Two-Sided Matching via Balanced Exchange: Tuition and Worker Exchanges

Umut M. Dur & M. Utku Ünver

North Carolina State & Boston College

March 25, 2015
Design of Matching Markets

In the last decade and a half, economists have worked on the design of matching markets

- Some influenced policy makers for adoption of new policies and institutions:
  - Doctor-residency matching (Roth 84, Roth & Peranson 99)
  - School choice (Balinski & Sönmez 98, Abdulkadiroğlu & Sönmez 03)
  - Kidney exchange (Roth, Sönmez, Ünver 04, 05, 07)
  - Signaling in Econ PhD Market (Coles, Kushnir, Niederle 13)
  - Course allocation (Sönmez & Ünver 10, Budish 11, Budish & Kessler 14)
  - Adoption of children (Vaughn, Akan, Kesten, Ünver 14)

- Some have not influenced the policy yet:
  - On-campus housing; Cadet-branch matching in the military;
  - Dynamic daycare and public housing assignment; Lobar live-donor lung and liver exchange ...

- Deeper understanding of how matching markets work
A new two-sided matching problem where eventual market outcome is linked to an initial status-quo matching, which may give firms and workers certain rights on how the future activity can play out.

Two new classes of assignment problems which mimic Tuition Exchanges and Student/Worker Exchanges

- The Tuition Exchange, Inc.
- US National and EU Erasmus Student Exchange
- Commonwealth Teacher Exchange
- International Clinical Exchange
- Employee rotation programs of departments of a company or institution: public school teacher rotation such as Turkey

Maintaining one-to-one balance between the outgoing and incoming students is the central issue for colleges.
In This Paper

- New axiom playing key role in success of these markets: **Balancedness**
- Procedure in use suffers from serious problems
  - Decentralized matching causing withdrawal of schools (tuition exchange): we identify the problems with stable market outcomes.
  - Bilateral agreements (student/worker exchanges) not being able to get all gains from exchange.
Propose a new mechanism: **two-sided top-trading cycles (2S-TTC)**

- Uses a variant TTC algorithm (Gale via Shapley and Scarf 74, and Abdulkadiroğlu and Sönmez 03), first use of TTC algorithm in a two-sided market
- The unique mechanism that satisfies student–strategy-proofness, balanced-efficiency, individually rationality, and a fairness criterion respect for internal priorities.
- Immune to admission and export quota manipulation by colleges.
- Any individually rational mechanism that matches more students is manipulable by students.
- When firms have 0-1 preferences over incoming workers, then it is also stable and strategy-proof.
What is Tuition Exchange?

- “The Tuition Exchange” (TuitionExchange.org) is a reciprocal scholarship program for children of faculty at more than 600 institutions.
- Dependent children of faculty are able to access tuition benefits in the other member institutions.
- Participating in tuition exchange programs enhances the packages at a nominal cost.
- Every year 20 new institutions join the program.
- On average 6,000 scholarships are awarded annually: $115 million awarded annually.
- No money transaction and tax.
- Schools prefer tuition exchange over direct compensation to protect themselves from “yearly demand shocks” (marginal cost of a student ≈ 1/4 of tuition; fixed costs dominate).
What is Tuition Exchange?

Each institution has agreed to maintain a balance between

- The number of awarded students sponsored by an institution: EXPORTS
- The number of scholarships awarded to students sponsored by other colleges: IMPORTS
- If EXPORTS exceed IMPORTS then

SUSPENDED
What is Tuition Exchange?

- Not all applicants are certified as eligible by the home institutions
  - Based on years of service
- Not all certified applicants are awarded
  - Scholarship receipts are chosen based on academic profile
What is Tuition Exchange?

Timeline

- **Summer 2013**: # of import and certified
- **November 2013**: apply to be certified
- **December 2013**: certified students apply for scholarship
- **January 2014**: deadline for college admission
- **April 2014**: final decision
Why Balancedness?

The Northwest Independent Colleges TE Program

Lewis & Clark, Reed, Puget Sound, Whitman, Willamette

Children of faculty members were allowed to attend one of the members tuition free upon admission.

