases (1) and (2). In case (3), we have \( \nu_l - p_1 \leq \nu_0 \) and \( \nu_l - p_2 \geq \nu_0, \) i.e., \( p_1 \geq \nu_l - \nu_0 \) and \( p_2 \leq \nu_l - \nu_0. \) In this setting, among all the customers who waited...
until the second period, those who see a markdown price \( p_2 \) would purchase in the second period while those who do not see a markdown would leave the market (by taking the outside option \( v_0 \)). Then, customer \( i \) purchases in period 1 if and only if \( v - p_1 + \xi_i \) \( \geq 1 - \xi_i \) \( v_0 + \xi_i \), which is simplified to \( p_1 \leq (v - \xi_i v_i)/(1 - \xi_i) - v_0 \).

We aim to identify the operating regime in which using the PM policy is optimal. It is necessary that a customer who obtained sample \( I_i = 1 \) prefers to purchase in the first period, and waits until the second period otherwise. Since \( \xi_i = \lambda \xi_0 + (1 - \lambda) I_i \), we require \( (v - \lambda \xi_0 v_i)/(1 - \lambda \xi_0) - v_0 < p_1 \leq (v - (\lambda \xi_0 + 1 - \lambda) v_i)/(\lambda (1 - \xi_0)) - v_0 \). Define the set

\[
\Omega_i \equiv \left\{ p_1, p_2 \mid \frac{v - \lambda \xi_0 v_i}{1 - \lambda \xi_0} - v_0 < p_1 \leq \frac{v - (\lambda \xi_0 + 1 - \lambda) v_i}{\lambda (1 - \xi_0)}, -v_0, v_L - v_0 \geq p_2 \geq c \right\}.
\]

The firm’s optimization problem under case (3) can be expressed as

\[
\max \Pi(\xi, p_1, p_2) \equiv \xi \{ p_1 - c - \xi (p_1 - p_2) \} + (1 - \xi) (p_2 - c) \xi
\]

s.t. \( \xi \in [0,1] \), \( (p_1, p_2) \in \Omega_i \),

where the first term in the objective function is the expected profit from the first period and the second term is that from the second period. Different from the OES when \( c > v_i - v_0 \) analyzed in Section 3.4, the firm now can also have two-period effective sales (TES). The firm can choose the OES strategy or the TES strategy to maximize its expected profit. Following the same approach as before, we can show the following result.

**Proposition 3.** Suppose \( c \leq v_L - v_0 \). We have the following:

(i) The optimal OES strategy is \( p_1^* = (v - v_0 - (\lambda \xi_0 + 1 - \lambda) c)/(\lambda - \lambda \xi_0) \), \( p_2^* = c \), and \( \xi^* = \frac{1}{2} \). The firm profit under this PM policy is \( 1/2 \) \((v - v_0 - c)/(\lambda - \lambda \xi_0)\). This PM policy is better than the NP policy if \( \lambda - \lambda \xi_0 < \lambda \).

(ii) The optimal TES strategy is \( p_1^* = (v - (\lambda \xi_0 + 1 - \lambda) v_i)/(\lambda - \lambda \xi_0) - v_0 \), \( p_2^* = v_L - v_0 \), and

\[
\xi^* = \frac{1}{2} \left\{ \frac{v_L - v_0 - c}{2(v - (\lambda \xi_0 + 1 - \lambda) v_L)/(\lambda - \lambda \xi_0) - v_0 - c} \right\}.
\]

The firm profit under this PM policy

\[
\Pi^* = \frac{1}{4} \left( \frac{v - (\lambda \xi_0 + 1 - \lambda) v_L}{\lambda - \lambda \xi_0} + v_L - 2v_0 - 2c \right)^2 \cdot \left( \frac{v - (\lambda \xi_0 + 1 - \lambda) v_L}{\lambda - \lambda \xi_0} - v_0 - c \right)^{-1}.
\]

This PM policy is better than the NP policy if \( \lambda - \lambda \xi_0 < (\sqrt{(v - v_1)^2 + (v - v_L)(v_L - v_0 - c) - (v - v_L))/2(v_L - v_0 - c)}) \).

(iii) If \( \lambda - \lambda \xi_0 < \frac{1}{4} \), the PM policy with the OES strategy is optimal; otherwise, the NP policy is optimal.

Proposition 3 is different from Proposition 2 in that the firm now has the option to choose the TES strategy so that there are customers who wait until the second period and purchase. This new phenomenon occurs since the customers still have a high valuation of the product in the markdown period. Under the TES strategy, the firm should offer a higher markdown probability and a higher markdown price than those under the OES strategy. The TES strategy can perform better than the NP policy, and part (ii) of Proposition 3 provides the condition. However, it turns out that the optimal price-matching strategy still only involves OES, which is the same as that in Proposition 2.

### 3.6. Generalization to \( S(m) \)

The \( S(1) \) model has the limitation that a customer only has access to a single sample before making a purchasing decision. In this section, we extend the \( S(1) \) model to a general \( S(m) \) model where customers make a decision based on \( m \) independent samples besides a prior belief of the probability of markdown \( \xi_{0i} \), where \( m = 1, 2, 3, \ldots \) These samples can be obtained through multiple customers from the previous generations.

We now analytically formalize the \( S(m) \) model. Denote

\[
\xi_i(m) \equiv \lambda \xi_0 + (1 - \lambda) \frac{1}{m} \sum_{j=1}^{m} I_{(i,j)}
\]

as the combination of the mean of the samples that customer \( i \) obtains and the prior belief, where \( I_{(i,j)} \) is the indicator for \( p_{1j} > p_{2j} \) of the \( j \)th sample.

