Book problems for practice (not to hand in): VI.5 p. 211-215 all

Problems to hand in:

Problem A. You have two straight 8-foot pieces of fence and want to make a triangular pen for your garden gnome, using the outside wall of your dorm for the third side. How do you make your pen have largest possible area?

Problem B. A circular wire of radius $a$ with a uniform charge creates an electrical force in space. On the line perpendicular to the ring, passing through the center of the ring, this force is

$$F = \frac{Qz}{(z^2 + a^2)^{3/2}},$$

where $z$ is the distance along the wire from the center of the ring, and $Q$ is a constant depending on the charge and the units used. Find the point $z \geq 0$ where the force is maximal. [The interval $[0, \infty)$ is not finite, but since $\lim_{z \to \infty} F(z) = 0$, the function $F(z)$ has a maximum on $[0, \infty)$.]

Problem C. Find the cheapest way to supply a nearby island with electricity from a power station on the mainland shore, using electric cable which costs $\ell$ dollars per meter on land, and $w$ dollars per meter underwater (so $0 < \ell < w$). The mainland shore is a straight line.

The setup: Let $(0, 0)$ be the point on the mainland nearest to the island, let $(a, 0)$ be the power station on the mainland shore and $(0, b)$ be the receiver point on the island shore, respectively. Finally let $(x, 0)$ be the point where the cable turns to go underwater to $(0, b)$. Your task is to find $x$ minimizing the cost of the cable.