- Balancedness was not required.
- Huge imbalances between the colleges.

It will stop accepting new applicants after Fall of 2015.
Why Balancedness?

Bilateral Agreements

- In student/worker exchange programs bilateral agreements are signed.
- If balancedness fails after a period of time the agreement is nullified.
Why Balancedness?
Time banks and favor currency holdings

- In “time banks” people make favors of each other.
- Marginal rate of substitution is one favor is equal to one favor (but not must).
- Baby-sitting, dog-sitting exchanges.
- Sweeney & Sweeney (77) reports the shutting down of a baby–sitting coop, as people are averse to spending their accumulated favor currencies (negative balance aversion). (See also Möbius 01 on dynamic favor exchange.)
A tuition exchange market consists of
- a set of **colleges** \( C = \{c_1, \ldots, c_m\} \)
- a set of **students** \( S = \bigcup_{c \in C} S_c \) where \( S_c \) is the set of students who are applying to be sponsored by college \( c \)
- an **admissions quota** vector \( q = (q_c)_{c \in C} \) where \( q_c \) is the maximum number of students who will be imported by college \( c \)
- an **eligibility quota** vector \( e = (e_c)_{c \in C} \) where \( e_c \) is the number of students certified as eligible by college \( c \)
- a list of **college internal priorities** \( \succ = (\succ_c)_{c \in C} \) \( (\succ_c \) is a linear order over \( S_c \)\); let \( r_c(s) \) be the ranking of student \( s \in S_c \) in \( \succ_c \).
- a list of **student and college preferences** \( \succsim = (\succsim_c, \succsim_s) = ((\succsim_c)_{c \in C}, (\succsim_s)_{s \in S}) \) over matchings. Students only care about their assignments.

Fixing \( C, \{S_c\}_{c \in C}, \succ \), a tuition exchange market is defined by \( [\alpha, \varepsilon, \succsim] \).
Model: Matchings

An outcome of a market \([q, e, \succsim]\) is a matching.

- A **matching** is a correspondence \(\mu : C \cup S \rightarrow C \cup S \cup c_0\) such that:

  - \(\mu(c) \subseteq S\) where \(|\mu(c)| \leq q_c\) for all \(c \in C\),
  - \(\mu(s) \subseteq C \cup c_0\) where \(|\mu(s)| = 1\) for all \(s \in S\),
  - If \(r_c(s) > e_c\) then \(\mu(s) = c_0\) for all \(s \in S_c\) (i.e., a student is eligible if and only if its internal priority does not exceed the cutoff.)

- Set of matchings \(\mathcal{M}\).

- A **(direct) mechanism** \(\varphi\) is a systematic way of selecting a matching for each market \([q, e, \succsim]\).
Property: Balancedness

Given a matching $\mu$,

- $X_c^\mu$: set of exports of college $c$; the eligible students in $S_c$ matched with other colleges.
- $M_c^\mu$: set of imports the eligible students of other colleges matched with $c$
- $b_c^\mu = |M_c^\mu| - |X_c^\mu|$: net balance of college $c$.
- $\mu$ is balanced if $b_c^\mu = 0$ for all $c \in C$. 

Dur & Ünver
Two-Sided Matching via Balanced Exchange
Model: College Preferences

- College preferences over admitted (groups of) students are **responsive** (Roth, 1985) (to ranking over individual students) and are denoted by a linear order $P_c$:
  - For any $J \subset S$ with $|J| < q_c$ and any $i, j \in S \setminus J$,
    - $(J \cup \{i\})P_cJ \iff iP_c\emptyset$
    - $(J \cup \{i\})P_c(J \cup \{j\}) \iff iP_cj$

- Colleges possibly also care about their net balance in the matching in addition to the admitted students.
  - For any two matchings $\nu$ and $\mu$ such that $b_c^\nu = b_c^\mu$ we have $\nu(c)P_c\mu(c) \implies \nu \succ_c \mu$
Other Desired Properties: Efficiency

A matching $\mu$ is **Pareto efficient** if it is not possible to find an alternative matching that makes

- all agents at least as well off,
- at least one agent better off.