It is clear that \( \lim_{m \to \infty} \xi_i(m) = \xi \) when \( \lambda = 0 \), i.e., the sample average becomes the population mean, and we are back to the rational-expectations case (recall Proposition 1). Hence, the rational-expectations model can be viewed as a special case of the anecdotal reasoning framework when \( m = \infty \). Thus, the \( S(m) \) framework is a generalization of the rational-expectations model commonly adopted in the extant literature. One advantage of the \( S(m) \) framework is that it uses a single parameter \( m \) to cover the full spectrum from the extreme of a single anecdote (i.e., \( S(1) \)) to the other extreme of rational expectations (i.e., \( S(\infty) \)). Hence, we may call \( m \) the level of rationality in customer expectations.

**Remark 1.** According to the \( S(m) \) model, consumers overinfer from their samples, especially when \( m \) is small. Consumers may behave this way for multiple reasons: scarce information (e.g., not enough samples available in the market), imperfect search (due to non-trivial search cost), cognitive limitations (e.g., a short span of memory), or computational limitations (e.g., taking sample average is much simpler than conducting complicated Bayesian updating). Experienced customers can suffer from this kind of bounded rationality as well.
Recall that the firm could choose between the OES and TES strategies. Note that the TES strategy is viable only when \( c \leq v_L - v_o \). Under the anecdotal reasoning based on \( m \) samples, if the firm adopts the OES strategy, customer \( i \) purchases in period 1 if and only if

\[
v - p_1 + \xi_i(m)(p_1 - p_2) \\
\geq \max\{(1 - \xi_i(m))(v_L - p_1) + \xi_i(m)(v_L - p_2), v_0\},
\]

while if the firm adopts the TES strategy, customer \( i \) purchases in period 1 if and only if

\[
v - p_1 + \xi_i(m)(p_1 - p_2) \geq (1 - \xi_i(m))v_0 + \xi_i(m)(v_L - p_2),
\]

similar to the (S(1)) model, where \( p_1 \geq p_2 \). Let \( \gamma_i(\xi, m) \) and \( \gamma'_i(\xi, m) \) be the fraction of customers that purchase in the regular period under the OES strategy and the TES strategy, respectively. Then, we have the result in Lemma A-2 that provides the explicit expressions for \( \gamma_i(\xi, m) \) and \( \gamma'_i(\xi, m) \). (See the online supplement.)

The firm chooses \((\xi, p_1, p_2)\) to maximize its expected profit. If the firm adopts the OES strategy, the expected profit function can be expressed as

\[
\Pi(\xi, p_1, p_2) = \gamma_1(\xi, m)[p_1 - c - \xi(p_1 - p_2)],
\]

subject to \( \xi \in [0, 1] \) and \((p_1, p_2) \in \Omega\) for an arbitrarily small \( \epsilon > 0 \). If the firm adopts the TES strategy when \( c \leq v_L - v_0 \), the expected profit function can be expressed as

\[
\Pi(\xi, p_1, p_2) = \gamma'_2(\xi, m)[p_1 - c - \xi(p_1 - p_2)] \\
\quad + [1 - \gamma'_2(\xi, m)]\xi(p_2 - c),
\]

subject to \( \xi \in [0, 1] \) and \((p_1, p_2) \in \Omega_2\) for an arbitrarily small \( \epsilon > 0 \). Solving this constrained optimization is generally challenging. However, we first have the following result:

**Proposition 4.** (i) If \( \lambda - \lambda \xi_0 < (m/(m + 1))^m(1/(m + 1)) \), it is optimal for the firm to use the PM policy. In particular, the OES strategy \( \xi = m/(m + 1) \), \( p_1 = (v - v_0 - (\lambda \xi_0 + 1 - \lambda)c)/((\lambda - \lambda \xi_0)) \), \( p_2 = c \) strictly dominates the no markdown strategy.

(ii) If \( c \leq v_L - v_0 \) and \(((1 - \xi)(m - \lambda \xi_0))/((1 - \xi)(1 - \xi)) > (v_L - v_0 - c)/(v_L - v_0) \), it is optimal to use the PM policy. In particular, the TES strategy \( \xi^* = \xi \), \( p_1 = (v - (\lambda \xi_0 + 1 - \lambda)v_L)/((\lambda - \lambda \xi_0)) \), \( p_2 = v_L - v_0 \) strictly dominates the no markdown strategy, where \( \xi \in [m - 1/m + 1, 1) \) uniquely solves

\[
\xi^m + m \xi^{m-1} - m \xi^{m-1} = \frac{v_L - v_0 - c}{(v - v_L)/((\lambda - \lambda \xi_0) + v_L - v_0 - c)}.
\]

Proposition 4 provides two sufficient conditions under which the PM policy is optimal. The results show that our main findings on the OES and TES strategies from the S(1) model are robust to the number of samples the customers obtain. We then study how to solve the firm’s optimization problems in (6) and (7), which both involve three continuous variables. We can first transform the optimization problem of the OES strategy to an equivalent problem: The firm chooses \((\xi, i, p_1)\) to maximize

\[
\Pi_m(\xi, i, p_1) = \left[1 - \sum_{n=0}^{i-1} B(n; m, \xi)\right] \\
\left\{\left[1 - \frac{m \xi}{i(1 - \lambda) + \lambda \xi_0 m}\right] p_1 + \frac{m(v - v_0)\xi}{i(1 - \lambda) + \lambda \xi_0 m} - c\right\},
\]

subject to

\[
\xi \in [0, 1], \quad p_1 \in \left[v - v_{0\iota} - \frac{m(v - v_0)\xi - [i(1 - \lambda) + \lambda \xi_0 m]c}{(1 - \lambda \xi_0)m - i(1 - \lambda)}\right],
\]

and \( i \in \{1, 2, 3, \ldots, m\} \). This problem is computationally simpler as it involves a finite search over \( i \) and optimizes over the continuous variables \( \xi \) and \( p_1 \). We state this result as Lemma A-3 in the online supplement.

