MATH 1102 Homework 5

Due Wednesday October 22, 2014

Book problems for practice (not to hand in):

VIII.1 p. 244 1-14.
VIII.2 p. 255 1-5, 7-19

Problems to hand in: (Be sure to write coherently, using sentences where possible, and say what you are computing or doing.)

**Problem A.** Compute the derivatives of the following functions \( f(x) \).

i) \( f(x) = \log(\cos x) \),  
ii) \( f(x) = e^{-x^2} \),  
iii) \( f(x) = x \log x \),  
iv) \( f(x) = e^{x(\sin x)} \),  
v) \( f(x) = 2^x \),  
vii) \( f(x) = 2^{\arctan x} \)  

**Solution.**

i) \( f'(x) = \frac{1}{\cos x} \cdot (\cos x)' = \frac{-\sin x}{\cos x} = -\tan x \).

**Problem B.** Consider the function \( f(x) = x^x \), defined for \( x > 0 \).

i) Compute \( f'(x) \) and find the point(s) \( x \) where \( f'(x) \) is positive, negative and zero.

ii) Compute \( f''(x) \) and find the point(s) \( x \) where \( f'(x) \) is positive, negative and zero.

iii) Use your information from i) and ii) to sketch the graph of \( x^x \).

**Problem C.** Suppose \( f(x) \) is a differentiable function such that \( f'(x) = 2f(x) \) and \( f(0) = 1 \). Prove that \( f(x) = e^{2x} \).

**Problem D.** Compute the limits

\[
\begin{align*}
\text{i) } \lim_{h \to 0} & \frac{\log(1 + h)}{h}, \\
\text{ii) } \lim_{h \to 0} & \frac{e^h - 1}{h}, \\
\text{iii) } \lim_{h \to 0} & \frac{2^h - 1}{h}.
\end{align*}
\]

Hint: consider the derivatives of \( \log(1 + x) \), \( e^x \), and \( 2^x \).

**Problem E.** Find the Taylor polynomial \( p(x) = c_0 + c_1 x + c_2 x^2 + \cdots \) of the function \( f(x) = \log(x + 1) \) and use it to give an approximation to \( \log(2) \) that is accurate to within \( 1/10 \).

**Problem F.** The **Hyperbolic trigonometric functions** \( \cosh(x) \) and \( \sinh(x) \) \(^1\) are defined by

\[ \cosh x = \frac{e^x + e^{-x}}{2}, \quad \sinh x = \frac{e^x - e^{-x}}{2}. \]

i) Show that \( \cosh^2 x - \sinh^2 x = 1 \). Explain why these functions are called “hyperbolic”.

ii) Show that \( (\cosh x)' = \sinh x \) and \( (\sinh x)' = \cosh x \).

\(^1\) pronounced “kosh” and “cinch”
iii) Compute the Taylor polynomials \( p(x) = c_0 + c_1 x + c_2 x^2 + \cdots \) for \( \cosh x \) and \( \sinh x \) and compare with the Taylor polynomials for \( \cos x \) and \( \sin x \).

**Problem G.** Let \( \text{arcsinh} \, x \) be the inverse function of \( \sinh x \). Compute the derivative of \( \text{arcsinh} \, x \).