A balanced matching is **balanced-efficient** if it is not Pareto dominated by another balanced matching.
Other Desired Properties: Non-manipulability

- A mechanism is **immune to preference manipulation by students (or colleges)** if it is always a weakly dominant strategy for each student (or college) to truthfully reveal her (or its) preferences over matchings for fixed quotas.

- A mechanism is **immune to quota manipulation by colleges** if for fixed college preferences, it is a weakly dominant strategy for each college to reveal its true admission and eligibility quotas.

- A mechanism is **student–strategy-proof** if it is immune to preference manipulation by students.

- A mechanism is **college–strategy-proof** if it is a weakly dominant strategy for a college to truthfully reveal its preferences and admission and eligibility quotas.

- A mechanism is **strategy-proof** if it is student–strategy-proof and college–strategy-proof.
The by-laws of many colleges regarding tuition exemption and exchange use priorities based on the seniority of the dependent faculty member/staff. This needs to be somehow respected.

A mechanism $\varphi$ respects internal priorities if for all colleges $c$, whenever a student $i \in S_c$ is assigned to a college in problem $[(q_c, q_{-c}), (e_c, e_{-c}), \succ]$ then $i$ is also assigned to a college in the problem $[(\tilde{q}_c, q_{-c}), (\tilde{e}_c, e_{-c}), \succ]$ where $\tilde{e}_c > e_c$ and $\tilde{q}_c \geq q_c$. 
The current decentralized market works as follows:

- Eligible students applications are sent to the colleges listed in their preference list.
- Each college ranks its applicants and sends acceptance letter to the best students without exceeding its quota.
- Students receive acceptance letter and reject all the offers except the best one.
- Each rejected colleges sends acceptance letter to the best students in the waiting list without exceeding its quota.
- Students receive acceptance letter and reject all the offers except the best one.

This procedure works like the first few steps of the college-proposing deferred acceptance algorithm.

**Benchmark decentralized market mechanism:** stable mechanisms.
We say a matching $\mu$ is **blocked by a college** $c \in C$ if there exists some $\mu' \in \mathcal{M}$ such that $\mu' \succ_c \mu$, $\mu'(s) = \mu(s)$ for all $s \in S \setminus \mu(c)$ and $\mu'(c) \subset \mu(c)$.

A matching $\mu$ is **blocked by a student** $s \in S$ if $c_0 \not\succ s \mu(c)$.

A matching $\mu$ is **individually rational** if it is not blocked by any individual college or student.

A matching $\mu \in \mathcal{M}$ is **blocked by college-student pair** $(c, s)$ if $c \not\succ s \mu(s)$ and $\mu' \succ_c \mu$ where $\mu' \in \mathcal{M}$ is obtained from $\mu$ by the **mutual deviation** of college $c$ and student $s$, that is, $s \in \mu'(c) \subseteq \mu(c) \cup s$, and $\mu'(s') = \mu(s')$ for all $s' \in S \setminus (\mu(c) \cup s)$.

A matching $\mu$ is **(pairwise) stable** if it is individually rational and not blocked by any college-student pair.
Stability and Market Shutting Down
Assumptions on College Preferences

Assumption (1)

For any $c \in C$ and $\mu, \nu \in M$,

1. (Preference increases with better admitted class and non-deteriorating balance) if $b^\mu_c \geq b^\nu_c$ and $\mu(c)P_c^*\nu(c)$ then $\mu \succ^c \nu$,

2. (Awarding unacceptable students exchange scholarships is not preferable) if there exists $s \in \nu(c) \setminus \mu(c)$, $\emptyset P_c s$ and $\nu(s') = \mu(s')$ for all $s' \in S \setminus s$ then $\mu \succ^c \nu$, and

3. (Unacceptability of own students for exchange scholarships) $\emptyset P_c s$ for all $s \in S_c$.