Denote \((\xi^*, i^*, p_1^*)\) as the optimal solution to the optimization problem (8). We have the following partial characterization of this optimal solution:

**Proposition 5.** Under the OES strategy, the optimal solution satisfies the following properties.

(i) If \( \xi^* < (i^*(1 - \lambda) + \lambda \xi_0 m)/m \), then \( p_1^* = (m(v - v_0) - [i^*(1 - \lambda) + \lambda \xi_0 m]c)/(i^*(1 - \lambda) + \lambda \xi_0 m - i^*(1 - \lambda)) \) and \( p_2^* = c \).

(ii) If \( \xi^* \geq (i^*(1 - \lambda) + \lambda \xi_0 m)/m \), then \( p_1^* = p_2^* = v_{0\iota} - v_0 \) and thus no markdown is optimal.

Proposition 5 implies that under the OES strategy, the profitability of posterior price matching crucially depends on the level of rationality \( m \): when markdown with price matching is optimal, we know that the first-period price has to be \( p_1^* = (m(v - v_0) - [i^*(1 - \lambda) + \lambda \xi_0 m]c)/(i^*(1 - \lambda) + \lambda \xi_0 m - i^*(1 - \lambda)) \) and the second-period price has to be \( p_2^* = c \). Then, the optimization problem (8) would reduce to a finite number of polynomial functions. This provides an efficient algorithm for practice.

We then consider the TES strategy when \( c \leq v_L - v_0 \). We have the following proposition to characterize how to solve the optimization problem.

**Proposition 6.** When \( c \leq v_L - v_0 \), under the TES strategy, the optimization problem can be transformed to an equivalent problem: the firm chooses \((\xi, i)\) to maximize

\[
\Pi_m(\xi, i) = \left[1 - \sum_{n=0}^{i-1} B(n; m, \xi)\right] (1 - \xi) \\
\left\{v - v_{0\iota} + \frac{(1 - \lambda)(i/m) + \lambda \xi_0 (v_0 - v_{0\iota})}{1 - (1 - \lambda)(i/m) - \lambda \xi_0} - c\right\} \\
+ \xi(v_L - v_0 - c),
\]

subject to \( \xi \in [0, 1] \) and \( i \in \{1, 2, 3, \ldots, m\} \). The optimal markdown strategy satisfies

\[
p_1^* = \frac{v - v_{0\iota} + \frac{(1 - \lambda)(i/m) + \lambda \xi_0 (v_0 - v_{0\iota})}{1 - (1 - \lambda)(i/m) - \lambda \xi_0}}{1 - (1 - \lambda)(i/m) - \lambda \xi_0}
\]

and \( p_2^* = v_L - v_0 \).
Proposition 6 indicates that the original problem under the TES strategy can be solved by a computationally simpler problem. In addition, the equivalent problem is simpler than that under the OES strategy because it involves only one continuous variable $\xi$ besides the finite search over $i$. Based on this proposition, an efficient algorithm could be carried out in practice.

4. Dynamic Markdown

In the basic model, we assumed that the firm commits to a fixed probability of markdowns. In this section, we relax this assumption by allowing the firm to dynamically adjust its markdown probability in each season. This becomes feasible due to the advance of information technology. Moreover, online retailers can even offer a personalized pricing strategy to each individual consumer through emails and by sending coupons to selected consumers. In this setting, although the firm may use a fixed probability to markdown a product in a given season, each consumer may obtain different markdown realizations/outcomes. Although it might still be hard to implement for some traditional brick-and-mortar stores, it becomes feasible for online retailers who have access to private consumer data (e.g., email or postal addresses). We are interested in answering the following questions: (1) Can posterior price matching still be optimal and under what conditions? (2) How should the firm dynamically set its markdown policy in the long run? We believe these questions are of both intellectual and practical importance: we would like to understand whether the main results in our basic model hold in a dynamic setting, and managers need guidelines on how to manage their markdowns while taking into account customer bounded rationality in practice.

