Math 202 Multivariable Calculus
Exam 2 SOLUTIONS

The first two questions are worth 10 points each, and rest are worth 20 points.

1. Calculate the line integral \( \int_c y^2 \, dx + x \, dy \), where \( c \) is the line segment from \((1,0)\) to \((2,3)\).

**SOLUTION:** \( c(t) = (1 + t, 3t), \quad 0 \leq t \leq 1. \) answer: \( 15/2 \)

2. Calculate the line integral \( \int_c x \, dx + y \, dy \), where \( c \) is the following path:

**SOLUTION:** \( Q_x - P_y = 0 \), so \( F \) is conservative. You could replace the squiggly path by a line, or find the potential: \( f(x,y) = (1/2)(x^2+y^2) \), and the line integral is \( f(2,3) - f(1,1) = 11/2 \).

3. Calculate the flux \( \int_c F \cdot N \, ds \) where \( F = (x,0) \), \( c \) is the ellipse \( \frac{x^2}{9} + \frac{y^2}{4} = 1 \).

**SOLUTION:**
\[
\int_c \mathbf{Q} \, dx - P \, dy = \int_c -x \, dy = -6\pi.
\]

4. Calculate the integral \( \iint_R x^2 y^2 \, dR \), where \( R \) is the unit disk centered at \((0,0)\).

**SOLUTION:**
\[
\iint_R x^2 y^2 \, dR = \int_0^{2\pi} \int_0^1 (r \cos \theta)^2(r \sin \theta)^2 r \, dr \, d\theta = \pi/24.
\]

5. Calculate the integral \( \iint_R y^2 \, dR \), where \( R \) is the parallelogram with vertices \((0,0), \ (2,0), \ (2,1), \ (4,1)\). (Use a linear mapping.)

**SOLUTION:** \( R(u,v) = (2u + 2v, v) \), Jacobian = 2,
\[
\iint_R y^2 \, dR = 2 \int_0^1 \int_0^1 v^2 \, du \, dv = \frac{2}{3}.
\]

6. Find the area of the four-sided polygon with vertices \((0,0), \ (1,0), \ (2,4), \ (3,3)\), by computing a certain line integral.

**SOLUTION:** We integrate \( x \, dy \) over the four sides. The side from \((0,0)\) to \((1,0)\) gives zero since \( dy = 0 \). The other sides are \( c_2(t) = (1 + 2t, 3t) \), \( c_3(t) = (3-t, 3+t) \), \( c_4(t) = (2-2t, 4-4t) \), all \( 0 \leq t \leq 1 \). The answer is \( 9/2 \).

MANY OF YOU DID NOT PARAMETRIZE THE SEGMENTS CORRECTLY.
LEARN THIS, PLEASE.
OTHERWISE, THE EXAM WENT VERY WELL.