Assumption (2)

(Negative Net Balance Aversion) College $c$ prefers $\mu \in M$ with $b^\mu = 0$ to all $\nu \in M$ with $b^\nu < 0$.
Theorem

*Under Assumption 1,*

- A stable matching exists.
- All stable matchings have the same net balance for all colleges.
- There may not be a stable and balanced matching in general.
- In a quota reporting game (when preferences are common knowledge) where market outcome is found by a stable mechanism:
  - If Assumption 2 also holds, the only best responses for a negative net balance college (under true quota revelation) dictate to decrease its eligibility quota.
  - When a college decreases its eligibility quota, the negative net balance of no college gets closer to zero.
Two-Sided Top-Trading-Cycles (2S-TTC) Mechanism works via the following variant of A&S TTC algorithm: Consider a problem \([q, e, \succsim]\): Assign two counters for import and eligible students to each college \(c \neq c_0\) and set them equal to \(q_c\) and \(e_c\).

- Each student points to her favorite college, which considers her acceptable, each college \(c \neq c_0\) points to the highest internal priority student, and \(c_0\) points to all students pointing to it.
- Every student in a cycle is assigned a seat at the college she is pointing to removed.
- The eligible student counter of each college whose student is in a cycle is reduced by one.
- The import counter of each college in a cycle is reduced by one only if the cycle includes at least two colleges. If either counter falls to zero, the college is removed.
- Continue with the remaining colleges and students.
Two-Sided Top Trading Cycles

Example

Let $C = \{a, b, c, d, e\}$, $S_a = \{1, 2\}$, $S_b = \{3, 4\}$, $S_c = \{5, 6\}$, $S_d = \{7, 8\}$, $S_e = \{9\}$. Let $e = (2, 2, 2, 2, 1)$ and $q = (2, 2, 2, 1, 1)$. The internal priority order is given as

\[
\begin{array}{cccccc}
 a & b & c & d & e \\
 1 & 3 & 6 & 7 & 9 \\
 2 & 4 & 5 & 8 \\
\end{array}
\]

The preference profiles of colleges and students are given as

\[
\begin{array}{cccccc}
 a & b & c & d & e \\
 3 & 5 & 2 & 2 & 2 \\
 4 & 1 & 3 & 3 & 3 \\
 5 & 6 & 4 & 4 & 8 \\
 9 & 2 & 9 & 9 & 7 \\
 7 & 7 & 7 & 5 & 5 \\
 6 & 6 & 6 & 6 & 6 \\
\end{array}
\]

\[
\begin{array}{cccccc}
 b & b & a & c \\
 c & c & c & a \\
 c & c & c & a \\
 c & c & c & a \\
 c & c & c & a \\
 c & c & c & a \\
 c & c & c & a \\
 c & c & c & a \\
 c & c & c & a \\
 c & c & c & a \\
\end{array}
\]
Two-Sided Top Trading Cycles

Example

$\emptyset P_a b_6$, Counters: $e = (2, 2, 2, 2, 1)$ $q = (2, 2, 2, 1, 1)$

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$b$</td>
<td>$b$</td>
<td>$a$</td>
<td>$c$</td>
<td>$b$</td>
<td>$a$</td>
<td>$c$</td>
<td>$e$</td>
<td>$c$</td>
</tr>
<tr>
<td>2</td>
<td>$c$</td>
<td>$c$</td>
<td>$c$</td>
<td>$c$</td>
<td>$a$</td>
<td>$a$</td>
<td>$b$</td>
<td>$a$</td>
<td>$c$</td>
</tr>
<tr>
<td></td>
<td>$c_0$</td>
<td>$c_0$</td>
<td>$c_0$</td>
<td>$c_0$</td>
<td>$c_0$</td>
<td>$c_0$</td>
<td>$c_0$</td>
<td>$c_0$</td>
<td>$c_0$</td>
</tr>
</tbody>
</table>