According to the analysis in the basic model, we know that the firm could only consider the OES strategy when $c > v_L - v_0$ but can choose either the OES strategy or the TES strategy when $c \leq v_L - v_0$. In this section, we will only present the case when $c \leq v_L - v_0$, which is analytically more challenging and interesting. The analysis for the case when $c > v_L - v_0$ is similar yet simpler, and hence is relegated to the online supplement for brevity. We formulate the firm’s dynamic markdown problem as a dynamic programming problem. The model setup is the same as in the basic model, except that the firm now has the flexibility to change its markdown probability $\xi_t$ in each season $t$, and consumers obtain samples from the buyers in the last season. This assumption applies to settings where each season is a long period of time; hence, it is natural for consumers to obtain the most recent samples. Given that the markdown probability in the previous season is $\xi_{t-1}$ and the markdown decision in season $t$ is $\xi_t$, under the OES strategy, the firm’s expected profit function in season $t$ is

$$\pi(\xi_{t-1}, \xi_t) = \xi_{t-1}[p_1 - c - \xi_t(p_1 - p_2)]$$

(10)

and under the TES strategy, the firm’s expected profit function in season $t$ is

$$\pi(\xi_{t-1}, \xi_t) = \xi_{t-1}[p_1 - c - \xi_t(p_1 - p_2)] + (1 - \xi_{t-1})\xi_t(p_2 - c),$$

(11)

where $\xi_{t-1}$ is the fraction of customers who purchase in the regular period of this season, $1 - \xi_{t-1}$ is the fraction of customers who wait for the markdown period of this season under the TES strategy, $\{p_1 - c - \xi_t(p_1 - p_2)\}$ is the firm’s unit profit margin in the regular period, and $\xi_t(p_2 - c)$ is the firm’s unit profit margin in the markdown period, following the $S(1)$ framework detailed in the basic model.

The objective of the firm is to dynamically set the markdown probability $\xi_t$ ($t = 1, 2, \ldots$) in each season to maximize the expected long-term infinite-horizon discounted profit $V(\xi_{t-1})$, given that customers experience the markdown probability $\xi_{t-1}$ in season $t-1$ and use anecdotal reasoning based on the samples from the previous season $t-1$ to infer the future markdown probability $\xi_t$ in season $t$. We assume that future profit is discounted by a discount factor $\delta_1 (0 < \delta_1 < 1)$. We have the optimality equation for the firm

$$V(\xi_{t-1}) = \max_{\xi_t \in [0,1]} \{\pi(\xi_{t-1}, \xi_t) + \delta_1 V(\xi_t)\}.$$  

(12)

Combining these equations, we obtain the Bellman equation that the firm solves

$$V(\xi_{t-1}) = \max_{\xi_t \in [0,1]} \{\xi_{t-1}[p_1 - c - \xi_t(p_1 - p_2)] + \delta_1 V(\xi_t)\}$$

(13)

under the OES strategy and

$$V(\xi_{t-1}) = \max_{\xi_t \in [0,1]} \{\xi_{t-1}[p_1 - c - \xi_t(p_1 - p_2)] + (1 - \xi_{t-1})\xi_t(p_2 - c) + \delta_1 V(\xi_t)\}$$

(14)

under the TES strategy.

In the next section, we investigate the optimal policy of this dynamic programming problem.

4.1. Optimal Markdown Policy

To characterize the structure of the optimal markdown policy, we first define a particular form of policies below:

**Definition 1.** A Cyclic Policy ($\xi_1, \xi_2$), where $\xi_1 \neq \xi_2 \in [0,1]$, is a policy where the firm cyclically sets markdown probability as $\xi_1$ and $\xi_2$ in the long run.
Remark 2. The cyclic policy defined here is similar in spirit to that in the literature, e.g., Conlisk et al. (1984) and Popescu and Wu (2007). However, there are major differences worth pointing out. In Conlisk et al. (1984), a new cohort of consumers enters the market in each season, interested in purchasing the product either immediately or later. Within each cohort, consumers vary in their tastes for the product. They show that the heterogeneity of tastes may lead the firm to vary its price periodically over time, despite consumer perfect rationality. The "length of the cycle" in their setting can be any finite integer value at least one, while in our setting it is exactly two. Popescu and Wu (2007), they define a cyclic policy broadly as any policy that admits no steady state (Popescu and Wu 2007, Proposition 3, p. 421). Our definition here is exact and only refers to the policies that admit precisely two steady states. In contrast, our setting does not generally admit state convergence due to the fact that our profit function has a concrete form and is submodular.

Interestingly, we can show in Proposition 7 that the optimal long-run stationary policy to be cyclic or converge to a single stationary point.

Proposition 7. Let \( \{\xi_t^i\} \) (\( t \geq 1 \)) denote the optimal decision path. Then, under the OES or TES strategy, the optimal stationary policy satisfies

(i) either \( \xi_t^i \leq \xi_{t+1}^i \) and \( \xi_{t+1}^i \geq \xi_{t+2}^i \) hold or \( \xi_t^i \geq \xi_{t+1}^i \) and \( \xi_{t+1}^i \leq \xi_{t+2}^i \) hold for any \( t = 1, 2, 3, \ldots, \infty \);

(ii) the two sub-paths \( \{\xi_{2t-1}^i\} \) and \( \{\xi_{2t}^i\} \) are monotone paths in opposite directions and converge to two steady points in the long run, and the two points can be the same.

As shown in Propositions 2, we use \( p_t^i = (v - v_0 - (\lambda \xi_0 + 1 - \lambda)c)/(\lambda - \lambda \xi_0) \) and \( p_t^j = c \) for the case of the OES strategy. The Bellman equation under this strategy becomes

\[
V(\xi_{t-1}^i) = \max_{\xi_t^i \in [0,1]} \left\{ \xi_{t-1}^i (1 - \xi_t^i) \frac{v - v_0 - c}{\lambda - \lambda \xi_0} + \delta_1 V(\xi_t^i) \right\}. \tag{15}
\]

As shown in Propositions 3, we use \( p_t^i = (v - (\lambda \xi_0 + 1 - \lambda)v_1)/(\lambda - \lambda \xi_0) - v_0 \) and \( p_t^j = v_L - v_0 \) for the case of the TES strategy. The Bellman equation under this strategy becomes

\[
V(\xi_{t-1}^i) = \max_{\xi_t^i \in [0,1]} \left\{ \xi_{t-1}^i (1 - \xi_t^i) \frac{v - v_0}{\lambda - \lambda \xi_0} + \delta_1 V(\xi_t^i) \right\}
+ [\xi_{t-1}^i + (1 - \xi_{t-1}^i) \xi_t^i](v_L - v_0 - c) + \delta_1 V(\xi_t^i). \tag{16}
\]

We next study the optimal markdown probability path for these two cases. Note that each customer can only rely on the prior belief \( \xi_0 \) to make the purchasing decision if the markdown probability is set to 0, since there will be no samples indicating the possibility of markdowns. It is easy to check that \( v - p_t^1 + \xi_0(p_t^1 - p_t^2) < v_0 \) for the OES strategy and \( v - p_t^1 + \xi_0(p_t^1 - p_t^2) > (1 - \xi_0)v_0 + \xi_0(v_L - p_t^2) \) for the TES strategy; therefore, no customers purchase in the first period any more. To guarantee the profitability for the long-term dynamic selling, we assume that the discounted factor satisfies \( \delta_1 > \delta \equiv 1/(2 - \xi) \), where \( \xi \) is strictly positive and sufficiently small. Otherwise, the discount factor is too small to generate a higher profit for the firm compared to a single-season selling. Thus, we can restrict the markdown probability \( \xi_t \) to the range \([\xi, 1]\) for a sufficiently small \( \xi > 0 \). We also assume that each customer can find a sample even when the fraction of customers who purchase in the previous season \( \xi \) is small. This assumption is reasonable, especially in today’s digital world where social media and online discussion forums are readily available. We fully characterize the optimal stationary policy in Proposition 8 below.

Proposition 8. (i) For the OES strategy the optimal markdown probability path converges to the cyclic policy \((\xi, 1)\), where \( \xi \) is strictly positive but sufficiently small; as long as \( \xi \) satisfies \( \xi < \min\{1/(1 + \delta_1), 2 - 1/\delta_1\} \), the optimal dynamic markdown probability path is still \((\xi, 1)\). If \( \lambda - \lambda \xi_0 < (1 - \xi)/(1 + \delta_1) \), this dynamic markdown PM policy is better than the NP policy (i.e., \( p_1 = p_2 = v - v_0 \)).

(ii) For the TES strategy the optimal markdown probability path converges to the cyclic policy \((\xi, 1)\), where \( \xi \) is strictly stationary but sufficiently small; as long as \( \xi \) satisfies \( \xi < \min\{1 - \xi(1 - \xi)/(1 + \delta_1), \bar{\xi}\} \), the optimal dynamic markdown probability path is still \((\xi, 1)\), where

\[
\bar{\xi} \equiv \frac{\delta_1(v - v_1)/(\lambda - \lambda \xi_0) + (1 + \delta_1)(v_L - v_0 - c)}{(1 + \delta_1)((v - v_1)/(\lambda - \lambda \xi_0) + v_L - v_0 - c)}.
\]

If \( \lambda - \lambda \xi_0 < (1 - \xi)/(1 + \delta_1) \), this dynamic markdown PM policy is better than the NP policy (i.e., \( p_1 = p_2 = v - v_0 \)).

(iii) If \( \lambda - \lambda \xi_0 < (1 - \xi)/(1 + \delta_1) \), the PM policy with the OES strategy is optimal; otherwise, the NP policy is optimal.

Proposition 8 shows that the cyclic policy is optimal if the firm could dynamically adjust the markdown probability. Why does the cyclic policy benefit the firm? The optimal policy is cyclic because of the submodularity of the firm profit function. If the markdown probability in the previous season is higher, then the firm has an incentive to decrease the markdown probability in the current season. The reason is that, a higher markdown probability in the previous season means a higher demand in the current season, which consequently leads to more profit losses due to more...
price-matching refund. In other words, the markdown probabilities in any neighboring seasons are strategic substitutes to each other (following the terminology from Bulow et al. 1985). This feature is in contrast to the existing operations management literature (e.g., Popescu and Wu 2007, Afkali and Popescu 2013 and references therein) where their profit functions are typically supermodular.

According to parts (i) and (ii) of Proposition 8, the firm should use the PM policy (i.e., either the OES or TES strategy) if \( \lambda - \lambda \xi_0 < (1 - \xi)/(1 + \delta_1) \). Furthermore, part (iii) of Proposition 8 also shows how to optimally choose between the OES and TES strategies. If each customer relies more on her anecdote to make her purchasing decision or she has a sufficiently high prior belief on the markdown probability, the firm should use the OES strategy; otherwise, the NP policy is optimal. This result is consistent with that in the basic model where the TES strategy is never optimal.

There are certainly infinitely many candidate cyclic policies \((\xi_1, \xi_2)\). The optimality of the policy \((\xi, 1)\) comes from solving the Euler equation and the Karush–Kuhn–Tucker (KKT) conditions for the dynamic programming problem. It turns out that it is profit-maximizing to set the neighboring markdown probabilities as far apart from each other as possible. It is indeed more profitable to set \( \xi \) as close to 0 as possible. However, as we show in Proposition 8 that there exists a region \( \xi \in (0, \min\{1/(1 + \delta_1), 2 - 1/\delta_1\}) \) in which the cyclic policy outperforms other markdown policies under the OES strategy and there exists a region \( \xi \in (0, \min\{1 - \xi(1 - \xi)(1 + \delta_1), \xi\}) \) in which the cyclic policy outperforms other markdown policies under the TES strategy. In other words, the firm has flexibility to increase the markdown probability \( \xi \) while still benefiting from customer bounded rationality.