Round 1
Two-Sided Top Trading Cycles

Example

\[ \emptyset P_a6, \text{ Counters: } e = (1,1,2,2,1) \ q = (1,1,2,1,1) \]

\[
\begin{array}{cccccc}
  a & b & c & d & e \\
  \hline
  1 & 3 & 6 & 7 & 9 \\
  2 & 4 & 5 & 8 \\
\end{array}
\]

\[
\begin{array}{cccccccccc}
  & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
  b & b & a & c & b & a & c & e & c \\
  c & c & c & a & a & b & a & c & d \\
  c_0 & c_0 & c_0 & c_0 & c_0 & c_0 & c_0 & c_0 & c_0 \\
\end{array}
\]

Round 1
Two-Sided Top Trading Cycles

Example

\( \emptyset P_a \{ 6 \}, \) Counters: \( e = (1,1,2,2,1) \) \( q = (1,1,2,1,1) \)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>e</td>
<td>e</td>
</tr>
<tr>
<td>1</td>
<td>c</td>
<td>c</td>
<td>c</td>
<td>a</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>d</td>
</tr>
<tr>
<td>2</td>
<td>c</td>
<td><em>c</em></td>
<td><em>c</em></td>
<td><em>c</em></td>
<td><em>c</em></td>
<td><em>c</em></td>
<td><em>c</em></td>
<td><em>c</em></td>
<td><em>c</em></td>
</tr>
</tbody>
</table>

Round 2

[Diagram showing the trading cycle with nodes and arrows]
Two-Sided Top Trading Cycles

Example

$\emptyset P_a 6$, Counters: $e = (1, 0, 1, 2, 1) \ q = (1, 0, 1, 1, 1)$

\[
\begin{array}{cccccc}
 a & b & c & d & e \\
\hline
 1 & 3 & 6 & 7 & 9 \\
 2 & 4 & 5 & 8 \\
\end{array}
\quad
\begin{array}{cccccccccc}
 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
 b & b & a & c & b & a & c & e & c \\
 c & c & c & a & a & b & a & c & d \\
 c_0 & c_0 & c_0 & c_0 & c_0 & c_0 & c_0 & c_0 & c_0 \\
\end{array}
\]

Round 2
Two-Sided Top Trading Cycles

Example

\[ \emptyset P_6, \text{ Counters: } e = (1, 0, 1, 2, 1) \quad q = (1, 0, 1, 1, 1) \]

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>6</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>e</td>
<td>c</td>
</tr>
<tr>
<td>c</td>
<td>c</td>
<td>c</td>
<td>a</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>d</td>
</tr>
<tr>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
</tr>
</tbody>
</table>

Round 3
Two-Sided Top Trading Cycles

Example

$\emptyset P_a \vec{6}$, Counters: $e = (0, 0, 0, 2, 1)$ $q = (0, 0, 0, 1, 1)$

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>e</td>
<td>c</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>c</td>
<td>c</td>
<td>a</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>d</td>
<td></td>
</tr>
<tr>
<td>$c_0$</td>
<td>$c_0$</td>
<td>$c_0$</td>
<td>$c_0$</td>
<td>$c_0$</td>
<td>$c_0$</td>
<td>$c_0$</td>
<td>$c_0$</td>
<td>$c_0$</td>
<td>$c_0$</td>
</tr>
</tbody>
</table>

Round 3
Two-Sided Top Trading Cycles

Example

\[ \emptyset P_a 6, \text{ Counters: } e = (0, 0, 0, 2, 1) \quad q = (0, 0, 0, 1, 1) \]

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>6</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>e</td>
<td>c</td>
</tr>
<tr>
<td>c</td>
<td>c</td>
<td>c</td>
<td>a</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>d</td>
</tr>
<tr>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
</tr>
</tbody>
</table>

Round 4
Two-Sided Top Trading Cycles

Example

\[ \emptyset P_a 6, \text{ Counters: } e = (0, 0, 0, 1, 1) \quad q = (0, 0, 0, 1, 1) \]

\[
\begin{array}{cccccc}
1 & 3 & 6 & 7 & 9 \\
2 & 4 & 5 & 8 \\
\end{array}
\]

\[
\begin{array}{cccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
b & b & a & c & b & a & c & e & c \\
c & c & c & a & a & b & a & c & d \\
c & c & c & c & c & c & c & c & c \\
\end{array}
\]