To better understand the nature of the cyclic policy \((\xi, 1)\), we resort to a numerical study. Figure 4 presents numerical examples to compare different policies when we vary the parameters \( \lambda \) and \( \xi_0 \). For the data \( v = 10, v_0 = 5, c = 2, \delta_1 = 0.9, \) left panel \( \xi_0 = 0.3 \) and right panel \( \lambda = 0.6 \). First, consistent with Proposition 8(i) for the OES strategy, the dynamic PM policy is better than the NP policy with a sufficiently low \( \lambda \) or a sufficiently high \( \xi_0 \). Second, it shows that for a relatively wide range of \( \xi \) (for instance, as high as 0.4), the dynamic PM policy can still perform better than the NP policy. Third, the advantage of dynamic markdowns over the NP policy can be significant (for example, the profit under the PM policy can be more than double that under the NP policy).

### 4.2. Multiple Samples

We next study the optimal dynamic markdown and PM policy when each customer can obtain \( m \) independent samples before making her purchase decision in each season. Given that the markdown probability in the previous season is \( \xi_{t-1} \) and the markdown decision in season \( t \) is \( \xi_t \), under the OES strategy, the firm’s expected profit in season \( t \) is

\[
\pi(\xi_{t-1}, \xi_t) = \gamma_1(\xi_{t-1}, m)[p_1 - c - \xi_t(p_1 - p_2)]
\]

and under the TES strategy, it is

\[
\pi(\xi_{t-1}, \xi_t) = \gamma_2(\xi_{t-1}, m)[p_1 - c - \xi_t(p_1 - p_2)] + [1 - \gamma_2(\xi_{t-1}, m)]\xi_t(p_2 - c),
\]

where \( p_1 > p_2 \), \( \gamma_1(\xi_{t-1}, m) \) and \( \gamma_2(\xi_{t-1}, m) \) are defined in Lemma A-2 (in the online supplement) and denote the fraction of customers who purchase in the first period of this season, \( [p_1 - c - \xi_t(p_1 - p_2)] \) is the firm’s unit

**Figure 4.** (Color online) Policy Comparisons

**Note.** Parameters are as follows: \( v = 10, v_0 = 5, c = 2, \) and \( \delta_1 = 0.9 \); left panel \( \xi_0 = 0.3 \) and right panel \( \lambda = 0.6 \).
profit margin in the regular period, and \( p_2 - c \) is the firm’s unit profit margin in the markdown period, following the \( S(m) \) framework.

Then, we have the optimality equation for the firm

\[
V(\xi_{t-1}) = \max_{\xi_t \in [0,1]} \{ \pi(\xi_{t-1}, \xi_t) + \delta_1 V(\xi_t) \}. \tag{19}
\]

Combining these equations, we obtain the Bellman equation that the firm solves

\[
V(\xi_{t-1}) = \max_{\xi_t \in [0,1]} \{ \gamma_1(\xi_{t-1}, m)[p_1 - c - \xi_t(p_1 - p_2)] + \delta_1 V(\xi_t) \}
\]

under the OES strategy and

\[
V(\xi_{t-1}) = \max_{\xi_t \in [0,1]} \{ \gamma_2(\xi_{t-1}, m)[p_1 - c - \xi_t(p_1 - p_2)] + [1 - \gamma_2(\xi_{t-1}, m)]\xi_t(p_2 - c) + \delta_1 V(\xi_t) \} \tag{20}
\]

under the TES strategy.

In Lemma A-4 (see the online supplement), we first show that the optimal long-run stationary policy is still cyclic or converges to a single stationary point when each customer has more samples, the same as the dynamic markdown policy under the \( S(1) \) framework (in Proposition 7). We next focus on the optimal markdown probability path under the PM policy. Under the OES strategy, we use the markdown strategy \( p^*_1 = (v - v_0 - (\lambda \xi_0 + 1 - \lambda)c)/(\lambda - \lambda \xi_0) \) and \( p^*_2 = c \), as shown in the basic model. The Bellman equation (20) becomes

\[
V(\xi_{t-1}) = \max_{\xi_t \in [0,1]} \left\{ \xi_{t-1}^{\pi_1} (1 - \xi_t) \left( \frac{v - v_0 - c}{\lambda - \lambda \xi_0} \right) + \delta_1 V(\xi_t) \right\}. \tag{22}
\]

Under the TES strategy, we use the markdown strategy \( p^*_1 = (v - (\lambda \xi_0 + 1 - \lambda)c)/(\lambda - \lambda \xi_0) - v_0 \) and \( p^*_2 = v_L - v_0 \), as shown in the basic model. The Bellman equation (21) becomes

\[
V(\xi_{t-1}) = \max_{\xi_t \in [0,1]} \left\{ \xi_{t-1}^{\pi_1} (1 - \xi_t) \left( \frac{v_L - v_0 - c}{\lambda - \lambda \xi_0} \right) + \xi_t (v_L - v_0 - c) + \delta_1 V(\xi_t) \right\}. \tag{23}
\]

Similar to the case of \( S(1) \), each customer can only rely on the prior belief \( \xi_0 \) to make the purchasing decision if the markdown probability is set to zero, since there will be no samples indicating the possibility of markdowns. It is straightforward to check that \( v - p^*_1 + \xi_0(p^*_1 - p^*_2) < v_0 \) for the OES strategy and \( v - p^*_1 + \xi_0(p^*_1 - p^*_2) < (1 - \xi_0)v_0 + \xi_0(v_L - p^*_2) \) for the TES strategy; therefore, no customers purchase in the first period any more. To guarantee the profitability for the long-term dynamic selling, we assume that the discounted factor satisfies \( \delta_1 > \delta \equiv 1/(1 + (1 - \xi) \xi^{m-1}) \) in what follows, where \( \xi \) is strictly positive and sufficiently small. Otherwise, the discount factor is too small to generate a higher profit for the firm compared to a single-season selling. Thus, we can restrict the markdown probability \( \xi_t \) to the range \( [\xi, 1] \) for a sufficiently small \( \xi > 0 \).

The same as the case of \( S(1) \) model, we assume that each customer can find a sample even when the fraction of customers who purchase in the previous season \( \xi \) is small.

We fully characterize the optimal stationary policy for the two cases in Proposition 9 below. We find that the optimal markdown probability policy is robust to the number of samples under each case. Similar to the \( S(1) \) framework, for the OES strategy, we can also show that the firm has flexibility to increase the markdown probability \( \xi \) in a range while still benefiting from customer bounded rationality. For ease of exposition, we define \( m_0 \) as an integer that satisfies \( (m_0 + 1)^{m+1} < 1/\delta - 1 < (m_0 - 1)^{m+1}/m_0^{m+1} \) if it exists. Let \( g(m) \) and \( \bar{a}(m) \) satisfying \( g(m) < a(m) \) be two solutions of the equation \( (1 - \xi) \xi^{m+1} = 1/\delta - 1 \).

**Proposition 9.** (i) For the OES strategy the optimal markdown probability path converges to the cyclic policy (\( \xi, 1 \)) when each customer has \( m \) samples, where \( \xi \) is strictly positive but sufficiently small. If there exists \( m_0 \) and \( m \leq m_0 \), then as long as \( \xi \) satisfies

\[
a(m) \equiv a(m) < \xi < b(m) = \min \left\{ \bar{a}(m), 1 - \left( \frac{\delta_1}{1 + \delta_1} \right)^m \right\}, \tag{24}
\]

the optimal dynamic markdown probability path is still \( (\xi, 1) \). If \( \lambda - \lambda \xi_0 < (1 - \xi)/(1 + \delta_1) \), this dynamic markdown PM policy is better than the NP policy (i.e., \( p_1 = p_2 = v - v_0 \)).

(ii) For the TES strategy the optimal markdown probability path converges to the cyclic policy (\( \xi, 1 \)) when each customer has \( m \) samples, where \( \xi \) is strictly positive but sufficiently small; as long as \( \xi \) satisfies \( \xi < \min\{1 - \xi \xi^{m+1} (1 + \delta_1), (1 + \delta_1) \} \), the optimal dynamic markdown probability path is still \( (\xi, 1) \), where \( \xi \) is the solution of the equation

\[
\xi^{m+1} - \delta_1 m \xi^{m-1} (1 - \xi) = \frac{v_L - v_0 - c}{v_L - v_0 - c}. \tag{25}
\]

If \( \lambda - \lambda \xi_0 < (1 - \xi)/(1 + \delta_1) \), this dynamic markdown PM policy is better than the NP policy (i.e., \( p_1 = p_2 = v - v_0 \)).

(iii) If \( \lambda - \lambda \xi_0 < (1 - \xi)/(1 + \delta_1) \), the PM policy with the OES strategy is optimal; otherwise, the NP policy is optimal.

Proposition 9 demonstrates the robustness of the cyclic policy with respect to the consumers’ level of rationality \( m \). To gain more insights about the nature of the cyclic policy (\( \xi, 1 \)) under the OES strategy, we conduct a numerical study. Figure 5 presents numerical examples for the range \( (a(m), b(m)) \) as we vary \( m \), where the discount factor \( \delta_1 = 0.95 \). Notably, it shows that the firm may not set a sufficiently small \( \xi \) if customers have more samples, i.e., become more rational. The reason is that, in this situation, the firm must
increase the markdown probability to attract more customers and guarantee the profitability for the long-term dynamic selling. In addition, we find that the range of $\xi$ is narrower as $m$ increases. This suggests that the dynamic markdown policy can be carried out with more flexibility when customers become more boundedly rational. Note that the long-term profit for the firm under the policy $(\xi, 1)$ is independent of the number of samples $m$. Thus, the profit comparison between the dynamic markdown policy and the NP policy is invariant to $m$. This suggests that, the firm can still make a significant profit by using the dynamic markdown policy when customers have multiple samples.

5. Conclusion, Limitations, and Discussions
The posterior PM policy has been widely adopted by many retailers in practice and received a lot of attention in the economics, marketing, and operations management literature. The extensive literature has offered several explanations why the PM policy benefits the seller. In this paper, we have provided a new behavioral explanation based on consumer bounded rationality in the sense of anecdotal reasoning. We demonstrated its significant role in the firm’s pricing and markdown decisions and offered managerial guidelines for the implementation of price matching when consumers have boundedly rational expectations.