Round 4
Two-Sided Top Trading Cycles

Example

\[ \emptyset P \mathbf{6}, \text{ Counters: } e = (0, 0, 0, 1, 1) \quad q = (0, 0, 0, 1, 1) \]

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>e</td>
<td>c</td>
</tr>
<tr>
<td>c</td>
<td>c</td>
<td>c</td>
<td>a</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>d</td>
</tr>
<tr>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
</tr>
</tbody>
</table>

Round 5
Two-Sided Top Trading Cycles

Example

\[ \emptyset P_a 6, \text{ Counters: } e = (0, 0, 0, 0, 0) q = (0, 0, 0, 0, 0) \]

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>e</td>
<td>c</td>
</tr>
<tr>
<td>c</td>
<td>c</td>
<td>c</td>
<td>c</td>
<td>a</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>c</td>
<td>d</td>
</tr>
<tr>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
<td>c_0</td>
</tr>
</tbody>
</table>

Round 5

\[ d \rightarrow 8 \]

\[ 9 \rightarrow e \]
Two-Sided Top Trading Cycles

Properties of 2S-TTC

Theorem

2S-TTC is balanced-efficient, respecting internal priorities, and individually rational.
There does not exist a mechanism which is balanced-efficient, individually rational, and immune to preference manipulation by colleges (even under Assumption 1).

2S-TTC is student–group–strategy-proof.
Relax Assumptions 1 and 2:

**Assumption (3)**

For any $\mu, \nu \in \mathcal{M}$ and $c \in C$, if $b^\mu_c = 0$, $b^\nu_c \leq 0$, and $\mu(c)P_c \nu(c)$ then $\mu \succ_c \nu$.

**Theorem**

Under Assumption 3 and when $e_c = |S_c|$ for all $c$, 2S-TTC is immune to quota manipulation by colleges.

- Given an acceptable set of students, colleges are indifferent between any of their rankings.
- Hence, the mechanism can be run through colleges only reporting acceptable students.
Two-Sided Top Trading Cycles

Properties of 2S-TTC

**Theorem**

*Under Assumption 3, 2S-TTC is the **unique** mechanism that is balanced-efficient, respecting internal priorities, individually rational, and student–strategy-proof.*

Related: Ma (94), Pápai (00), Pycia & Ünver (09), Morrill (11), Abdulkadiroğlu & Che (11), Dur (12)
Proposition

Any balanced and individually rational mechanism, which
- assigns at least the same number of students as 2S-TTC
- selects an allocation in which more student is assigned whenever exists,

is not strategy-proof for students.
In temporary exchange programs, firm preferences can be coarser.

Suppose firms find workers either acceptable or not:

- US National exchange and EU Erasmus Exchange
- International Clinical Exchange: Medical students
- Commonwealth Tuition Exchange
- Staff Rotation Programs: Teacher rotation under Ministry of Education; employee rotation for multinationals.

Initial employees are acceptable.

If a firm’s employee is not matched to a different firm in the market then she remains matched to her home firm.
Temporary Student and Worker Exchanges

Assumption (3*)

For any $c \in C$ and $\mu, \nu \in \mathcal{M}$, if $b^\mu_c = b^\nu_c$ and
\[ \{|s \in \mu(c) : s \not\in P_c\emptyset| \geq \{|s \in \nu(c) : s \not\in P_c\emptyset| \text{ then } \mu \succ_c \nu. \]

Theorem

Under Assumption 3*, 2S-TTC is

- a balanced–efficient, individually rational, \textbf{strategy-proof}, and \textbf{stable} mechanism that also respects internal priorities; and
- the unique balanced–efficient, individually rational, and student–strategy-proof mechanism that respects internal priorities.
Future Work

- Implementation
- Dynamic tuition exchange 2S-TTCC
- Erasmus student exchange and diversity, 2S-TTCC (Dur, Kesten, Ünver, in progress)