A frequently asked question (and a common criticism of economic models that depart from the standard rational-choice paradigm) is whether the bounded-rationality model can be replicated/replaced by a standard model of rationality with incomplete information (e.g., through Bayesian learning). This is an important and fundamental question that has been discussed thoroughly in the economics literature. In particular, the paper Spiegler (2011b) and Chapter 13 of the textbook Spiegler (2011a), shows that, although such a replacement sounds plausible in the context of anecdotal reasoning, it suffers from a number of issues, such as changed assumptions about the external environment (e.g., we must assume that firms have multiple payoff-relevant types), new unobservable parameters (e.g., we are required to introduce new parameters that describe the distribution of firm types, etc.), and the ambiguity on whether and when the rationalizing model replicates the original model’s key predictions. Moreover, people are typically not Bayesians. It has been shown and argued that, in uncertain situations, they do not update their choices in light of incoming information about the probability of outcomes in the manner predicted by calculations from probability theory, such as the well-known Bayesian learning rule (see, e.g., Edwards 1968, Kahneman and Tversky 1979, 1984, Piattelli-Palmarini 2011, and Jones 1999).

Our model is kept simple to purposely isolate and focus on the role of consumer bounded rationality. The main disadvantage is that it may be too stylized to capture all details necessary for practical decision-support systems. For instance, we assume that the firm has perfect knowledge about its consumers (in terms of their prior $\xi_0$, weight for the prior $\lambda$, and sample size $m$). However, obtaining such perfect information in practice could be a daunting task for the firm. In those cases, it would be of interest to investigate the impact of parameter estimation errors. One can also incorporate other realistic factors into our model, such as adaptive learning across seasons about the markdown probability, heterogeneous sample sizes, consumer heterogeneity, consumer risk aversion, demand uncertainty, inventory considerations (see Lai et al. 2010), and other types of consumer bounded rationality (as an example, see the online supplement for consumer imperfect memory). Such a model may be used as a decision support tool in practice. We believe that our main qualitative finding would remain in those settings, since the firm’s key trade-off between higher profit margins from consumers who obtained positive/favorable anecdotes and lost sales from consumers who obtained negative/unfavorable anecdotes would still be present.

We hope that this paper, being among the first in this area, stimulates more future research (either theoretical, experimental, or empirical) on how the firm should manage its PM policies in the presence of consumer bounded rationality.

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Notice that, for concreteness and ease of exposition, we assume that customers are new. However, our model can work for experienced or repeat customers as well, as long as they exhibit bounded rationality by overinferring from their own past experiences/anecdotes.

Notice that $v_i - v_0$ may be interpreted as the “salvage value” of selling the product to consumers in the markdown period as it is the maximum price they are willing to pay in the markdown period if the firm does not have access to an external salvaging market. It is commonly assumed that the salvage value is lower than the production cost (see, e.g., Tereyagolţu and Veeraraghavan 2012 and references therein).

As aforementioned, we purposely ignore the impact of capacity/inventory constraint to isolate the role of bounded rationality. Indeed, as analyzed in the online supplement, the possibility of stockout in the second period does not change our main result.

Consider that the firm sets the markdown probability as $\xi \in [0, 1]$ before setting the probability as 0. Thus, the firm achieves a profit $\xi(v_i - v_0)/(\lambda - \lambda_0)$ under the OES strategy. If the firm sets the markdown probability as $\xi$ instead of 0, the firm can guarantee a long-term profit:

$$\xi(1 - \xi) \frac{v_i - v_0}{\lambda - \lambda_0} + \frac{1}{\delta_1} \xi(1 - \xi) \frac{v_i - v_0}{\lambda - \lambda_0} + \frac{1}{\delta_1}.$$

To ensure that the long-term profit is larger, we have $\delta_1/(1 - \delta_1) > \xi(1 - \xi)$. To make sure that this inequality holds for any $\xi$, we have $\delta_1/(1 - \delta_1) > 1/(1 - \xi)$, i.e., $\delta_1 > 1/(1 - \xi)$. Similarly, we could also prove that the long-term profit is larger under the OES strategy if

$$\delta_1 > \frac{1}{(1 + \xi)\xi^{\xi - 1}}(v_i - v_0)/(\lambda - \lambda_0).$$

If the firm sets the markdown probability as $\xi \in [0, 1]$ before setting the probability as 0. Thus, the firm achieves a profit $\xi^\infty(v_i - v_0)/(\lambda - \lambda_0)$. If the firm sets the markdown probability as $\xi$ instead of 0, the firm can guarantee a long-term profit:

$$\xi^\infty(1 - \xi) \frac{v_i - v_0}{\lambda - \lambda_0} + \frac{1}{\delta_1} \xi^\infty(1 - \xi) \frac{v_i - v_0}{\lambda - \lambda_0}.$$

To ensure that the long-term profit is larger, we have $\delta_1/(1 - \delta_1) > \xi^\infty/(1 - \xi)^{\xi - 1}$. To make sure that this inequality holds for any $\xi$, we have $\delta_1/(1 - \delta_1) > 1/(1 - \xi)^{\xi - 1}$, i.e., $\delta_1 > 1/(1 - \xi)^{\xi - 1}$. Similarly, we can prove that the long-term profit is larger under the OES strategy if

$$\delta_1 > \frac{1}{(1 + \xi)\xi^{\xi - 1}}(v_i - v_0)/(\lambda - \lambda_0) + \xi^\infty.$$